MATHEMATICAL INVESTIGATIONS IN THE CLASSROOM: A CONTEXT FOR THE DEVELOPMENT OF PROFESSIONAL KNOWLEDGE OF MATHEMATICS TEACHERS

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This article reports part of a study developed with the purpose of understanding professional knowledge of teachers involved in the development of investigative tasks in the classroom. The study adopted the interpretative paradigm and elaborated a case study of a first-grade teacher. This teacher worked regularly in collaboration with one of the researchers, selecting, planning and developing investigative tasks with her students in the classroom, and also reflecting on her teaching practice with investigations. The analysis of the data allows us to identify several aspects in which the professional knowledge of the teacher was deepened and broadened. It is also possible to identify the main factors contributing to the development of the teacher's professional knowledge.

Keywords: Professional knowledge; investigative tasks; teacher practice and refection

PURPOSE OF THE STUDY

Investigative tasks promote a rich mathematical experience for the students and provide them with a significant intellectual challenge (NCTM, 2000). These tasks have an open nature and can be approached in very different ways, depending on the work of the solver. This kind of task can contribute to providing students with the mathematical learning experiences recommended by curricular orientations of many countries. In Portugal, they are explicitly suggested in the National Curriculum of Mathematics and in the recent syllabus (2007) of Mathematics for Basic Education.

However, investigative tasks can be very complex and their development in the classroom represents a serious challenge for many mathematics teachers (Oliveira *et al.*, 1999). It is important to understand the professional knowledge of teachers involved in the development of investigative tasks in the classroom, namely for first-grade teachers, since the majority of Portuguese teachers of this grade are not familiar with them. This was the main purpose of the study reported in this article. We investigated the practice of a teacher who participated in a collaborative context with the researcher (second author). Our purpose was to identify the components of professional knowledge she applied to develop investigative tasks in the classroom, and also to identify the influences of that work in the development of her professional knowledge.

THEORETICAL FRAMEWORK

Teachers have particular knowledge they apply in their day to day practices of teaching mathematics (Schön, 1992; Ponte & Chapman, 2006; Sowder, 2007). This professional knowledge is multifaceted and can be organized into different components (Elbaz, 1983; Sowder, 2007). According to Ponte (1995) and Canavarro (2004), there are four interrelated components of professional knowledge that assists the teacher directly when he/she prepares and conducts mathematical lessons: knowledge of mathematics, knowledge of students and their learning processes, knowledge of curriculum and knowledge of instructional process.

The knowledge of mathematics includes knowledge of specific topics, an overview of mathematics, as a science and as a school discipline, the perspective on its nature and its relation to reality (Ponte, 1995). This is essential knowledge for teaching. Having poor mathematical knowledge can inhibit the action of the teacher in the classroom, particularly the mathematical tasks that he/she presents to the students (Leikin & Levav-Waynberg, 2007).

Knowledge of students and their learning processes consists of the knowledge of characteristics of the students, aspects that motivate their learning, and how they develop and acquire knowledge from the learning situations (Canavarro, 2004).

Curricular knowledge includes knowledge of the purpose and guidelines of the curriculum, specific knowledge of the contents of the syllabus that the teacher teaches and of the syllabus of the years before and after, and also includes knowledge about the approaches, strategies and materials proposed (Ponte, 1995).

Finally, the knowledge of instructional process refers to the knowledge directly related to the organization and concretization of classroom practice. It refers to the phases of planning, conducting and evaluating teaching and learning (Canavarro, 2003). This knowledge has a decisive influence on the kind of teaching that the teacher puts into practice (Sowder, 2007).

Research in mathematics education has been explaining how the professional knowledge of the teacher influences the way he/she develops investigative tasks with his/her students (Oliveira *et al.*, 1999). This knowledge is reflected in two aspects of the teacher's work during the development of investigative tasks, which are mutually dependent: the mathematical aspects concerning the investigative task in question – when a teacher assesses the mathematical scope of the task, when he/she is involved in mathematical reasoning in front of the students, when he/she establishes connections between the knowledge present in the task and other mathematical concepts; and the didactical aspects that are fundamental to accomplish the objectives of the investigative activity. These are present when the teacher creates or selects the task, when he/she plans the lesson, when he/she conducts its development, when he/she has to handle unexpected situations of uncertainty, when

he/she has to understand the thought processes ofdifferent students and to promote collective reflection of the class (Ponte *et al.*, 1999).

