A temperature-dependent damage model for ductile crack initiation and propagation with finite strains

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OUTLINE

- A. Brief introduction on finite strain models
- B. First- and second theory of elasto-plasticity
- C. Temperature-dependent finite strain model of ductile fracture
- D. Some discretization issues
- E. Simulation results with and without temperature, first or second theory

Introduction

Quoting NAGHDI ("Critical Review of the State of Plasticity" (1990)):

...there is some degree of disagreement on nearly all of the main constitutive ingredients and features of plasticity in the presence of finite deformations ... Some of these issues of disagreements are of basic and fundamental importance.

Today, thanks to intensive study of the **physics of micro-mechanics**, and the progress in **computational power** and **numerical methods**, better understanding has been reached.

Introduction

We propose:

- A simulation code (SIMPLAS) developped by Pedro Areias (Univ. Evora, Portugal) and co-workers able to
- Perform 3D simulations of
  - temperature-dependent
  - small or finite strain elasto-plasticity
  - with initiation and propagation of damage and cracks

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- Perform 3D simulations of
  - temperature-dependent
  - small or finite strain elasto-plasticity
  - with initiation and propagation of damage and cracks
- Main assumptions
  - existence of an elastic range with a yield limit
  - elastic isomorphism
  - rate independance

Main theoretical issue: the two decompositions

- The concept of *additive decomposition* (Green Naghdi (1965)) of the strain into symmetric "elastic" and "plastic" parts: how are these two parts defined individualy? how is the dependence on the arbitrary reference configuration resolved?

- The concept of *multiplicative decomposition* with *stress- and defect-free intermediate reference configuration* (Lee, Mandel (1965-70))

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- In general finite strain elasto-plasticity requires the choice of a **privileged reference configuration** which is related to the **postulated plastic laws**

- Rational thermodynamics was yet not able to answer definitively this key theoretical issue

A. Additive decomposition and Hypo-elasticity

- ASSUMPTION A1: $d := \nabla^S v = d^e + d^p$
  - consider objective strain rates: $\overset{\star}{\epsilon}$, $\overset{L}{\epsilon}$ (Lie), $\overset{\circ}{\epsilon}$ (upper Oldroyd)
  - non-corrotational: $d = \overset{\circ}{\epsilon^G}$ with $\epsilon^G := \nabla^S u + \frac{1}{2} \frac{\partial u}{\partial X} \left(\frac{\partial u}{\partial X} \right)^T$
  - corrotational: $\dot{\tau}_{\mathcal{R}} = \mathcal{R}^T \overset{\star}{\tau} \mathcal{R}$ with $\left\{ \begin{array}{c} \overset{\star}{\tau} = \dot{\tau} + \tau \dot{\Omega} - \dot{\Omega} \tau \\ \dot{\mathcal{R}} = \dot{\Omega} \mathcal{R}, \quad \mathcal{R}(0) = I \end{array} \right.$

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- ASSUMPTION A2: $\left\{\begin{array}{c} \overset{L}{\tau} = \mathcal{C}(d - d^p) \\ \dot{\tau}_{\mathcal{R}} = \mathcal{S}(d_{\mathcal{R}} - d^p_{\mathcal{R}}) \end{array}\right.$

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- ASSUMPTION A3 (isotropy): $\mathcal{S} = 2\mu I_4 + \lambda I \otimes I = \frac{\partial \tau_R}{\partial \epsilon^e}$
  - with $\begin{cases} \tau_{\mathcal{R}} & = & 2\mu\epsilon^e + \lambda I \operatorname{tr} \epsilon^e \\ \epsilon^e & = & \int_0^t \mathcal{R}^T d^e \mathcal{R} dt' \\ \mathcal{C}_{ijkl} & = & \mathcal{S}_{ijkl} - \frac{1}{2}\left(I_{ik}\tau_{jl} + I_{il}\tau_{jk} + I_{jl}\tau_{ik} + I_{jk}\tau_{il}\right) \end{cases}$

B. Multiplicative decomposition

- ASSUMPTION B1: There exists
  - A *local* plastic process (ex. dislocations at the crack tip)
  - A plastic deformation and *local* reference configuration F^p
  - A *local* current deformation $F^e := F(F^p)^{-1}$ s.t. $(F^e)^{-1}$ is stress-free

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- Model variables
  - left Cauchy-Green tensor: $b^e := F^e(F^e)^T$
  - plastic velocity gradient: $l^p := \dot{F}^p(F^p)^{-1} = -\frac{1}{2} \overset{L}{b^e} (b^e)^{-1}$

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- ASSUMPTION B2: Mandel's principle of maximal dissipation
  - dissipation: $\mathcal{D} = \tau \cdot l^p - \bar{\epsilon}^p \dot{p}$
  - equivalent plastic strain: $\bar{\epsilon}^p$ s.t.: $\tau \cdot l^p = \bar{\tau}\bar{\epsilon}^p$
  - maximality $\Longrightarrow (l^p)^S = \dot{\gamma}\frac{\partial \Phi}{\partial \tau}$
  - $\dot{\gamma} = \dot{p}$
  - flow law Φ

