PARAMETRIC IMMUNIZATION IN BOND PORTFOLIO MANAGEMENT

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Abstract. In this paper, we evaluate the relative immunization performance of the multifactor parametric interest rate risk model based on the Nelson-Siegel-Svensson specification of the yield curve with that of standard benchmark investment strategies, using European Central Bank yield curve data in the period between January 3, 2005 and December 31, 2011. In addition, we examine the role of portfolio design in the success of immunization strategies, particularly the role of the maturity bond. Considering multiperiod tests, the goal is to assess, in a highly volatile interest rate period, whether the use of the multifactor parametric immunization model contributes to improve immunization performance when compared to traditional single-factor duration strategies and whether duration-matching portfolios constrained to include a bond maturing near the end of the holding period prove to be an appropriate immunization strategy. Empirical results show that: (i) immunization models (single- and multi-factor) remove most of the interest rate risk underlying a naïve or maturity strategy; (ii) duration-matching portfolios constrained to include the maturity bond and formed using a single-factor model outperform the traditional duration-matching portfolio set up using a ladder portfolio and provide appropriate protection against interest rate risk; (iii) the multifactor parametric model outperforms all the other non-duration and duration-matching strategies, behaving almost like a perfect immunization asset; (iv) these results are consistent to changes in the rebalancing frequency of bond portfolios.

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1. Introduction

Interest rate risk immunization, which may be defined as the protection of the nominal value of a portfolio against changes in the term structure of interest rates, is a well-known area of portfolio management. The term "immunization" describes the steps taken by an asset and liability manager to build up and manage a bond portfolio in such a way that this portfolio reaches a predetermined goal. That goal can be either to guarantee a set of future payments (e.g., future life insurance benefits or pension payments), to obtain a certain rate of return for the investment or, in certain cases, to replicate the performance of a bond market index.

Immunization models (also known in the literature as interest-rate risk or duration models) control risk through first-order (duration) and second-order (convexity) measures. These measures capture the sensitiveness of bond-returns to changes in one or more interest rate risk factors. For a given change in the yield curve, the estimate of the change in bond price is typically approximated by multiplying the duration (and eventually the convexity) by the change in the yield curve factor.

The classical approach to immunization employs duration measures derived analytically from prior assumptions regarding specific changes in the term structure of interest rates. For instance, the duration measure developed by Fisher and Weil (1971) assumes that a parallel and instantaneous shift in the term structure of interest rates occurs immediately after the bond portfolio is build up. In this case, the recipe was basically to build up a portfolio such that its duration was equal to the investor's horizon. In order to take into account the fact that interest rates do not

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always move in a parallel way, a number of alternative models considering non-parallel shifts were proposed by Bierwag (1977), Khang (1979) and Babbel (1983) or, in an equilibrium setting, by Cox et al. (1979), Ingersoll et al. (1978), Brennan and Schwartz (1983), Nelson and Schaefer (1983) and Wu (2000), among many others.

A number of techniques have been developed to reduce the immunization risk resulting from un-expected adverse term structure changes when setting up immunizing portfolios. Fong and Vasicek (1983, 1984) developed the M-Squared model in order to minimize the immunization risk due to non-parallel (slope) shifts in the term structure of interest rates. The authors show in particular that by setting the duration of a bond portfolio equal to its planning horizon and by minimizing a quadratic cash flow dispersion measure, the immunization risk due to adverse term structure shifts can be reduced. Alternative immunization risk (dispersion) measures were proposed by Nawalkha and Chambers (1996), Balbás and Ibáñez (1998) and Balbás et al. (2002).

An alternative line of attack to the problem of immunization involves the use of parametric duration models. In this kind of formulation, which has its roots in the work of Cooper (1977), all that is assumed is that at each moment in time the term structure of interest rates adheres to a particular functional form, which expresses itself as a function of time and a limited number of shape parameters. In this line of thought, provided that the mathematical function fits accurately most yield curves all interest rate movements can be expressed in terms of changes in one or more shape parameters that characterize this function. In other words, it is apparent that in this kind of models the interest rate risk uncertainty is reflected by the unknown nature of future parameter values. Differentiating the bond price with respect to each shape parameter we obtain a vector of parametric interest rate risk measures. Choosing a particular functional form involves obviously some pricing errors. The difference is that in this case the errors can be quantified and controlled systematically, as long as we are able to choose the appropriate specification for the yield curve, where by appropriate we mean the one that minimizes immunization risk.