Oliveira *et al.* (1999) stress that the selection and planning of investigative tasks is a complex activity because it requires from the teacher a broad knowledge about their students, their knowledge, interests and abilities. It also requires a solid knowledge of curriculum objectives, materials and resources they have available. The conduct of the class with investigative tasks is also very important, requiring many roles of the teacher (to challenge students, to evaluate their progress, to reason mathematically, to recall or provide relevant information, to support the progress of students and promote their reflection). In addition, the teacher should be aware of the type of interactions he/she has with students in order to promote an investigative ambience (to encourage the discussion of different opinions, the critical sense, reflection and debate). The teacher should also provide, and teach students how to use, technological tools that help them in investigative work (Oliveira *et al.* 1999; Ponte *et al.*, 1999).

The professional knowledge of the teacher is a dynamic knowledge, evolving from teaching practice and the reflection of the teacher about his/her practice. This reflection can be encouraged in the context of the school teacher, through collaborations of the teacher in complicity with colleagues. These collaborations are especially interesting when teachers experiment with the development of new and challenging tasks or new ways of working with students in the classroom (Sowder, 2007).

METHODOLOGY AND CONTEXT OF THE STUDY

As previously explained, this study is intended to contribute to the understanding of the professional knowledge of the teachers involved in the development of investigative tasks in the classroom (Patrício, 2010). The researchers adopted the interpretative paradigm and a qualitative approach, and elaborated case studies of teachers. The option for this design was because the study was intended for provide a explanation of a well-delimited phenomena, embedded in context, for which we seek thick description from the standpoint of the subjects being investigated (Erickson, 1986; Merriam, 1991).

The researchers decided to consider a teacher of the first grade because the practice of these teachers is still poorly documented with regard to the development of investigative tasks in classroom. The teacher should be an experienced one and denote propensity for innovation and investment – these conditions offer some guarantee of considerable professional knowledge and a willingness to embark on a new experience. This is the case for the first-grade teacher Petra, who is the focus of this paper. She is a teacher of 40 years, teaching a fourth-grade class (9 year-old students). Petra had never developed investigative tasks in their lessons. The total inexperience of the teacher with these tasks recommended the collaboration with the

one of the researchers (the second author). That collaboration created conditions to operationalise this study, allowing more reliable and complete data collection, and representing an opportunity for professional development for the teacher.

The collaborative work took about two months. Petra had the opportunity to know and solve several investigative tasks and to discuss their potential with the researcher. The tasks were open and they could be developed in a more or less complete ways by the students. In the next paragraph, we give an example of one investigative task that Petra selected to work on with her students:

Dividing by 11, 111, ...

Look at the figures of the non integer part of the decimal representation of the following fractions:

3/11 9/11 18/11 47/11 52/11 125/11

Can you find any pattern in it?

Is it possible to estimate what the figures are of the non-integer part of the decimal representation of any fraction with denominator 11? Please explain.

And what happens if the denominator is 111? Write your conclusions.

You can also investigate with 1111…

In this collaborative work, Petra also responded to the challenge of the researcher to develop investigative tasks with her students in the classroom. She carried out five two-hour lessons with different investigative tasks, one per lesson.

The role of the researcher was to provide a collection of investigative tasks for the teacher, to support her in selection and in the planning of the lessons, and to promote her reflection on teaching practices. This collaboration emphasized discussion of the plans of the lessons that the teacher elaborated to each class with investigative tasks. The plans were organized according to a script provided by the researcher. The script considers three phases to the development of the lessons: a first phase of presentation of the task to the students; a second phase of autonomous work by the students, individually or in group; and a third phase of a collective discussion of the work done by the students. The script also includes the provision of a list of questions to ask the students in order to promote their mathematical thinking while performing the task.

In the classroom, when the teacher was developing the investigative tasks with her students, the researcher took the role of a non-participant observer and collected data. The teacher's classroom practice was particularly relevant for data collection, considering its different moments (preparation, conduction and reflection). After each lesson with investigative classes, the researcher and the teacher always spent some time reflecting on the teaching practices. They focused on mathematical episodes that took place during the lesson, on the mathematical learning of the

students, and they also tried to identify factors of influence in the way the tasks were developed.

