SIMPLAS @ CMAF

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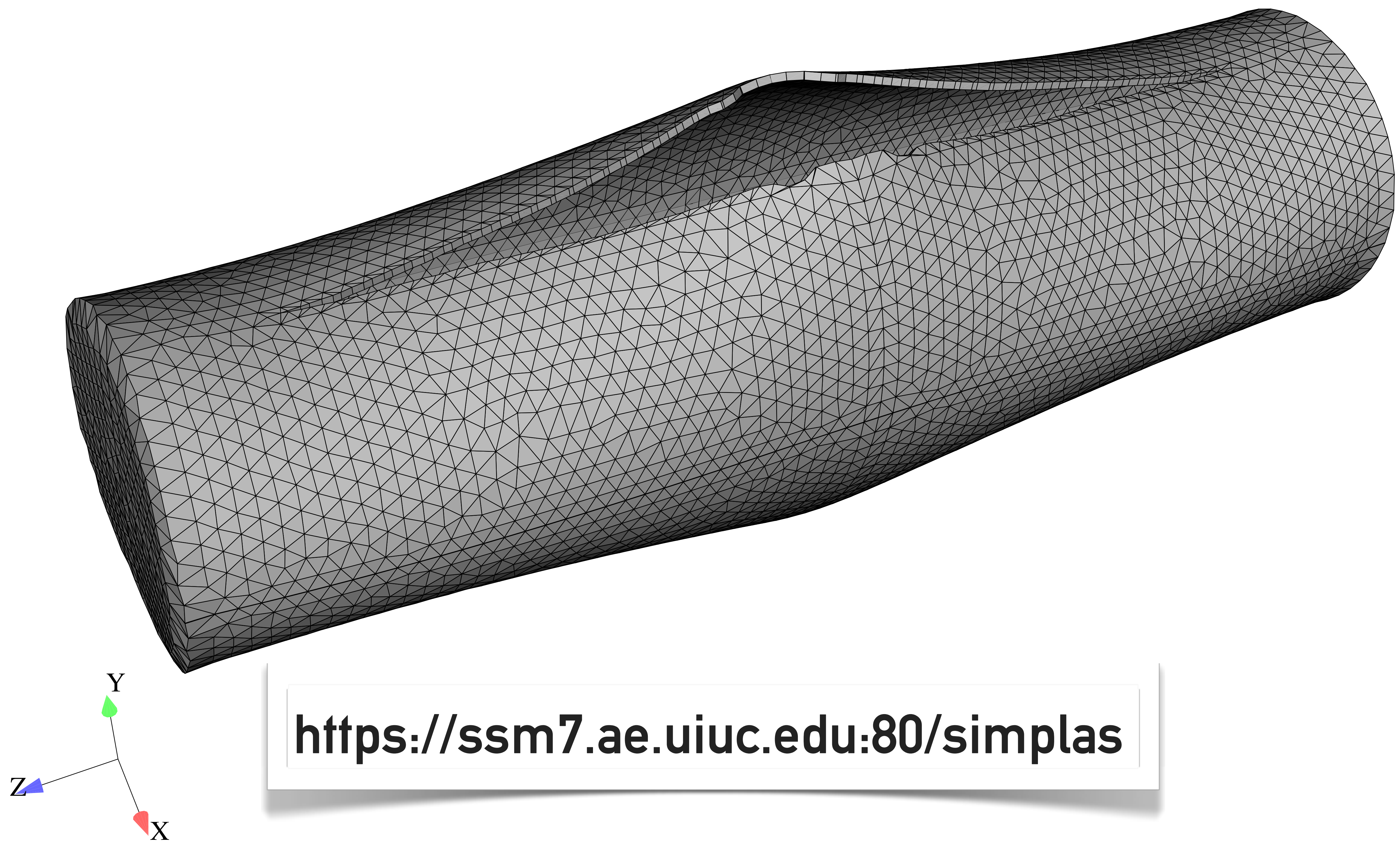


https://ssm7.ae.uiuc.edu:80/simplas

CFRAC 2011, 6-8 June, Barcelona, Spain

in memory of JAC Martins

Goals, methodologies and tools

Goals:

•To produce a definite computational tool allowing a systematic reproduction of results for quasi-brittle and ductile fracture in finite strains[+].

•Create a underlying framework where each physical law (Cauchy equilibrium, Maxwell's equations, heat transfer, etc) is *automatically* used with time-tested (and published) discretization technologies.

•Allow the testing and validation of new constitutive laws, thermal coupling, electro-magnetic coupling.

•Allow an automated incorporation of technical requirements such as:

- Plane stress condition.
- Non-local state variables.

•Introduce and test general heuristics and solution control.

•Incorporate new technologies in shell and beam elements prone to fracture.

Methodologies and tools:

•Consistently linearize all equations and perform preliminary tests (isoerror maps, convergence radius, etc).

•Use Chen-Mangasarian replacement functions for complementarity conditions (elasto-plasticity, contact and friction, cohesive laws).

•Make extensive use of the ACEGEN add-on to Mathematica.

•Use of a in-house sparse library along with a graph database (also in-house).

•Continue to develop SIMPLAS wrapped in a C++ graph database.

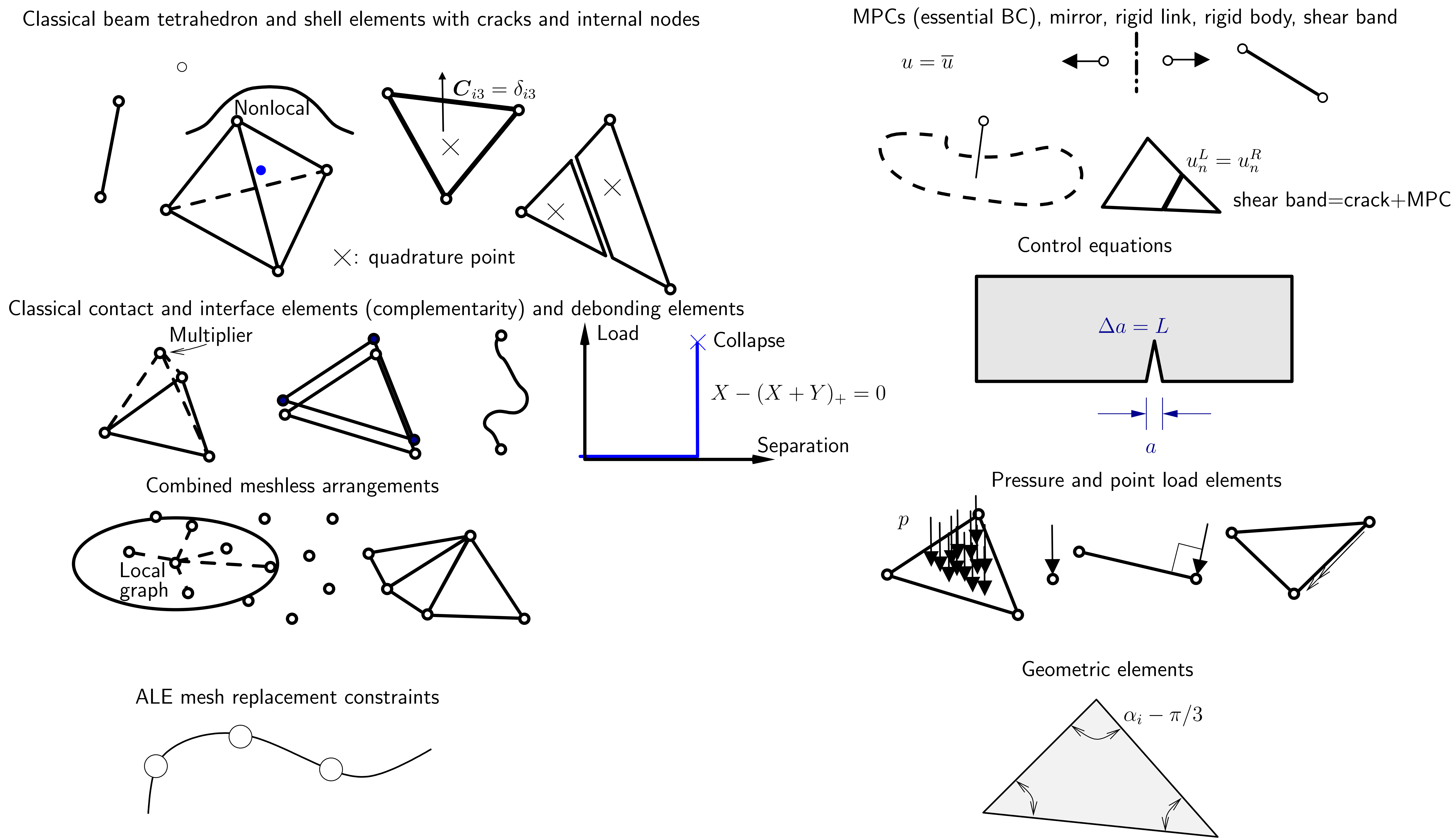
•Use ALE and geometric elements.

•*Avoid enrichment or "enhancement" techniques*

[+]We find that ductile fracture is the most complex problem that can be dealt with Newton's method, hence the motivation

Global perspective of our approach

All components of a discrete "engineering" system are either additive (e.g. elements or cliques) or multiplicative (e.g. boundary conditions or multiple-point constraints).
Components may introduce non-smoothness to the system.