In this paper we investigate the empirical immunization performance of the parametric interest rate risk model proposed by Bravo (2007) with that of standard benchmark investment strategies, using European Central Bank yield curve data in the period between January 3, 2005 and December 31, 2011. In addition, we examine the role of portfolio design in the success of immunization strategies, particularly the role of the maturity bond. Considering multiperiod tests, the aim is to assess, in a highly volatile interest rate period, whether the use of the multifactor parametric immunization model contributes to improve immunization performance when compared to traditional single-factor duration strategies and whether duration-matching portfolios constrained to include a bond maturing near the end of the holding period prove to be an appropriate immunization strategy.

Our results show that the multifactor parametric duration vector model outperforms all the other non-duration and duration-matching strategies, behaving almost like a perfect immunization asset. Duration-matching portfolios constrained to include the maturity bond and formed using a single-factor model outperform the traditional duration-matching portfolio set up using a ladder portfolio and provide appropriate protection against interest rate risk. These results are consistent to changes in the rebalancing frequency of bond portfolios.

The outline of the remaining part of the chapter is as follows. In Section 2, we briefly describe the parametric interest rate risk model and discuss its implications for portfolio design, namely the importance of first-order conditions and that second-order conditions for immunization. In Section 3 we offer some insights into the portfolio design and testing methodology used to perform the immunization tests and discuss the results of the empirical study. The last section summarizes our conclusions.

2. Theoretical Framework

The foundations of the parametric immunization model tested in this paper are found in the works of Willner (1996) and Bravo (2007). In this model, the interest rate risk factors correspond to the set of parameters of the mathematical function proposed by Svensson (1994) to represent the yield curve, widely used by practitioners and major central banks.

2.1. Term Structure Specification

Svensson (1994) proposed a mathematical characterization of the yield curve based on the following parametric specification of the instantaneous forward rate, *f(t,a):*

$$f(t,\mathrm{a}) = a_0 + a_1 e^{-\frac{1}{a_4}} + a_2 \left(\frac{t}{a_4} e^{-\frac{1}{a_4}} \right) + a_3 \left(\frac{t}{a_5} e^{-\frac{1}{a_5}} \right) \tag{1}$$

where *f(t,a)* is a function of both the time to maturity t and a (line) vector of parameters a=$(a_0,a_1,a_2,a_3,a_4,a_5)$ to be estimated by solving a non-linear optimization procedure to data observed on a trade day, with $(a_0,a_4,a_5)>0$.

Svensson's model extends the Nelson and Siegel's (1987) functional form by adding a potential extra hump in the forward curve to increase the flexibility of the curves and to improve the fit, particularly on the short segment of the yield curve, has proven to be more adequate in estimating the term structure of interest rates.

From (1) the continuously compounded zero-coupon curve *r(t,a)* can be derived noting that $r(t,\mathrm{a}) = \frac{1}{t}\int_0^t f(t,\mathrm{a})dt$:

$$r(t,\mathrm{a}) = a_0 + a_1 \frac{a_4}{t}\left(1 - e^{-\frac{1}{a_4}}\right) + a_2 \frac{a_4}{t}\left(1 - e^{-\frac{1}{a_4}}\left(1 + \frac{t}{a_4}\right)\right) + a_3 \frac{a_5}{t}\left(1 - e^{-\frac{1}{a_5}}\left(1 + \frac{t}{a_5}\right)\right) \tag{2}$$

whereas the discount function *d(t,***a**) is defined as:

$$d(t,\mathrm{a}) = exp\left[-r(t,\mathrm{a})t\right] \tag{3}$$

Each parameter in (2) has a particular impact on the shape of the forward rate curve. Parameter a_0, which represents the asymptotic value of $f(t,\mathrm{a})$ (i.e., $\lim_{t\to\infty} f(t,\mathrm{a})=a_0$), can actually be regarded as a long-term (consol) interest rate. Parameter a_1 defines the speed with which the curve tends towards its long-term value. The yield curve will be upward sloping if $a_1<0$ and downward-sloping if $a_1>0$. The higher the absolute value of a_1 the steeper the yield curve. Notice also that the sum of a_0 and a_1 corresponds to the instantaneous forward rate with an infinitesimal maturity ($\lim_{t\to 0} f(t,\mathrm{a})=a_0+a_1$), i.e., it defines the intercept of the curve. Parameters a_2 and a_3 have similar meaning and influence the shape of the yield curve. They determine the magnitude and the direction of the first and second humps, respectively. For example, if a_2 is positive, a hump will occur at a_4 whereas, if a_2 is negative, a U-shape value will emerge at a_4. Parameters a_4 and a_5, which are always positive, have similar roles and define the position of the first and second humps, respectively.

The Svensson model is very intuitive since parameters a_0, $a1$, a_2 and a_3 (the interest rate risk factors in our model) can be interpreted as representing, respectively, parallel displacements, slope changes and curvature shifts in the yield curve, given that scale coefficients are fixed.

2.2. Constructing Immunized Portfolios

In this section we resume the main results derived in Bravo (2007) for the parametric immunization model. Consider an investor who has a position in a number L of default-free bonds and denote by n_l the number of type l bonds in the portfolio. Let c_{lt} denote the nominal cash flow (in monetary units) received from bond l (l=1,...,L) at time t (t=1,...,N). Let t=0 be the current date, and H a known, finite investment horizon, measured in years. Assuming that the initial term structure is known and described by the parametric function (2), which assigns a spot rate to each payment date t, the present value (at time 0) of this bond portfolio, P_0(a), is given by:

$$\begin{aligned} P_0(\mathrm{a}) &= \sum_{l=1}^{L} n_l B_0^l(\mathrm{a}) = \sum_{l=1}^{L} n_l c_{lt} \exp\left[-r(t,\mathrm{a})t\right] \\ &= \sum_{l=1}^{L}\sum_{t=1}^{N} n_l c_{lt}^{\,-\left\{a_0 + a_1\frac{a_4}{t}\left(1-e^{-\frac{1}{a_4}}\right)+a_2\frac{a_4}{t}\left(1-e^{-\frac{1}{a_4}}\left(1+\frac{t}{a_4}\right)\right)+a_3\frac{a_5}{t}\left(1-e^{-\frac{1}{a_5}}\left(1+\frac{t}{a_5}\right)\right)\right\}t} \end{aligned} \tag{4}$$

where we have stressed the functional relationship between the bond (and portfolio) price B^l(a) and the initial vector a=$(a_0,a_1,a_2,a_3,a_4,a_5)^T$ of parameters of the forward rate function.

For simplicity of exposition, consider now that the investor is interested only in his wealth position at some future time H (where H might represent, for example, the due date on a single liability payment or a target investment horizon). The value of this portfolio at time H, under the expectations hypothesis of the term structure assuming no change in the yield curve, will be:

$$P_H(\mathrm{a}) = P_0(\mathrm{a})\exp\left[-r(H,\mathrm{a})H\right] \tag{5}$$

Suppose now that at time τ, immediately after the investor purchased the portfolio, the spot rate function has undergone a variation, which may be viewed here as a vector dA of multiple random shifts and represent both parallel and nonparallel shifts, such that the new term structure, represented again by Svensson's model, is r (t,A)=r(t,a+dA), with A=$(A_0,...,A_5)^T$ denoting the new vector of coefficients of the spot rate function estimated at time τ. The traditional definition of immunization (e.g. Fisher and Weil, 1971) for the case of a single liability establishes that a portfolio of default-free bonds is said to be immunized against any type of interest rate shifts if its accumulated value at the end of the planning horizon is at least as great as the target value, where the target value is defined as the portfolio value at the horizon date under the scenario of no change in the spot (and forward) rates. Stated more formally, by immunization we mean selection of a bond portfolio such that the actual future value of the income stream P_H(A) at time H will exceed the initially expected value $P_H(\mathbf{a})$ i.e., $P_H(\mathrm{A}) \geq P_H(\mathbf{a})$ (or equivalently, $P_H = P_H(\mathrm{A}) - P_H(\mathrm{a}) \geq 0$), if the interest rates r(t,A) shift to their new value r_τ(t,A).