The techniques and instruments used for collecting data were those generally recommended (Merriam, 1991): the interviews (the researcher conducted two formal long interviews at the beginning and at the end of the study; five informal interviews during the preparation of the investigative tasks developed in classroom; five informal interviews at the end of the lessons with investigative tasks – all fully transcribed); the observation of the teacher classroom practice (the researcher observed five classes with the investigative tasks, all audio-recorded and fully transcribed); the documental analysis (the researcher analysed all the lesson plans elaborated by the teacher).

THE TEACHER AND THE INVESTIGATIVE TASKS – RESULTS OF THE STUDY

Petra accepted with great enthusiasm, dedication and responsibility the invitation to participate in the study because she anticipated that she could learn and develop herself professionally. She was teaching a fourth grade class (9 year-old students).

Petra had never developed investigative tasks in her lessons, neither knew their characteristics. From the beginning, she revealed a great appreciation for the nature of the investigative tasks and the work they provide for students in the classroom. This is very consistent with what she valued in the teaching of mathematics: "Mathematics, for me, is to teach to think, is developing the reasoning, is awakening to the problems."

Her lesson plans were very detailed. She worked hard trying to explore all the mathematical possibilities she could imagine. For each task, she elaborated an extended list of questions she could pose to her students to help them to progress. She explained that she wanted to take full advantage of the potential of the tasks and also to reduce the degree of unpredictability of the work with the students.

The teacher practice with investigative tasks was developed according to the three phases previously described. Her main purpose was to promote the development of students mathematical reasoning, but she tried to establish connections with the concepts emerging from the tasks. For example, for the task *Dividing by 11, 111…,* she wanted the students "to develop confidence in exploring mathematics in an autonomous way; to explore and use patterns on division by 11; to learn about periodic decimal representations."

In the post-lesson interviews and in the final interview of overall reflection, Petra identified several aspects that she considered gains in her experience with the investigative tasks. In the following sections, we synthesize these aspects, organized by the four components of professional knowledge that we adopted, despite the fact that some of them could be included in more then one component.

Mathematical knowledge

Petra acknowledges having learned mathematical knowledge. When planning for the lessons, she forced herself to get into deep mathematical exploration of the tasks and she involved herself in investigative work.

> I spent the weekends searching… and studying for the mathematical knowledge to tackle the tasks… And then I interrogate myself: "what can you investigate more on this?" And I get deeper but I never find everything, of course! (…)

This work revealed some "gaps" in her mathematical knowledge. She consulted some books, colleagues and the researcher trying to overcome the difficulties:

> I learned a lot of mathematical knowledge, is true… Because I had to search for things I didn't know at all and others I was not sure… I also asked other colleagues, I asked you ... Because sometimes I had some doubts... the investigation of the rational numbers, for example. I had never worked with infinite decimals, with periodic decimals, my decimals were always finite! And exploring their period, hum… when I was solving the task myself with the calculator, I was not sure about the numbers it was showing me… that's when I called you!"

Petra also experienced some difficulties with a task in the domain of probability. She considered that her knowledge was too informal to explore the task correctly. She had to learn how to count all the possible cases in a complex situation – for example, how to differentiate between 1-6 and 6-1when throwing two dice.

Curricular knowledge

Petra said that she has extended her repertoire of the types of tasks that she considers appropriate to work with students in the context of current curriculum guidelines. She confessed that she always liked open tasks and she was a great enthusiast of mathematical problems. But after this experience, she came to privilege the investigative tasks:

> Now I'm a fan of investigations, seriously! I am fan! I think they have enormous potential... This year was the unknown, it was my first experience with this kind of task… I also like problems very much, but the investigative tasks are more challenging and have more potential for the learning of the students.

Petra said that the investigative tasks contribute to several aspects of mathematical competence that students should develop:

> Because they are complex in nature, mathematical investigations provide a greater intellectual challenge and promote the development of more complex capacities of students, as mathematical reasoning, mathematical communication and critical thinking.

Petra also stressed that the investigative tasks, instead of constituting something marginal to the school curriculum, helped to promote compliance with the mathematics syllabus in an interesting way:

> With the investigative tasks you facilitate the achievement of the curriculum and of the mathematics syllabus, one thing is inseparable from the other... I think it only helps! You're not compromising anything… You are addressing mathematics in a much more interesting way and this is how it is expected to be addressed.

Students and learning process knowledge

Petra considered that investigative tasks have great potential to motivate the students to the learning of mathematics because they provide contexts for meaningful learning. The teacher also recognised that students may have an important role as creators of knowledge when developing investigative tasks:

> And I think the investigative tasks have great power to motivate the students to mathematics – they like, they get involved and the knowledge that comes from this involvement is great. I'm sure they made significant learning. And they are creating knowledge, they are learning - and this is great!