Fracture problems in finite strains

Ingredients:

- Element technology:
  - Plane stress with thickness field (Comp. Mech).
  - Plane strain and 3D with pressure unknowns (inf-sup verified) (CMAME and IJNME).
  - Fully finite strain exact shell (6 Dofs with physical drilling) (Comp. Mech).
- Geometrical element:
  - 2D (Comp. Mech).
  - Shell (to be submitted).
  - 3D (not yet implemented).
- Constitutive modeling:
  - Correct multiplicative plasticity with Chen-Mangasarian replacements (IJNME and to be submitted).
  - Multiple-surface approach for ductile damage (to be submitted).
- Solution control and multiple-point constraints.
  - Clique processor and sparse library

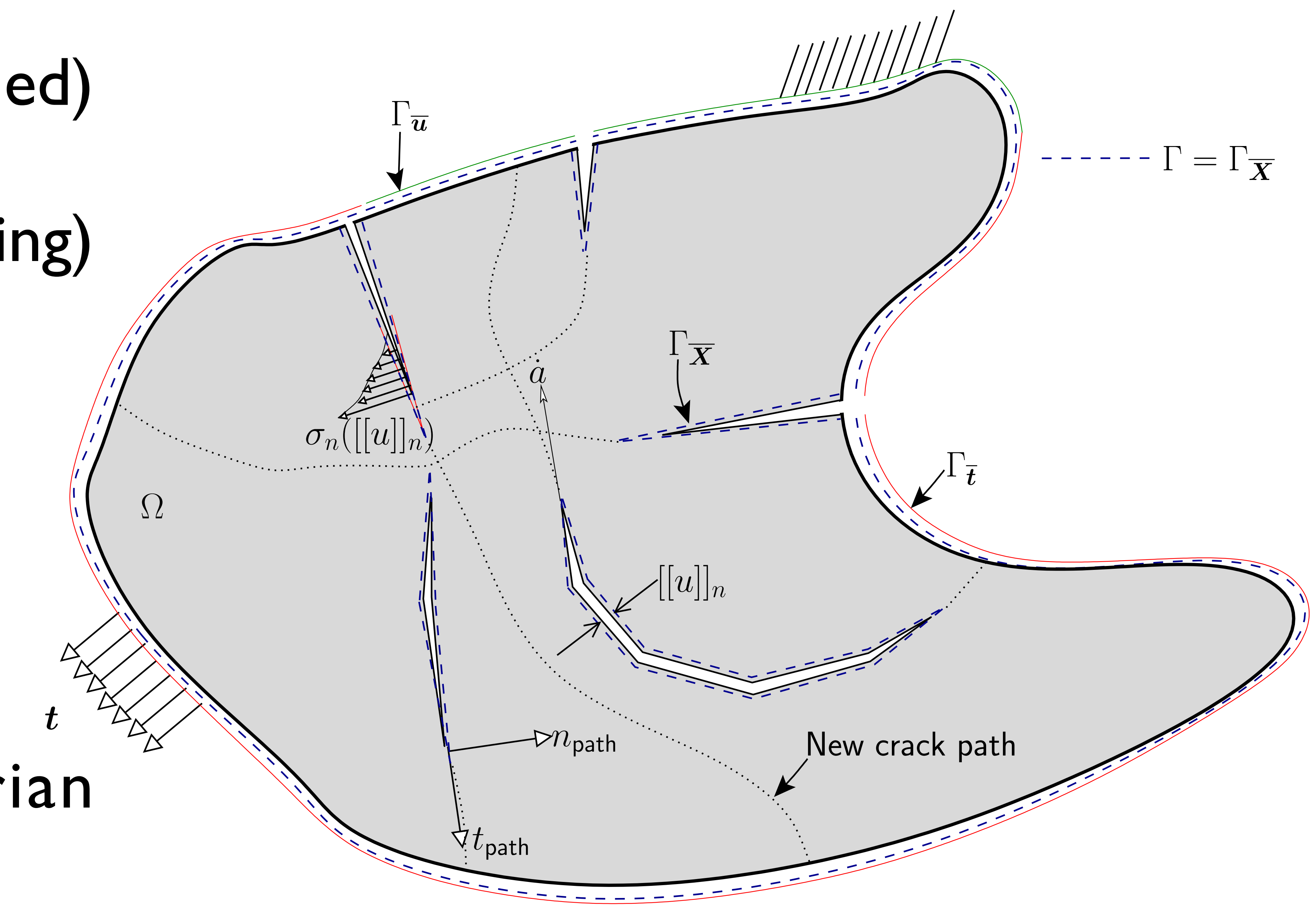
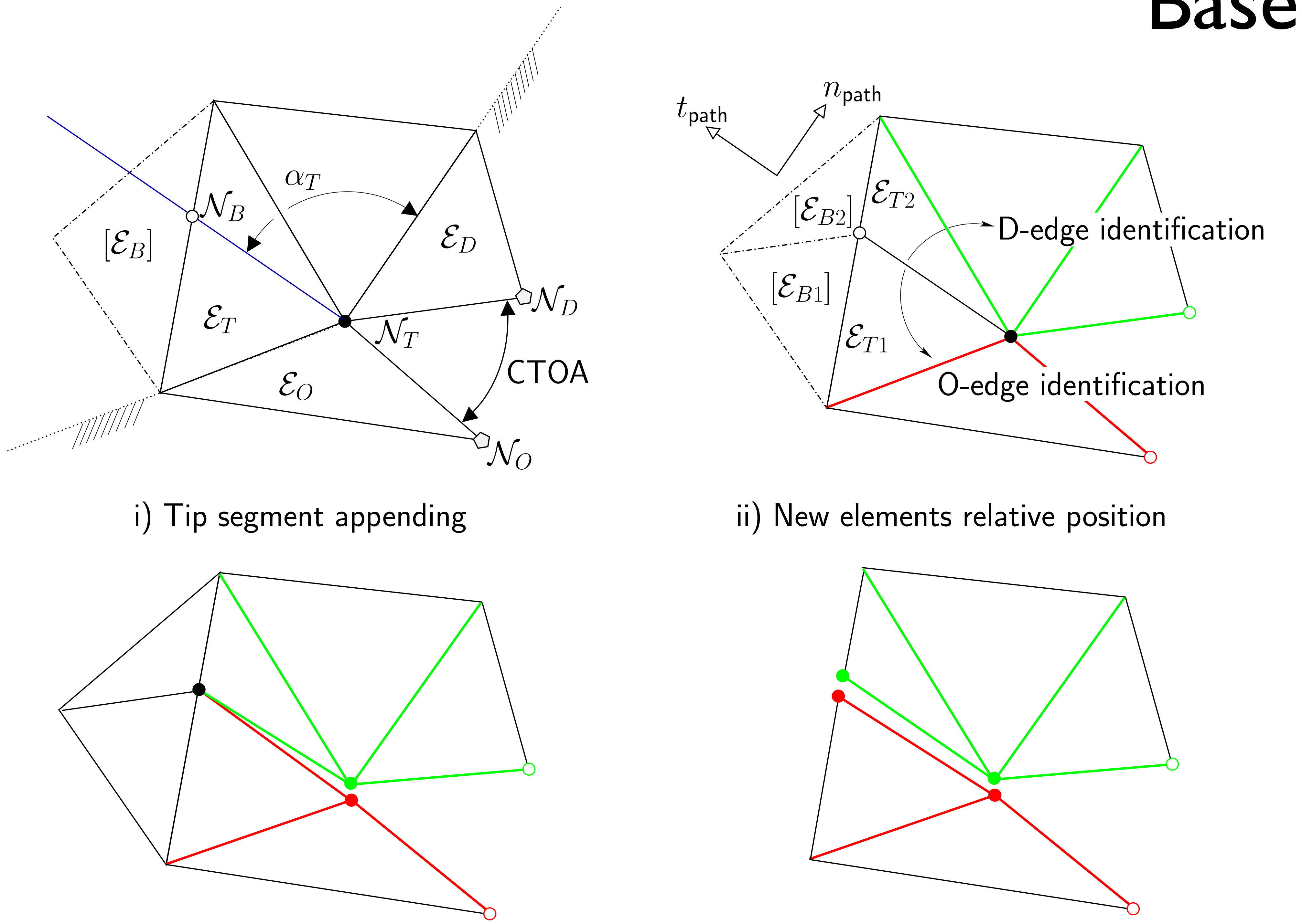