Assuming that P_H(A) is a multivariate (twice continuously differentiable) and considering a Taylor series expansion of P_H(A) around the initial vector of parameter, it can be shown, according with Bravo (2007), that the first-order conditions for bond portfolio immunization are obtained from a gradient vector equal to zero, whose generic representation is

$$D^{(P)}(k,\mathrm{A}) = H\frac{\partial r(H,\mathrm{A})}{\partial A_k} \qquad (k=0,\ldots,5) \tag{6}$$

Each of the conditions in (6) defines an immunization condition for a different type of yield curve shift. For instance, selecting a bond portfolio such that its $D^{(P)}(0,\mathrm{A})$ is set equal to $H\frac{\partial r(H,\mathrm{A})}{\partial A_0}$ protects the investment against a parallel shift in the yield curve, i.e., the traditional approach to immunization is a particular case of the parametric model.

Formally, the immunization program is

$$\begin{aligned} Min \quad & \sum_{t=1}^{L} w_i^2 \\ s.t. \quad & \sum_{t=1}^{L} w_i = 1 \\ & D^{(P)}(k,A) = D^{(Target)}(k,A) \end{aligned} \tag{7}$$

Where $D^{(Target)}(k,A)$ denotes a target parametric duration vector for the k risk factors (k=0,…,5), i.e., is such that the first order conditions for bond portfolio immunization are met

3. Portfolio design and methodology

The yield curve series used in the present article comprises the daily (business days) European Central Bank (ECB) estimates of spot, forward and par yield curves, and their corresponding time series, obtained by applying the Svensson (1994) extension of the Nelson and Siegel (1987) parametric model to "AAA-rated" euro area central government bonds (referred to the AAA Euro sovereign bonds), i.e., debt securities with the most favourable credit risk assessment, in the period between January 3, 2005 and December 31, 2011. The dataset used in this study includes also the time-series of parameter estimates for Svensson's model in the same period.

3.1. Portfolio Design

In Table 1 we summarize the five alternative investment strategies tested in this empirical study. The five strategies are separated in two different groups according to the inclusion or not of duration constraints in the immunization program, i.e., according to whether constraints regarding interest rate risk are imposed. The first strategy in the non-duration matching strategies group, termed Naïve Strategy, corresponds to an equally weighted bond portfolio including all available bonds that involves no attempt to target a specific return on investment at the investment horizon and, therefore, serves mainly to indicate the direction and magnitude of interest rate changes.

The second strategy in this group termed Maturity Strategy (Mat), acts also as a benchmark and is defined by a bond portfolio comprising all available bonds with maturity equal to the investment horizon set up considering the maximum diversification objective function defined above. Although they ignore coupon flows, maturity strategies have been used since Fisher and Weil (1971) as a first approximation to an immunization strategy.

Table 1: Definition of Investment Strategies

Strategy name	Definition
Non-Duration matching strategies	
Naïve	Equally weighted bond portfolio including all available bonds
Maturity Bond	portfolio with maturity equal to the investment horizon
Duration matching strategies	
NSS Parametric Immunization	Immunization using the first four Nelson-Siegel-Svensson parametric duration measures
Duration	Classical immunization strategy using a ladder portfolio
Maturity-Barbell	Two-bond portfolio combining the bond chosen for the maturity strategy and the bond with largest duration available. Short-selling is permitted.