Model equations (1): balance equation

- Plane stress (**Unknown** displacement u, $\forall \tilde{u} \in H^1_0(\Omega_{0t}, \mathbb{R}^3)$)

$$\int_{\Omega_{0t}} \tau\left(F(u); T\right) \cdot \nabla \tilde{u} dV = \int_{\Gamma^N_{0t}} G \cdot \tilde{u} dS + \int_{\Omega_{0t}} F \cdot \tilde{u} dV$$

$$\tau_{33} = 0, \quad \text{and at convergence:} \quad \mathcal{C} = \frac{\partial \tau}{\partial \epsilon} - \frac{\partial \tau_{33}}{\partial \epsilon_{33}} \frac{\partial \tau}{\partial \epsilon_{33}} \otimes \frac{\partial \tau_{33}}{\partial \epsilon}$$

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- Plane strain (**Unknowns:** displacement u and the pressure p)

$$\begin{aligned} \int_{\Omega_{0t}} \left(\tau^d + pI\right) \cdot \nabla \tilde{u} dV &= \int_{\Gamma_{0t}^N} G \cdot \tilde{u} dS + \int_{\Omega_{0t}} F \cdot \tilde{u} dV \\ \int_{\Omega_{0t}} \left(\frac{1}{3} \mathrm{tr}\ \tau - p\right) \tilde{\theta} dV &= 0 \end{aligned}$$

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\end{aligned}$$

- Heat equation (**Temperature**, Taylor-Quinney coeficient α)

$$\int_{\Omega_{0t}} \left(\rho C_p \dot{T} \tilde{T} + k \nabla T \cdot \nabla \tilde{T}\right) dV = \int_{\Omega_{0t}} \rho \alpha \tau \bar{\epsilon}^p \theta dV$$

Model equations (2): plasticity

Nonsmooth & nonlinear equations (**Unknowns:** b^e (or $\dot{\epsilon}^p$) and $\dot{\gamma}$)

- **Flow laws:**

A. Additive:
$$\dot{\epsilon}^p = \dot{\gamma}\frac{\partial \Phi}{\partial \tau}$$

B. Multiplicative:
$$\left(-\frac{1}{2}\,\overset{L}{b^e}\,(b^e)^{-1}\right)^S = d^p = \dot{\gamma}\frac{\partial \Phi}{\partial \tau}$$

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- **Temperature-dependent yield functions:**

$$\Phi = \sum_i \Phi^i, \quad \Phi^i = \sigma^i_{eq} - (1-f)(1 - \frac{T - T_{room}}{T_{melt} - T_{room}})\sigma^i_y$$

  - void fraction f: $f = \left(\frac{\bar{\epsilon}^f}{\epsilon_f}\right)^2$
  - $$\begin{cases} A.Additive : \bar{\epsilon}^f = \bar{\epsilon}^p = \int_0^t ||\dot{\epsilon}^p(t')||dt' \\ B.Multiplicative : \bar{\epsilon}^f = arg\max_{\epsilon_i}\left(\det[\frac{1}{2}(I - b^{-1}) - \epsilon_i I]\right) \end{cases}$$
  - equivalent stress σ_{eq}: $\sigma_{eq} = J_2 + c_1 f < p >$
  - yield stress $\sigma_y(f, \epsilon^p)$

Computation of the tangent modulus

Specific Heat Supply:

$$\dot{Q} = \alpha \tau \bar{\epsilon}^p$$

with **Taylor-Quinney** coefficient $\alpha = 0.8$.

Remark: $(1 - \alpha)\tau\bar{\epsilon}^p$ *goes to dislocations at the crack tip.*

Quasi-incompressible **Neo-Hookean** stress-strain constitutive law: $\tau = \tau(b^e, T)$.

Tangent Modulus (computed with *AceGen* module (cf. J. Korelc))

$$\begin{bmatrix} \frac{\partial \dot{Q}}{\partial T} & \frac{\partial \dot{Q}}{\partial F} \\ \frac{\partial \tau}{\partial T} & \frac{\partial \tau}{\partial F} \end{bmatrix}$$

Smoothing the complementarity condition

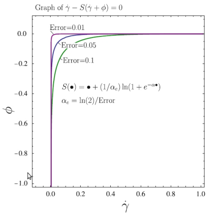
- The **complementarity conditions** are nonlinear and nonsmooth: $\Phi \leq 0, \quad \dot{\gamma} \geq 0, \quad \Phi\dot{\gamma} = 0, \quad \dot{\Phi}\dot{\gamma} = 0$
- It can be replaced by a **first-order nonlinear ODE:**

$$c\dot{\gamma} = [c\dot{\gamma} + \Phi(T, \gamma, \beta)]_+$$

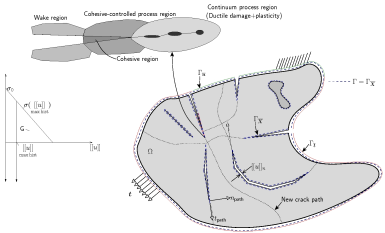
where $[x]_+ = \max\{x, 0\}$

- The "plus" function $[\cdot]_+$ is replaced by the smooth ramp function (Chen and Magasarian, 1996):

$$S(x) \quad = \quad x + \frac{1}{\beta} \ln\left(1 + \exp(-\beta x)\right)$$



OUR PROBLEM: DUCTILE FRACTURE



• Metals at room temperature fail in a ductile fashion with large and plastic (ie. permanent) deformations

- **Wake region:** already cracked: discontinuities, jumps, mesh separation, etc.
- **Process region:** microscopic failure (nucleation, growth and coalescence of microvoids), crack lips cohesion, field continuity, macroscopic damage variable
- **Away from the crack:** Non-linear elasto-plasticity.