Fig. Relevant ingredients

Base technology

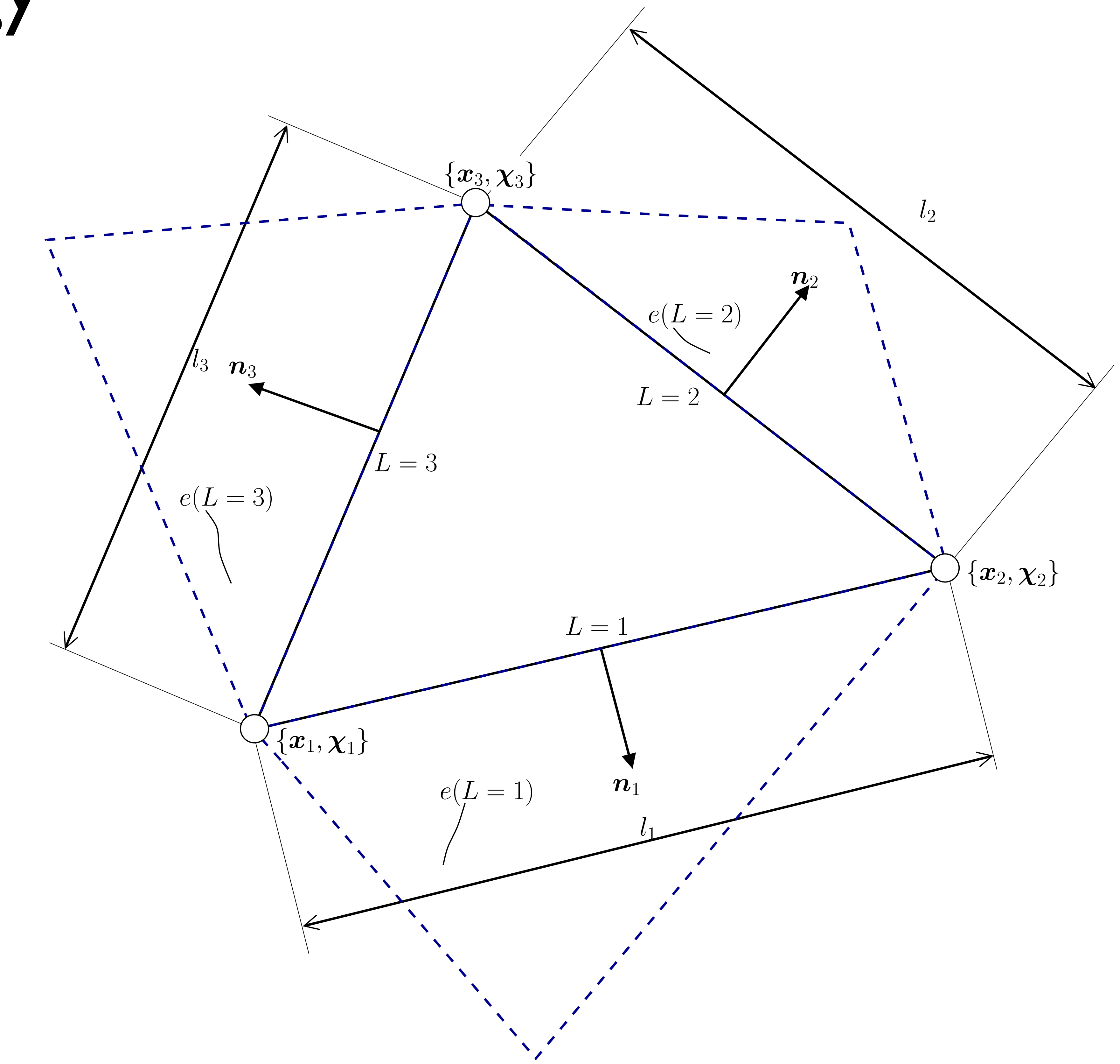


i) Tip segment appending

ii) New elements relative position

iii-a) Node splitting with subsequent element

iii-b) Node splitting without subsequent element



$$\boldsymbol{f}^{\alpha} = \frac{\partial \Pi_{\text{angle}}}{\partial \boldsymbol{\chi}_v}$$

$$\boldsymbol{K}^{\alpha} = \frac{\partial^2 \Pi_{\text{angle}}}{\partial \boldsymbol{\chi}_v \partial \boldsymbol{\chi}_v}$$

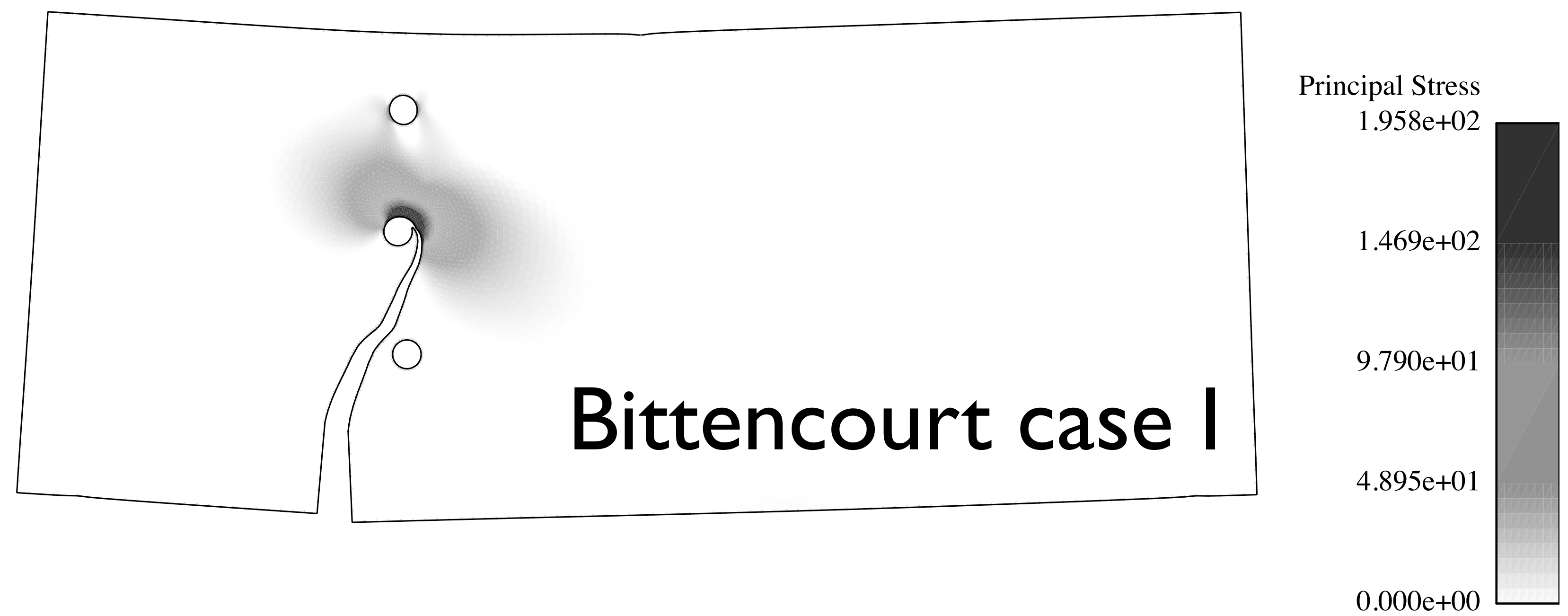
Geometric element

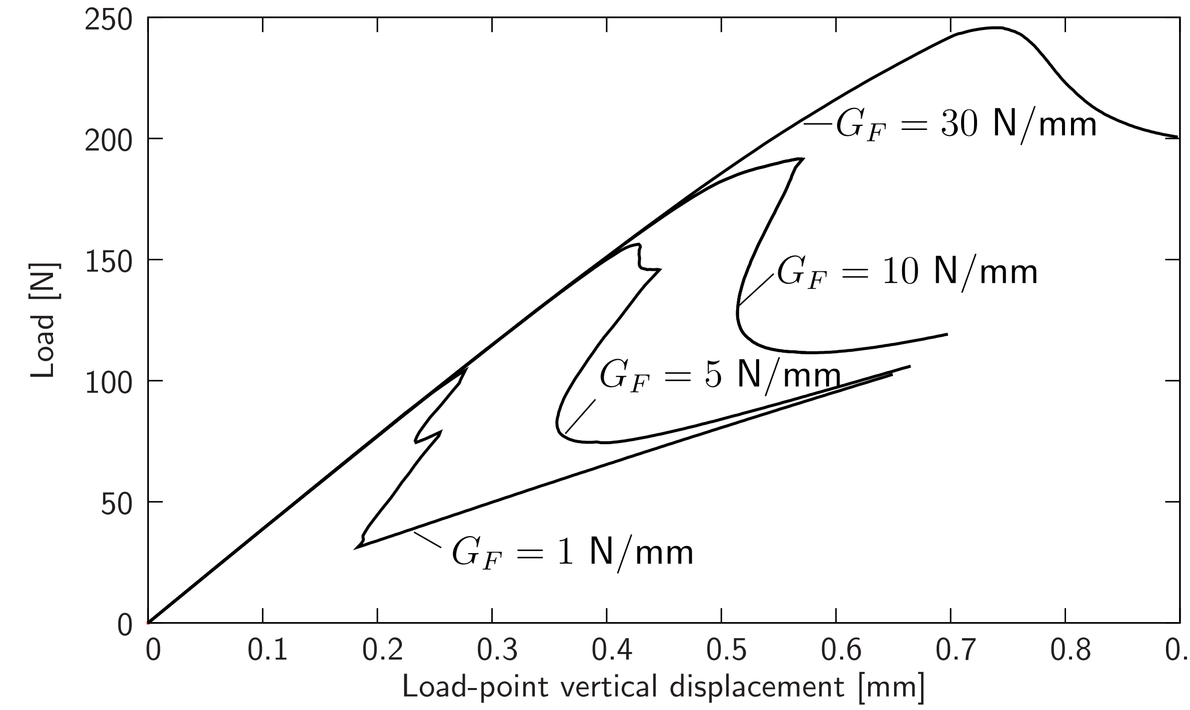
Relevant quantities in Godunov scheme

$$\boldsymbol{F}^e_o = \boldsymbol{F}^e + \frac{1}{2A_e} \sum_{L=1}^{3} \left\{ \left[l_L \boldsymbol{n}_L \cdot \left(\Delta \boldsymbol{x}_{\overline{L}_3} + \Delta \boldsymbol{x}_{\overline{L+1}_3} - \Delta \boldsymbol{\chi}_{\overline{L}_3} - \Delta \boldsymbol{\chi}_{\overline{L+1}_3} \right) \right]_+ \left(\boldsymbol{F}^{e(L)} - \boldsymbol{F}^e \right) \right\}$$