The first investment strategy in the duration matching strategies group, termed NSS Parametric Immunization (NSS), is designed to test the immunization performance of the Nelson-Siegel-Svensson parametric immunization model presented in Section (2) considering the first four parametric duration measures. The second investment strategy, termed duration (Dur), corresponds to the classical immunization strategy set up considering a single interest rate risk (Fisher-Weil) duration measure and a ladder portfolio. The third strategy, termed Maturity-Barbell (MBar), is defined by a two-bond portfolio combining the bond with maturity closest to the horizon date and the bond with largest duration (and maturity) available. (Fisher-Weil) duration of the portfolio is set equal to the horizon date. Negative positions are allowed. The Maturity-Barbell strategy is designed to ascertain the importance of portfolio design in immunization performance, namely the role of the maturity bond in the immunized portfolio. The good performance of maturity portfolios in most empirical studies is still an open issue in the immunization literature.

3.2. Testing Methodology

The immunization tests performed here use ECB term structure data (Nelson-Siegel-Svensson parameter estimates and corresponding estimates of zero-coupon yields for maturities ranging from 1 month through ten years on an annual basis over the seven-year period ranging from the first trading day in 2005 (January 3) to the last trading day in 2011 (December 30). Starting in January 2005, a total of 30 different coupon bond prices are constructed and simulated using zero-coupon yields observed in January 3, 2005 considering ten different maturities (1, 2, 3, …, 10 years) and three different coupon rates (2%, 4%, 6%) for each maturity. The central coupon rate specified in this study (4%) is in line with the average annual par yields (or rates) observed in the sample period. By considering bonds with coupon rates above and below the average market value we offer a more realistic setting for the immunization tests performed.

The testing procedure is as follows. Using January 3, 2005 as a starting point, portfolios were set up on the first business day of each month. Given that our sample period is short when compared with immunization studies carried out in other bond markets, we adopt an initial planning period of three years. Besides facilitating the comparability of our results with similar studies conducted for European bond markets (see, e.g., Soto, 2001;2004), this allows us to evaluate whether immunization is more effective as the investment horizons shortens.

Subsequently, we test the impact of the planning horizon on performance by reporting results for a shorter (e.g., one-year) horizon.

In order to provide a sufficient number of simulations in which to test the model, overlapping holding periods were identified. Thus, in the period January 2005-December 2011, 48 three-year overlapping investment periods were constructed, each starting one month after the previous period on the first business day of each month. For the purposes of multiperiod immunization, all portfolios are rebalanced annually. Coupon payments are reinvested with the same strategy on the first reallocating date available after the payment. At the end of the planning horizon, the portfolios are liquidated. The actual returns of all of the five investment strategies are compared with the target risk-free return, defined as the promised yield to maturity on a hypothetical H-year zero-coupon bond observed at the beginning of the planning period. Deviations of the actual return from the target return are employed to assess the effectiveness of each particular immunization strategy.

The interest rate risk hedging performance is summarized by three dispersion measures: Mean Absolute Differences (MAD) between the realized returns and zero-coupon rate at the time portfolios were set up, Root-Mean-Squared Differences (RMSD) and a downside risk measure termed Root-Fishburn-Risk-Measure (RFRM) (see Fishburn, 1977). The three dispersion measures are given, for each investment strategy, by:

$$MAD_H = \frac{1}{S}\sum_{i=1}^{S}\left|r_{r,i}^{H} - r_{e,i}^{H}\right| \tag{8}$$

$$RMSD_H = \left(\frac{1}{S}\sum_{i=1}^{S}\left(r_{r,i}^{H} - r_{e,i}^{H}\right)^2\right)^{\frac{1}{2}} \tag{9}$$

$$RFRM_H = \left(\frac{1}{S}\sum_{i=1,\, r_{r,i}^{H} < r_{e,i}^{H}}^{S}\left(r_{r,i}^{H} - r_{e,i}^{H}\right)^2\right)^{\frac{1}{2}} \tag{10}$$

where S denotes the number of simulations, $r_{r,i}^{H}$ is the actual (or effective) return of the portfolio over the ith period of length H, and $r_{e,i}^{H}$ is the estimated (target) return for the same period. In order to compare the risk of the Naïve and maturity strategies with all the immunization strategies we calculate and report a RMSD-index (I_{RMSD}).