Instructional process knowledge

The planning of investigative tasks allowed Petra to acquire a new vision about what makes a lesson plan effective: to consider other aspects besides the content, such as ideas of how to conduct the class or of the good mathematical questions to pose to the students. Petra concluded that a careful plan is essential to explore and conduct the lesson in order to take full advantage of all the potential of the tasks:

> I learned so much! Look, I learned to plan properly, because I think this it the right way (...) It was ideal that we could always do so but it is too time consuming… When you have a major concern in planning – in the choice of tasks, the choice of materials, the good questions to make them think, the knowledge that could result from the task… – it is obvious that the development of the task is much better and the results can be very positive.

The experience with the investigative tasks in the classroom allowed Petra to rethink various aspects of her practice, although the teacher was used to adopting methodologies that value the role of students and was familiar with students working in groups. The idea that it is very important to limit the support that the teacher provides to students while they are working on the tasks was strengthened. During the developmental phase of investigative tasks, Petra began to avoid giving answers and/or validating the assumptions of the students – otherwise she would prevent the development of mathematical communication and reasoning:

> But… I think I can do it better, I still have the tendency to give them the answer…I need to still my tongue… I can not tell them everything, I just can tell a small part of what I am used to…

The ideas of Petra, related to the final lesson phase of discussion, also seem to have changed. The teacher began to attribute more importance to discussion and she reviewed her role in its conduct. She decided to try not to summarize, during that

phase, as each student disclosed the work they had done. Instead, she became aware of the importance of promoting the discussion and reflection of all students:

> You know another thing I have come to notice? Is that when they pass to the communication phase, they reveal some lack of interest… this is because I validate all the assumptions that they ask me during the work group… So, when they present their work to the class, they are already quite sure of what they say… and the others also know that everything is ok, so… there is no discussion among the groups! The next task, I would like to do something different: I will not validate anything. I will give them the task, I will ask them to do all the work and after that the class will validate... or not… I want to bring the discussion to class. They will present and after… "So, what do others think about this? Is it right, is it wrong? Why?"

THE DEVELOPMENT OF PROFESSIONAL KNOWLEDGE – FINAL CONCLUSIONS

This study shows how professional knowledge of a first-grade teacher was revealed in the development of investigative tasks in the classroom. It also shows the contributions that the teacher took through this experience to deepen and broaden her professional knowledge in various components.

It is noteworthy that this experience involved the teacher in the search for a greater understanding and deepening of her mathematical knowledge. The exploration of the investigative tasks was quite demanding for the teacher, because she wanted to be prepared to deal with the multiple assumptions that students could propose and that do not always appear so obvious to the teacher. The teacher's mathematical preparation is crucial to the development of his/her teaching practices (Leikin & Levav-Waynberg, 2007). And, the more demanding these practices are, the more complete and deep the mathematical knowledge needs to be.

This study also emphasizes that this experience provided the teacher the opportunity for reflection on some aspects of her knowledge of the instructional process. On the one hand, the teacher recognized the value of careful planning, referring to it as crucial when exploring open tasks and promoting the discussion of students in the mathematics classroom (Franke, Kazemi & Battey, 2007). On the other hand, the teacher acknowledged the advantage of introducing changes in the way she conducted the class, either at the phase of the development of the tasks by the students, or at the final phase of collective discussion. She considered it to be necessary to moderate the intervention of the groups in order to achieve more shared discussions and challenge the students to validate genuine conjectures and conclusions of other colleagues (Oliveira *et al.*, 1999).

Three main factors contributed to the development of teacher professional knowledge. One was the contact and work with challenging and powerful mathematical tasks with great potential for the mathematical experience of the teacher herself and of her students, with consequences in the way she usually

conducted the classroom (Stein & Smith, 1998). Another factor was the teacher's perception of the acceptance of the investigative tasks by the students, and of the mathematical experience and mathematical learning that they revealed – a factor of great influence in the professional development of teachers (Canavarro & Rocha, 2008; Sowder, 2007). A third factor was the focus on the teacher's teaching practice provided by the opportunity of collaborative work with the researcher, who introduced the tasks, discussed the plans of the lessons, supported implementation in the classroom and encouraged reflection on practice, based on the analysis of the episodes from the classroom (Canavarro & Rocha, 2008; Sowder, 2007).

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