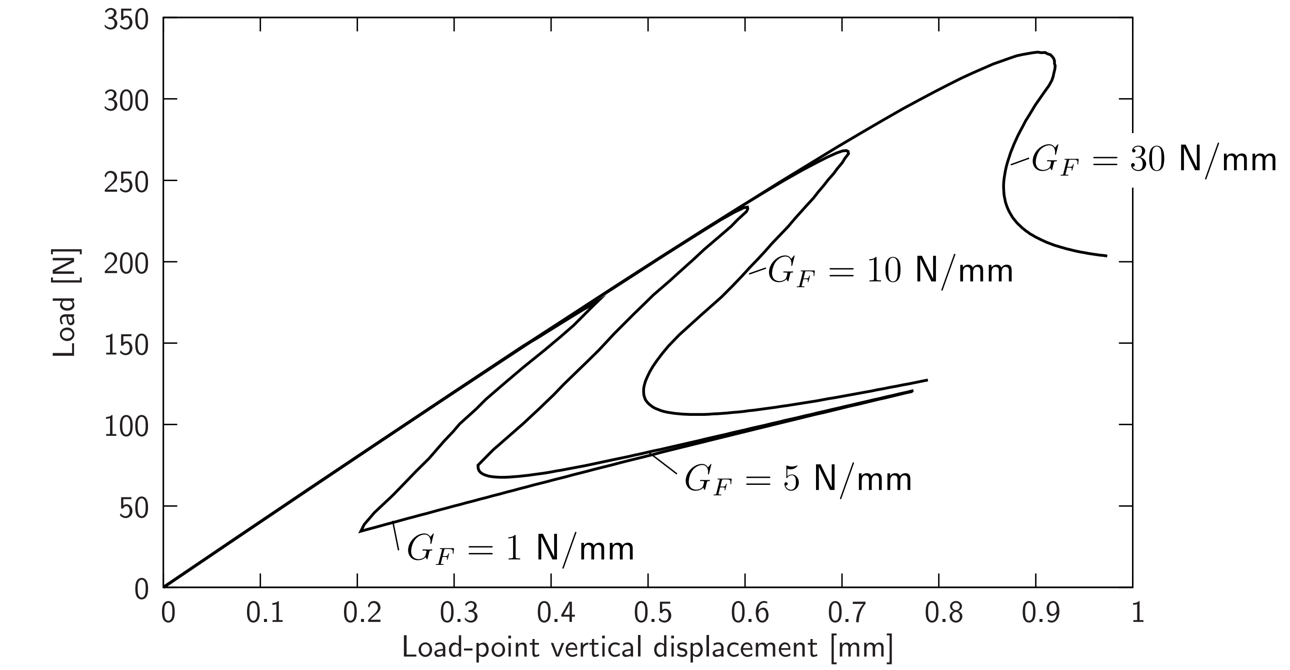
Advection steps

Base technology - results (quasi-brittle)