Numerically CRACK INITIATION AND PROPAGATION. REMESHING.

- **Propagation:**
  - as soon as the damage parameter (=*void fraction* f) becomes critical
  - at the crack tip along Ma & Sutton (1999) criterion
  - numerically: remeshing, node splitting
- **Initiation:**
  - as soon as f becomes critical
  - at the boundary
  - normal to the principal stress direction

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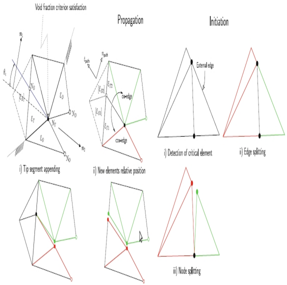
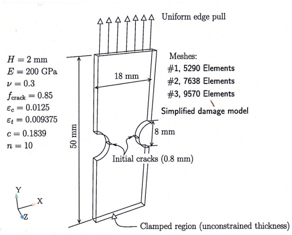
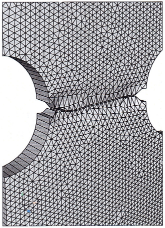
- **Tip remeshing:** when crack advances, the mesh is altered and the historical variables $(b^e, \bar{\epsilon}^P, F_{old}, f)$ are mapped to the new mesh



SIMULATION RESULTS: second theory. **A. THE GROOVED PLATE**(1)



(a) Grooved specimen



(b) Final cracked mesh with the formation of a particle, mesh #3. Extrusion was performed with the thickness field to produce a 3D image.

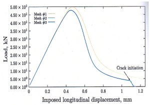


Fig. 5 Grooved specimen: load versus imposed displacement

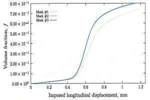
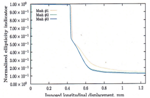


Fig. 6 Grooved specimen: volume fraction at the centroid versus imposed displacement



SIMULATION RESULTS. **A. THE GROOVED PLATE**(2)

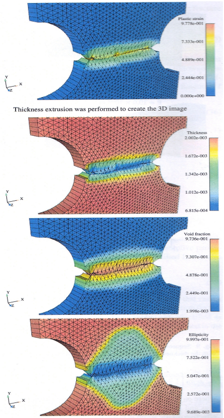


Fig. 8 Grooved specimen: effective plastic strain, thickness distribution, void fraction and ellipticity indicator for mesh #3. Extrusion was performed with the thickness field to produce 3D images

SIMULATION RESULTS. **B. PLATE WITH RIGID INCLUSIONS**

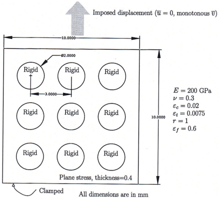


Fig. 15 Plate containing 9 rigid *circular* inclusions. No use of symmetry is made

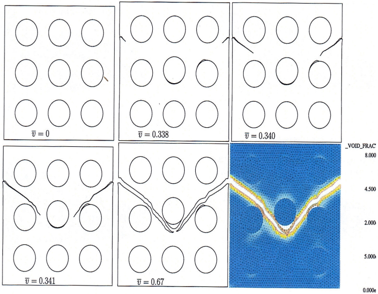


Fig. 16 Plate with *circular* inclusions: sequence of crack stages and void fraction contour plot over the final deformed mesh ($\overline{v} = 0.67$). Crack initiation, propagation and particle decohesion occur with the same criterion. Node-to-node decohesion elemer the contour plot picture

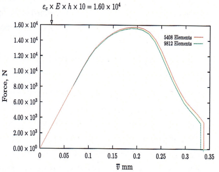
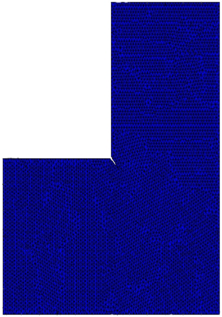


Fig. 17 Plate with *circular* inclusions: load/imposed displacement results

REF. Areias, Van Goethem and Pires: Comp. Mech. (2011)

SIMULATION RESULTS: first theory with temperature.
A. THE CANTILEVER with initial crack



OBSERVE THE EVOLUTION OF THE CRACK with the following fields:

- the damage parameter (void fraction)
- the equivalent plastic strain
- the temperature

Perspective of the model

- $3D$ temperature-dependent ductile crack
- crack propagation on shells
- dislocation emission at the crack tip
- single crystal plasticity

THANK YOU FOR YOUR ATTENTION