The use of the root-mean-squared differences may lead to misleading rankings of immunization performance for two reasons: (i) when there are large outliers; (ii) when the statistical distribution of the effective rate of return is not symmetrical, in which case the strategy with smaller quadratic deviations may not be the best performer to meet (or exceed) the target. For this reason and in order to assess the statistical significance of the immunization performance, we report results for a well-known non-parametric test -- the Sign Test -- which has previously been used in the immunization literature by Fooladi and Roberts (1992), Bierwag et al. (1993), Nawalkha et al. (2003) and Bravo and Silva (2006). The Sign Test determines whether the percentage of simulations in which each strategy outperforms the benchmark strategy (naïve or maturity strategy) is significantly different from the null hypothesis of a randomly-expected outcome of 50%.

3.3. *Results of portfolio simulations*

Table (2) presents y statistics on the performance of the five investment strategies over 48 horizons of three years in the sample period 2005-2011 with annual rebalancing. The second column shows the average annual realized return $r_{r,i}^{H}$ obtained by each strategy. The third, fourth and fifth columns show, respectively, the average, maximum and minimum deviation (in basis points) of the actual return of a given immunization strategy from the expected return.

The sixth, seventh and eight column report the immunization performance measures (in basis points) and column ninth calculates a RMSD-index (I_{RMSD}) comparing the risk of the maturity strategy with all of the other investment strategies. Duration matching strategies are said to outperform maturity matching when its absolute difference between promised and actual return is smaller. Asterisks accompanying the percentage of cases duration-matching or Naïve strategies outperform indicate that the strategy performs statistically different from the maturing strategy at 1% (***), 5% (**) or 10% (*) levels by means of a normal approximation to the binomial sign test.

Table 2: Immunization results with annual rebalancing over three-year horizons, 2005-2011

Investment strategy	Average $r_{r,i}^{H}$	$r_{r,i}^{H} - r_{e,i}^{H}$ b.p.			RMSD	RFRM	MAD	I_{RMSD}	% Outperforms Maturity
		Aver	Max	Min					
Naïve	4,03%	61,7	168,5	-84,0	88,3	19,8	75,0	177,7%	35,4%*
Maturity	3,10%	-31,0	76,8	-134,0	49,7	47,1	42,3	100,0%	NA
NSS	3,42%	0,9	30,8	-72,7	14,8	13,2	8,4	29,8%	93,8%***
Duration	3,15%	-26,9	70,3	-133,9	44,4	42,0	37,1	89,4%	64,6%*
M-Barbell	3,33%	-43,9	11,1	-197,0	26,3	24,8	15,7	53,0%	79,2%***

The results reported above can be summarized by saying that they offer evidence in favour of using a multifactor parametric immunization model in order to guarantee a return close to the target, reducing by 70.2% the risk inherent to a maturity strategy, even when traditional duration-matching strategies are forced to include the maturity bond. However, the good performance of maturity-bond portfolios should be taken into account when considering the need for more frequent reallocations and corresponding transaction costs implicit in parametric duration vector models.

4. Conclusion

This paper has analyzed the immunization performance of the parametric duration vector model proposed by Bravo (2007) along with those of several duration-matching and non-duration matching strategies, taking into consideration the role of portfolio design in the success of immunization. Consistent with previous studies on immunization using multiperiod multifactor models, the results obtained considering daily (business days) European Central Bank yield curve data in the period between January 3, 2005 and December 31, 2011 - using different reallocation intervals and different portfolio design approaches -- suggest that immunization models (single- and multi-factor) remove most of the interest rate risk underlying a more naïve or maturity strategy.

Our empirical results show that the multifactor model capturing shifts in the level, slope and curvature of the term structure behaves almost like a perfect immunization asset and should be sufficient to guarantee a return close to the target. Traditional immunization strategies constrained to include a maturity bond outperform the traditional duration-matching portfolio set up using a ladder portfolio but contrary to similar studies conducted in other countries did not show its superiority when compared with the multifactor parametric model. Varying the rebalancing frequency reveals that these results are robust.

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