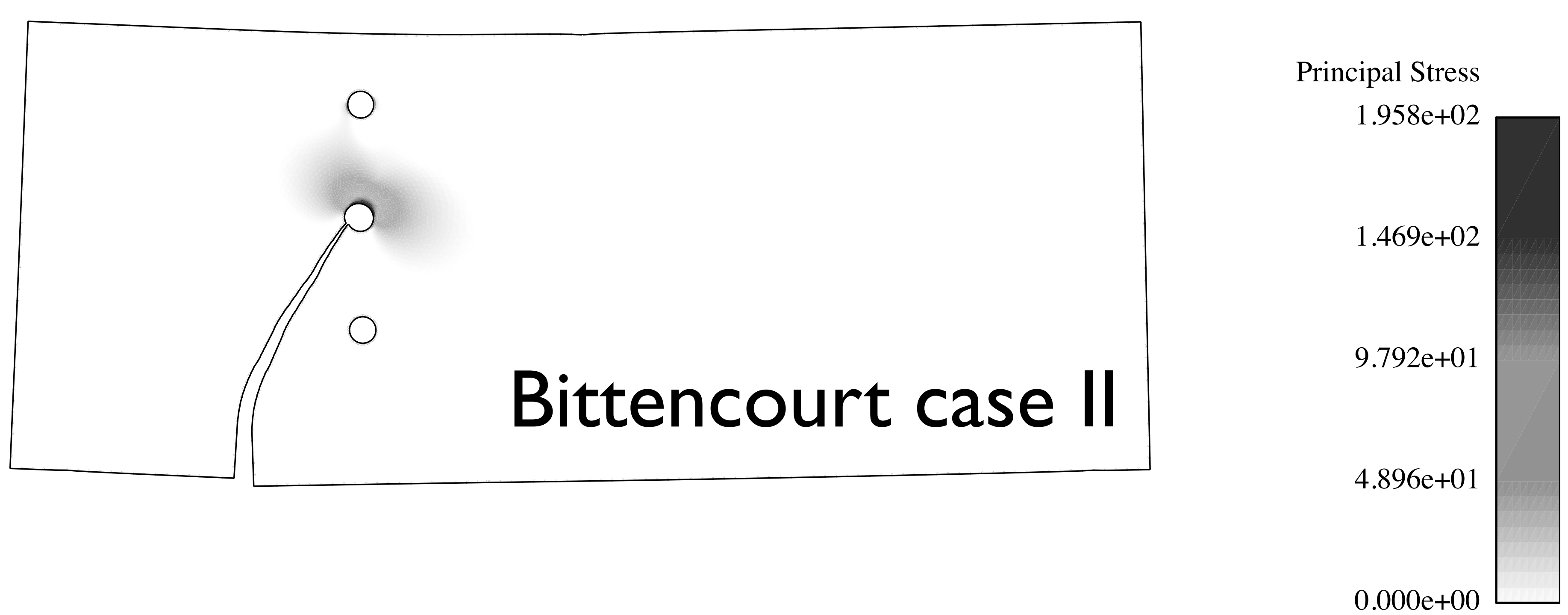


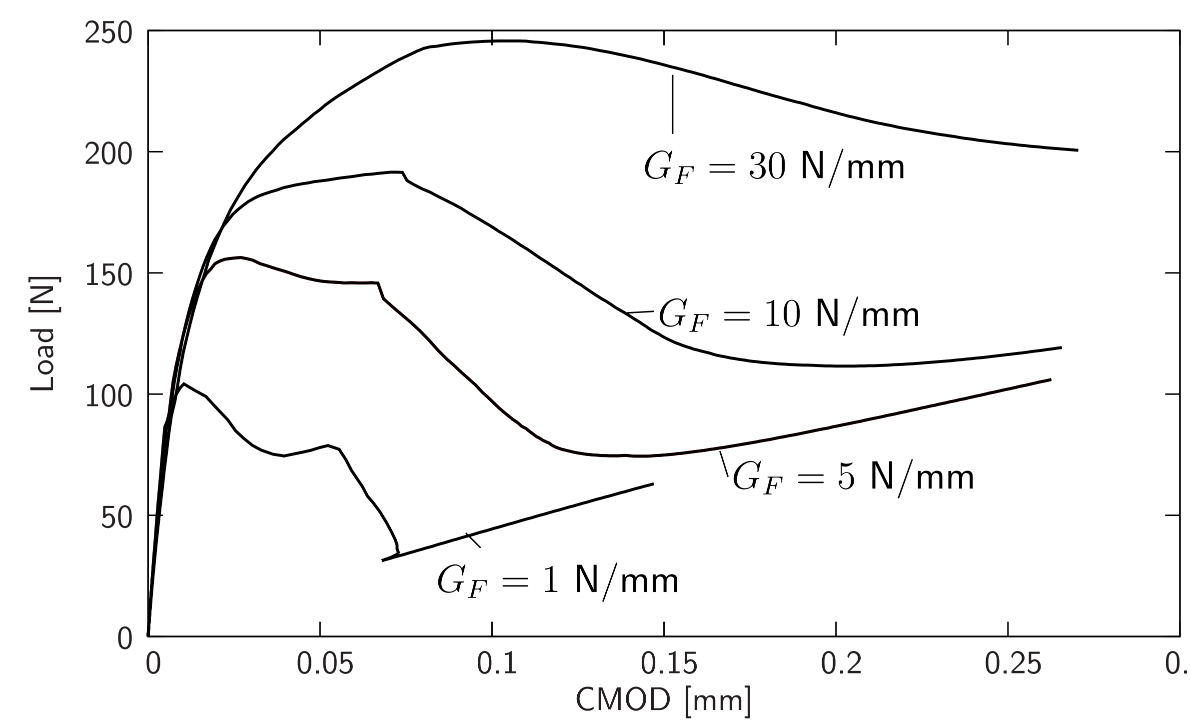


(a) Specimen #1, vertical displacement

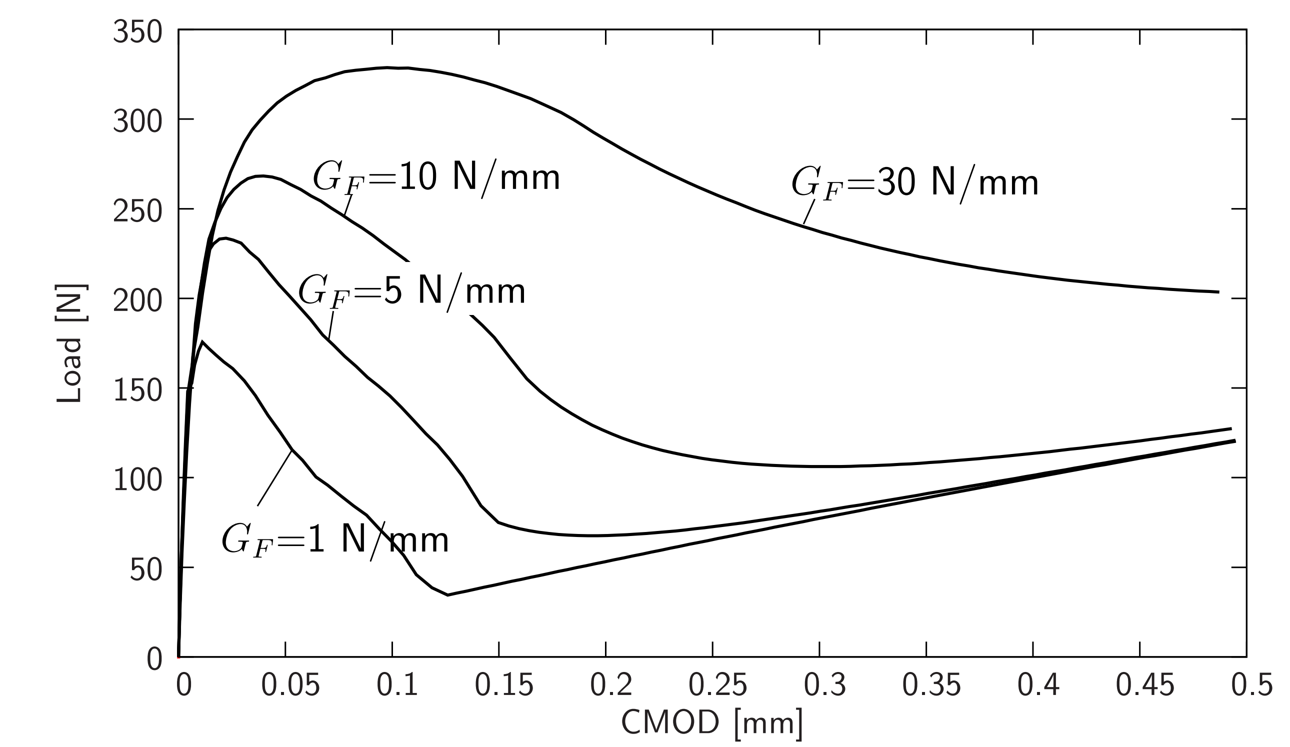


(a) Specimen #2, vertical displacement

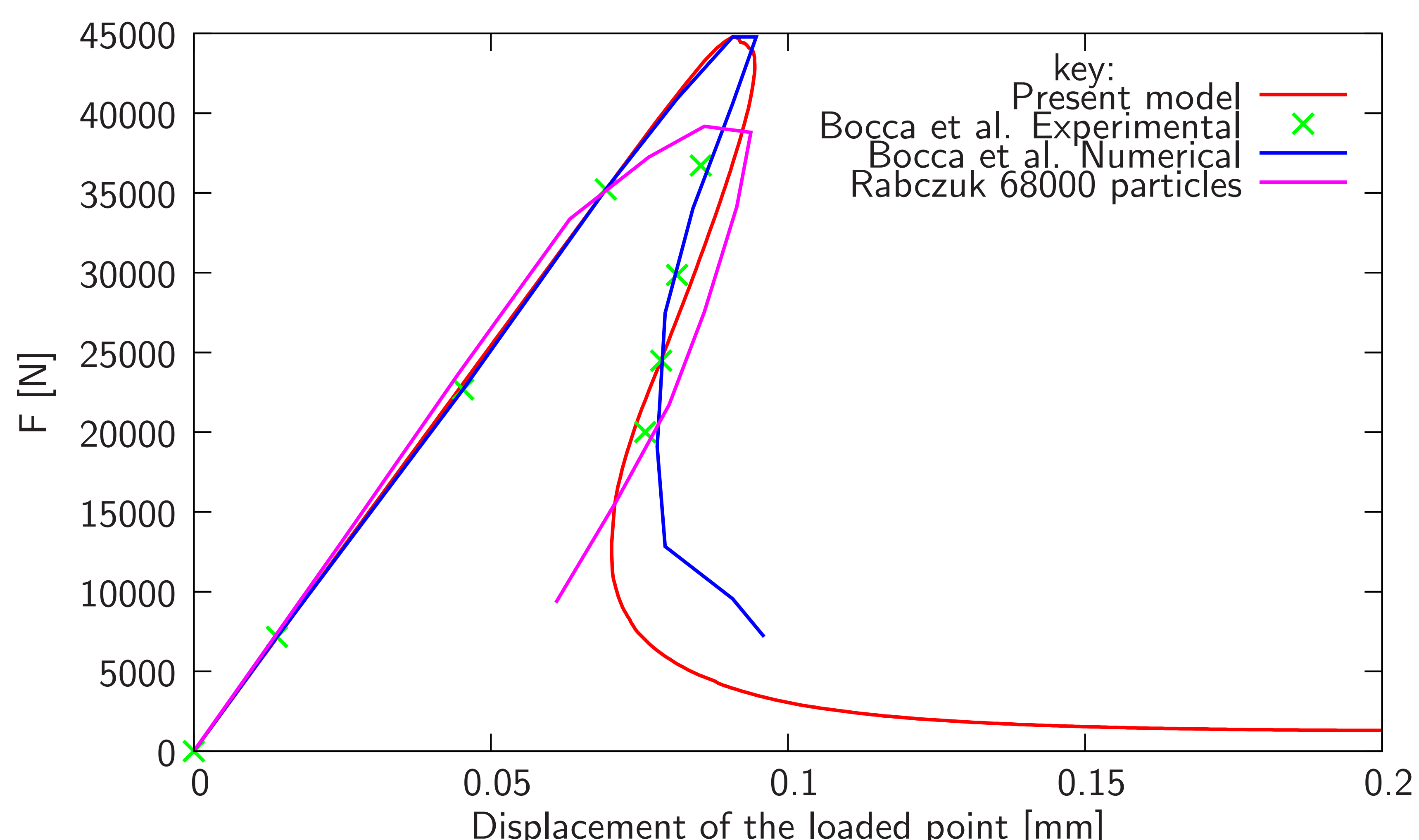




(b) Specimen #1, CMOD

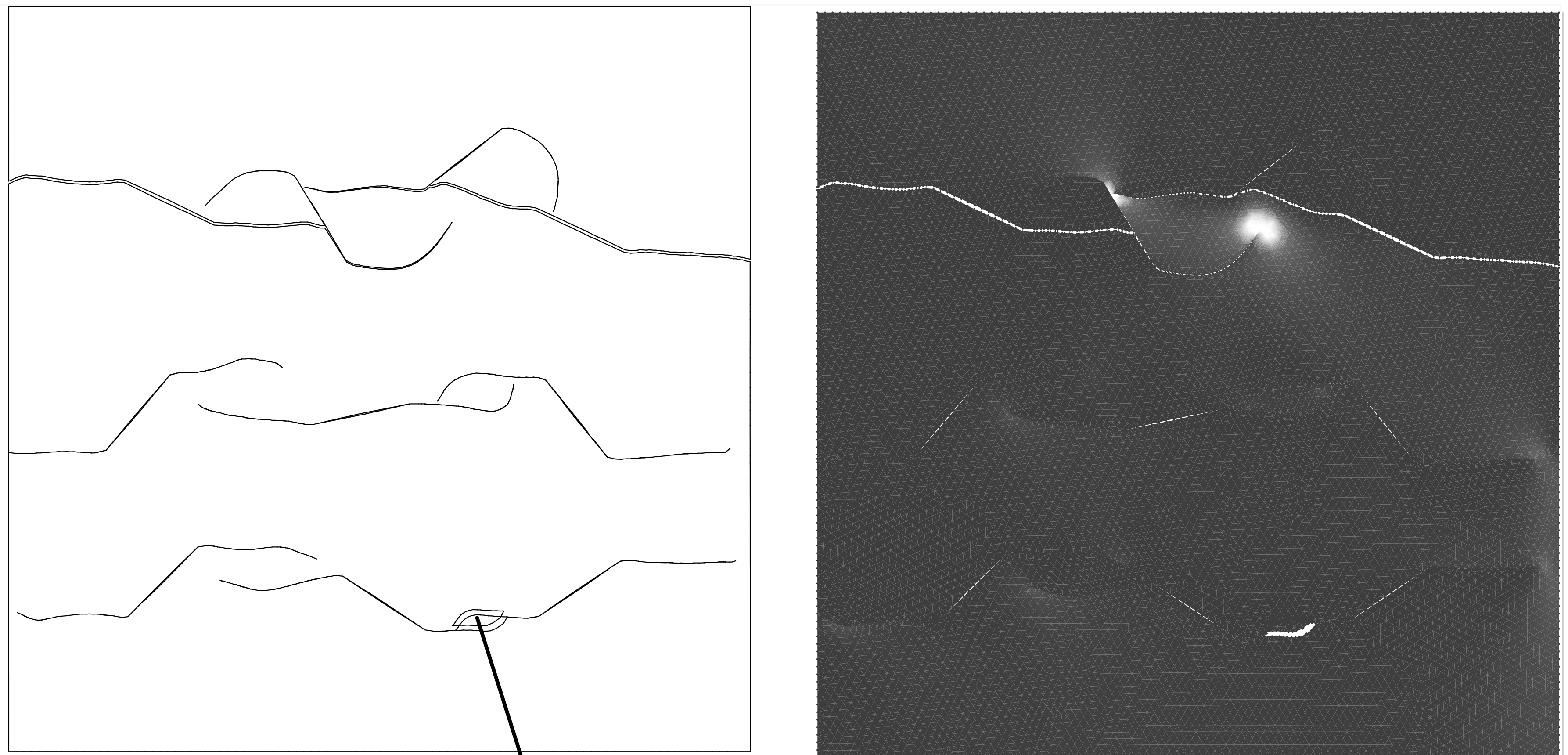


(b) Specimen #2, CMOD



<u>Many literature problems solved with success</u>

Also with simultaneous crack growth



Particle formation ($5\times$ magnified)

Any hyperelastic law along with any plasticity model.

Yield criterion	Number of yield surfaces	Equivalent stresses
von-Mises	1	$\sigma_{\mathrm{eq}_1} = \sqrt{I_1^2 - 3I_2}$
Tresca	6	$\sigma_{\mathrm{eq}_k} = \widetilde{\tau}_i - \widetilde{\tau}_j, \quad i \neq j$
Ductile damage	2	$\sigma_{eq_1} = \frac{\sqrt{I_1^2 - 3I_2} - fc_1 I_1}{1-f}$ $\sigma_{eq_2} = \frac{\sqrt{I_1^2 - 3I_2}}{1-f}$

$I_1 = \mathrm{tr}\boldsymbol{\tau}, \quad I_2 = \frac{1}{2}\left[(\mathrm{tr}\boldsymbol{\tau})^2 - \mathrm{tr}\boldsymbol{\tau}^2\right], \quad I_3 = \det\boldsymbol{\tau}$

However... **much** tougher than quasi-brittle is *ductile* fracture

Finite strain plasticity as we see it

$$\boldsymbol{\tau} = 2\frac{\mathrm{d}\psi_b}{\mathrm{d}\boldsymbol{b}_e}\boldsymbol{b}_e = 2\boldsymbol{b}_e\frac{\mathrm{d}\psi_b}{\mathrm{d}\boldsymbol{b}_e}$$

$$\frac{[\mathrm{d}\boldsymbol{b}]_{ij}}{[\mathrm{d}\boldsymbol{F}]_{mn}} = \delta_{im}[\boldsymbol{F}]_{jl} + \delta_{jm}[\boldsymbol{F}]_{in}$$

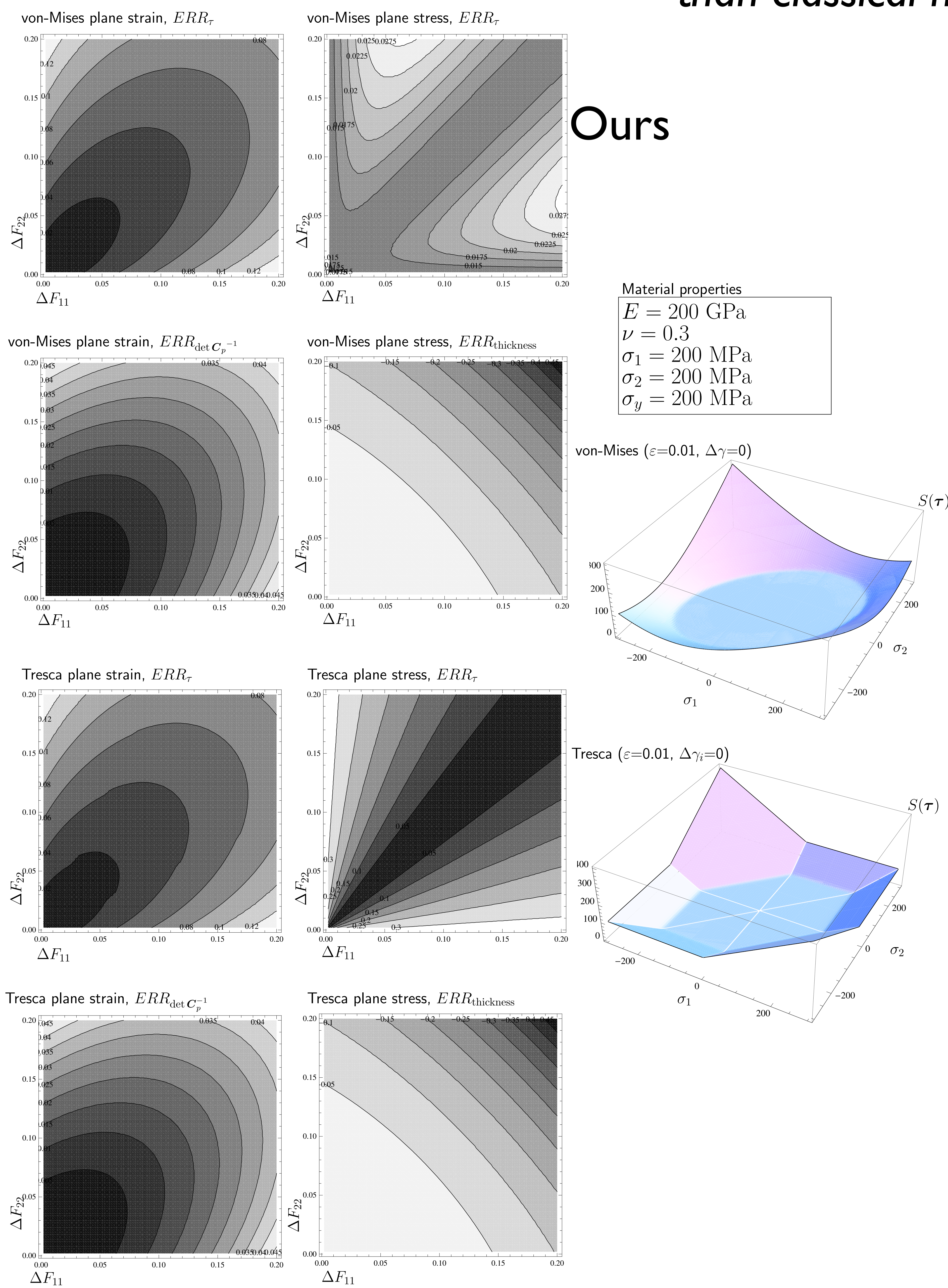
$$\begin{aligned}
\overset{\star}{\boldsymbol{b}}_{eV} &= -4\sum_{i=1}^{n_s}\dot{\gamma}_i\boldsymbol{A}^{-1}\boldsymbol{n}_i \\
\dot{\boldsymbol{v}} &= -\sum_{i=1}^{n_s}\dot{\gamma}_i\boldsymbol{\varphi}_i \\
\boldsymbol{\tau} &= 2\frac{\mathrm{d}\psi_b}{\mathrm{d}\boldsymbol{b}_e}\boldsymbol{b}_e \\
\mu\dot{\gamma}_i - \langle\mu\dot{\gamma}_i + \phi_i\rangle &= 0
\end{aligned}$$

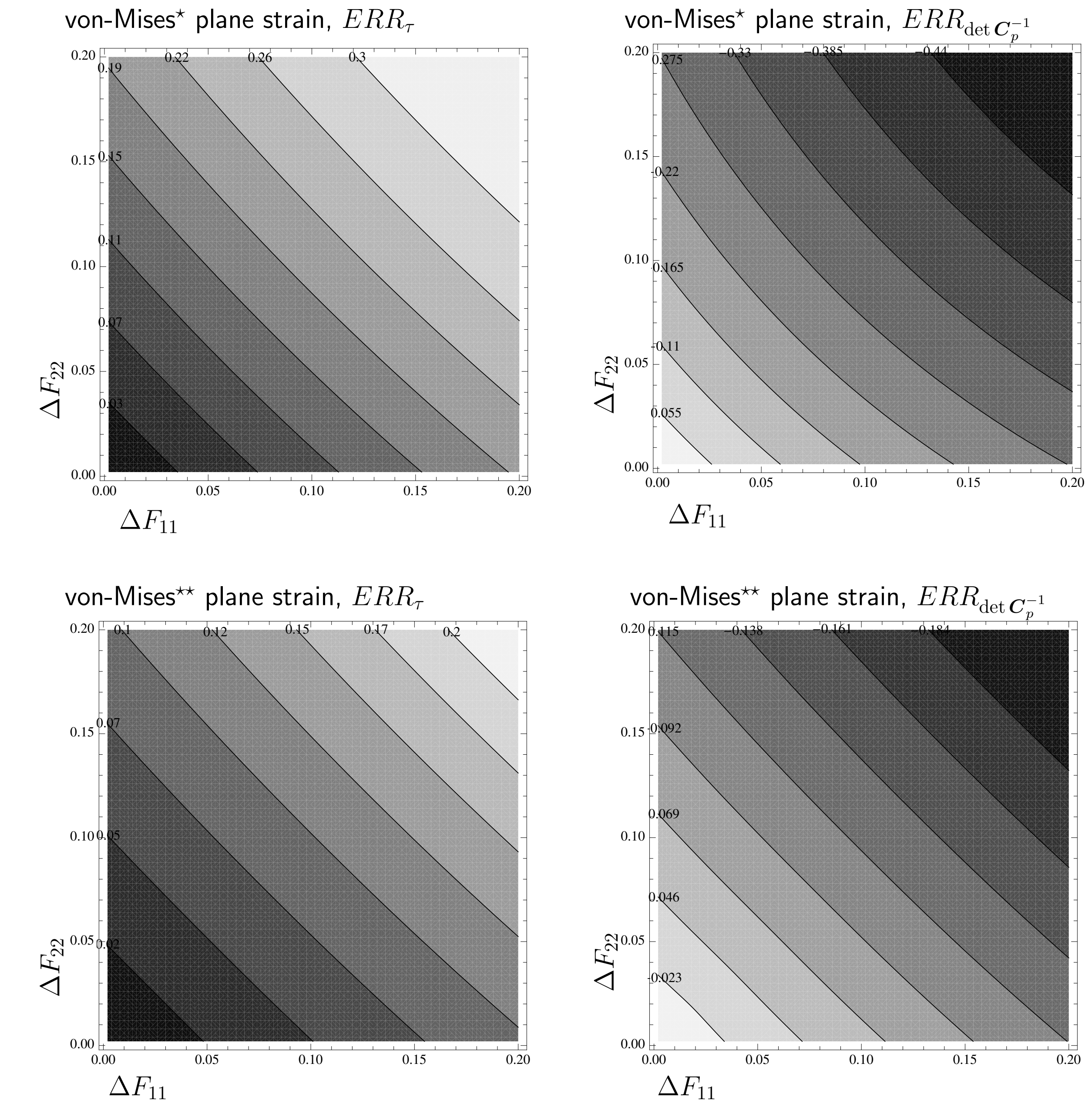
What the books do not describe

$$\mathrm{d}\varepsilon^i_{p_{n+1}} = \frac{\boldsymbol{n}_i : \boldsymbol{\tau}}{\sigma_{eq_i}}\mathrm{d}\Delta\gamma_i + \left[\frac{\Delta\gamma_i}{\sigma_{eq_i}}\left(\boldsymbol{\tau} : \frac{\mathrm{d}\boldsymbol{n}_i}{\mathrm{d}\boldsymbol{\tau}} + \boldsymbol{n}_i\right) - \frac{\boldsymbol{n}_i : \boldsymbol{\tau}}{\sigma_{eq}}\right] : \frac{\partial\boldsymbol{\tau}}{\partial\boldsymbol{b}_e} : \mathrm{d}\boldsymbol{b}_e$$

$$4\boldsymbol{d}_{pV} = -\boldsymbol{A}\overset{\star}{\boldsymbol{b}}_{eV}$$

No requirement for active set strategies, and no "return-mapping" and *much better accuracy than classical methods, including Simo's...*



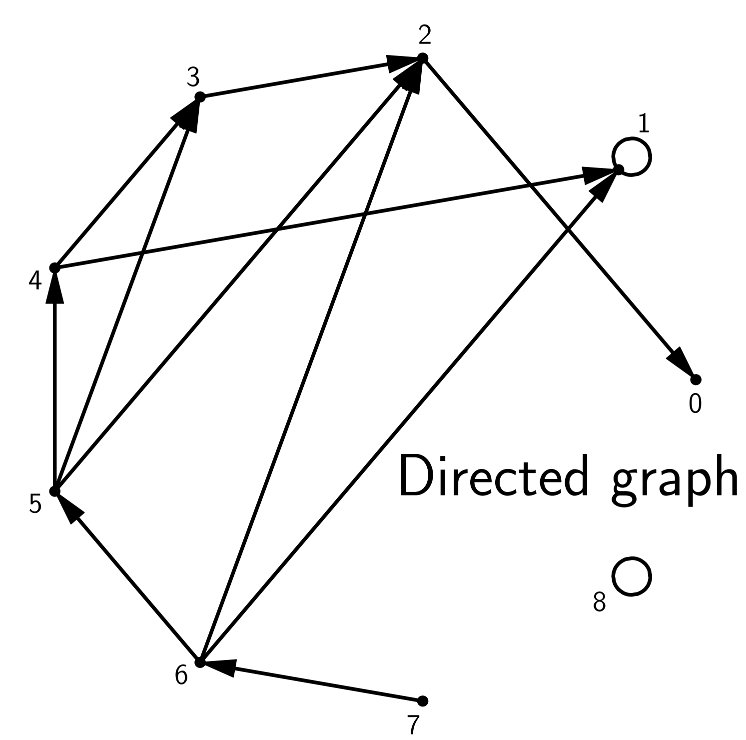


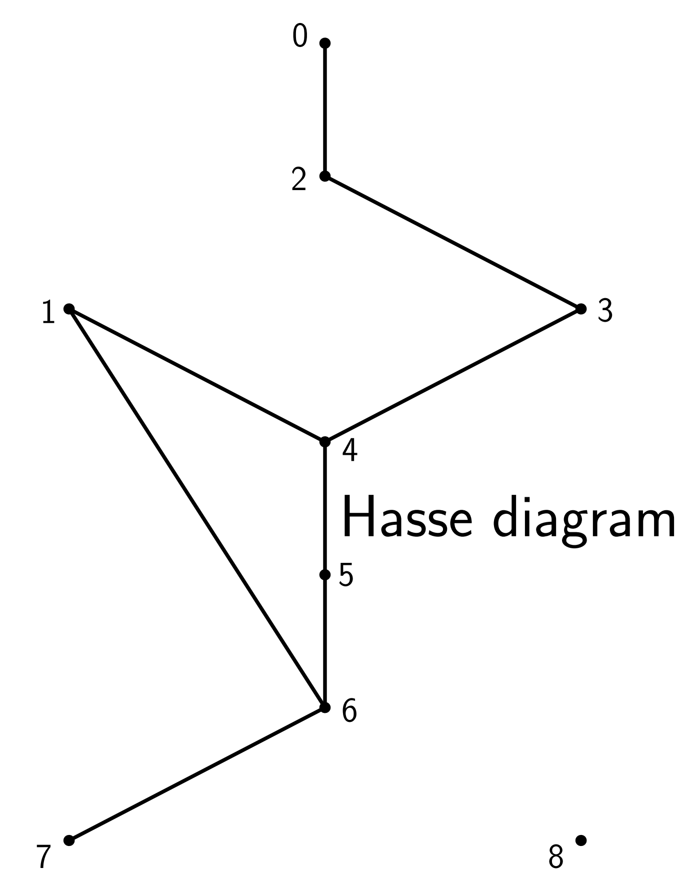
Simo 1988 (*) and 1992 (**)

Multiple-point constraints (*control, ALE repositioning, ...*)

$$\boldsymbol{T}_\star^T \left(\sum_{i=1}^{n_e} \boldsymbol{K}_i^e \right) \boldsymbol{T}_\star + \boldsymbol{T}_\star^T \left\{ - \sum_{j=1}^{m} \left[\left(\sum_{k=1}^{n_e} \boldsymbol{f}_k^e \right)^T (\boldsymbol{c}_j \boldsymbol{g}_j'') \right] \right\} \boldsymbol{T}_\star \mathrm{d}\boldsymbol{a}_r \quad = \quad - \boldsymbol{T}_\star^T \left(\sum_{j=1}^{n_e} \boldsymbol{f}_j^e \right) - \boldsymbol{T}_\star^T \left(\sum_{l=1}^{n_e} \boldsymbol{K}_{\boldsymbol{j}}^{\boldsymbol{e}} \right) \boldsymbol{b}_\star$$

Very hard to implement efficiently - a combination of clique and sparse data structures





Acyclic test
Topological sort

	Unroll of nested DOFs	Collapse (and sum)
$1:1$	$1:\mathbf{1}$	$1:1$
$2:0$	$2:\mathbf{0}$	$2:0$
$3:2$	$3:2,\mathbf{0}$	$3:0$
$4:1,3$	$4:1,\mathbf{1}\vert 3,\mathbf{0}$	$4:1,0$
$5:3,4,2$	$5:3,\mathbf{0}\vert 4,\mathbf{1}\vert 4,\mathbf{0}\vert 2,\mathbf{0}$	$5:0,1$
$6:2,1,5$	$6:2,\mathbf{0}\vert 1,\mathbf{1}\vert 5,\mathbf{0}\vert 5,\mathbf{1}\vert 5,\mathbf{0}\vert 5,\mathbf{0}$	$6:0,1$
$7:6$	$7:6,\mathbf{0}\vert 6,\mathbf{1}\vert 6,\mathbf{0}\vert 6,\mathbf{1}\vert 6,\mathbf{0}\vert 6,\mathbf{0}$	$7:0,1$
$8:8$	$8:\mathbf{8}$	$8:8$

Degree-of-freedom:list of masters

Mixed elements

Given $t \in \mathbb{R}_0^+$ $\boldsymbol{t} \in [L^2(\Gamma_{0t}^N)]^2$ and $\boldsymbol{b} \in [L^2(\Omega_{0t})]^2$, find $\boldsymbol{u} \in \left[H^1(\Omega_{0t})\right]^2$ (with non-homogeneous boundary conditions on Γ_{0t}^D) and $p \in L^2(\Omega_{0t})$ such that $\forall \widetilde{\boldsymbol{u}} \in \left[H^1(\Omega_{0t})\right]^2$ (with homogeneous boundary conditions on Γ_{0t}^D) and $\forall \widetilde{\theta} \in L^2(\Omega_{0t})$:

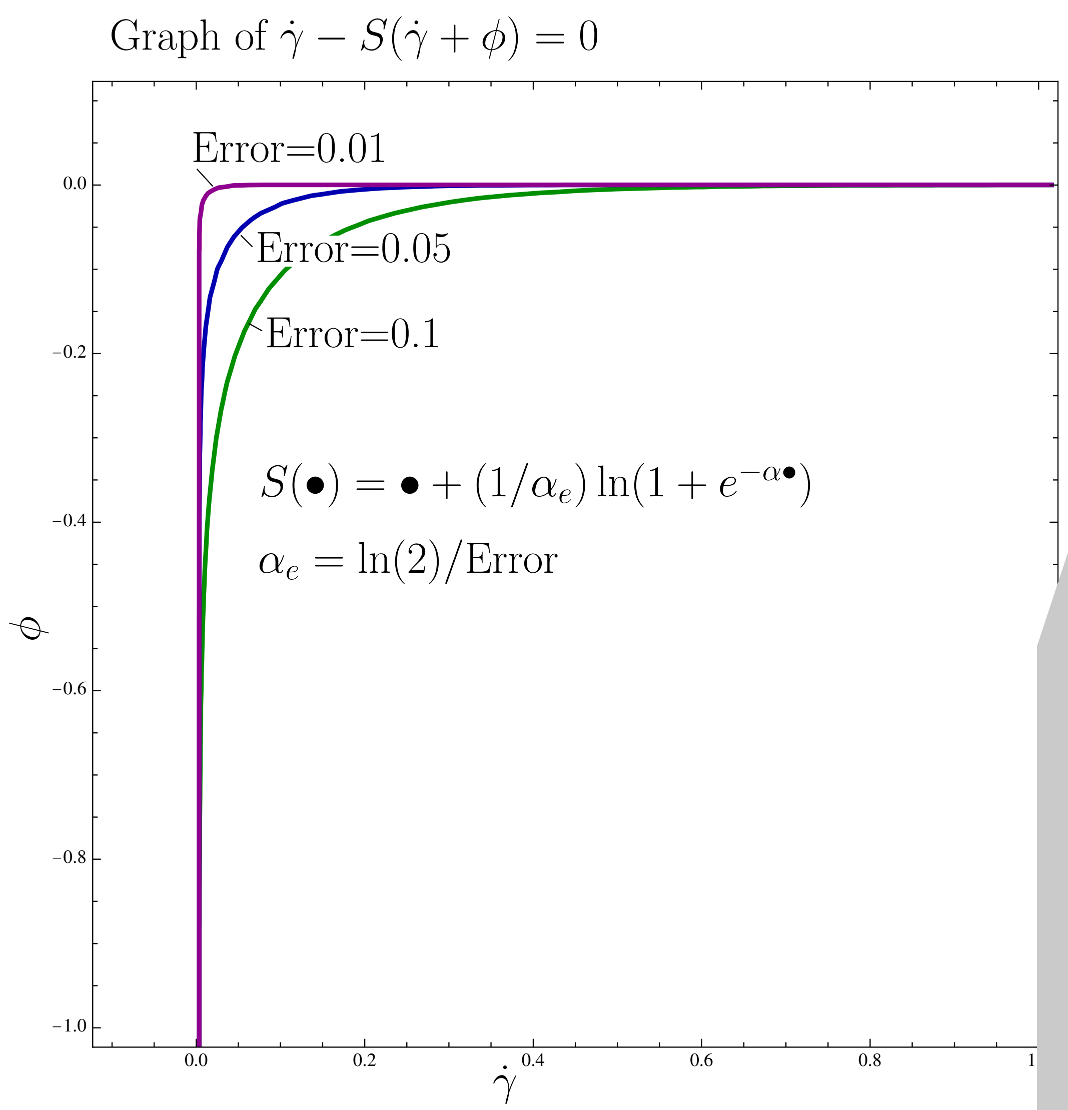
$$\begin{aligned}
\int_{\Omega_{0t}} \{\mathscr{P} : \boldsymbol{\tau}_c\left[\boldsymbol{F}(\boldsymbol{u}), t\right] + p\boldsymbol{I}\} : \nabla\widetilde{\boldsymbol{u}}\, \mathrm{d}\Omega_{0t} &= \int_{\Gamma_{0t}^N} \boldsymbol{t} \cdot \widetilde{\boldsymbol{u}}\, \mathrm{d}\Gamma_{0t} + \int_{\Omega_{0t}} \boldsymbol{b} \cdot \widetilde{\boldsymbol{u}}\, \mathrm{d}\Omega_{0t} \\
\int_{\Omega_{0t}} \left\{\frac{1}{3}\boldsymbol{\tau}_c : \boldsymbol{I} - p\right\} \widetilde{\theta}\, \mathrm{d}\Omega_{0t} &= 0
\end{aligned}$$

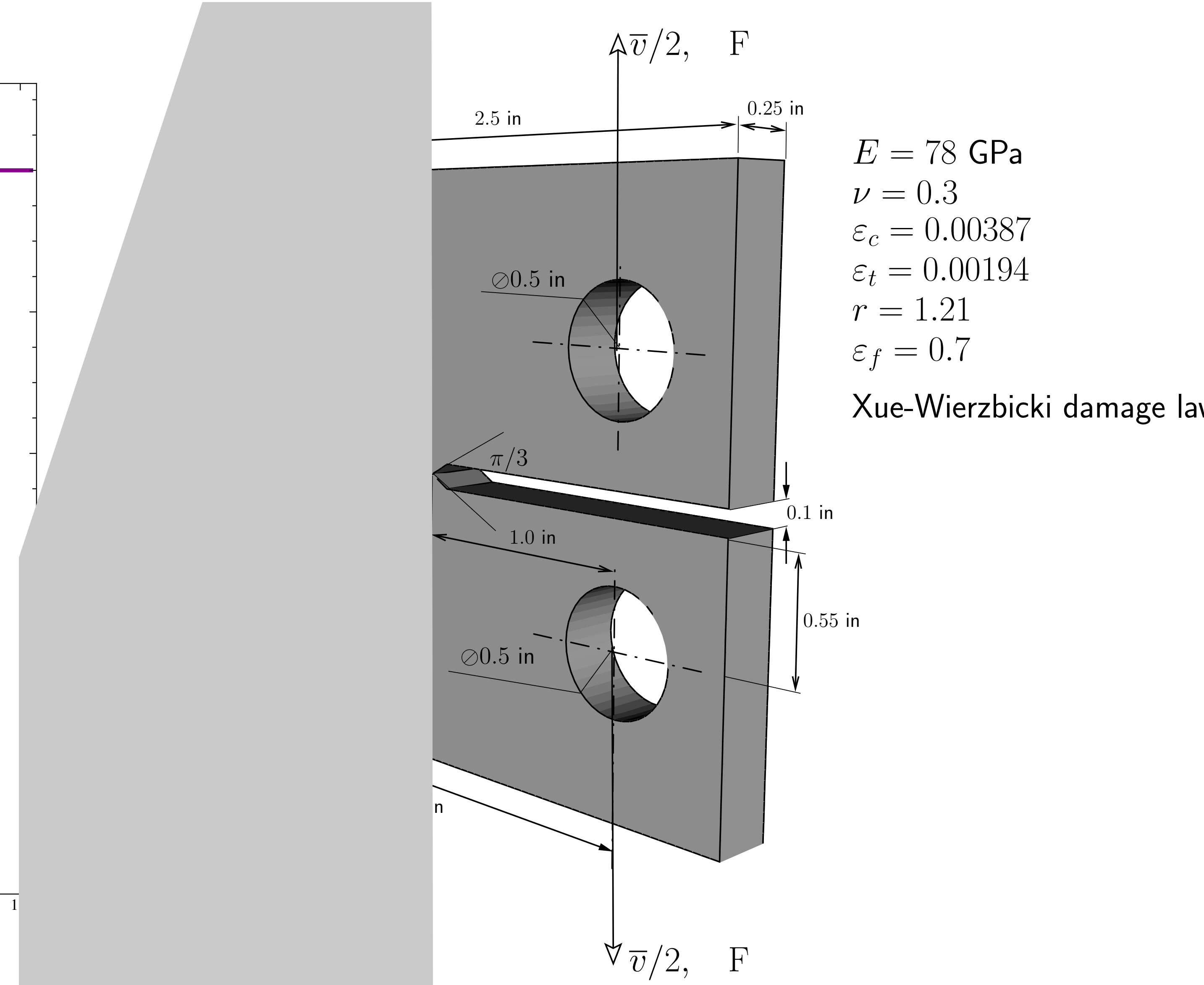
$$\int_{\Omega_{0t}} \{(\mathscr{P} : \dot{\boldsymbol{\tau}}_c) : \nabla\widetilde{\boldsymbol{u}} - (\mathscr{P} : \boldsymbol{\tau}_c + p\boldsymbol{I}) : (\nabla\widetilde{\boldsymbol{u}}\nabla\dot{\boldsymbol{u}})\}\, d\Omega_{0t} + \int_{\Omega_{0t}} \dot{p}\boldsymbol{I} : \nabla\widetilde{\boldsymbol{u}}\, \mathrm{d}\Omega_{0t} = \delta\dot{W}_{ut}$$

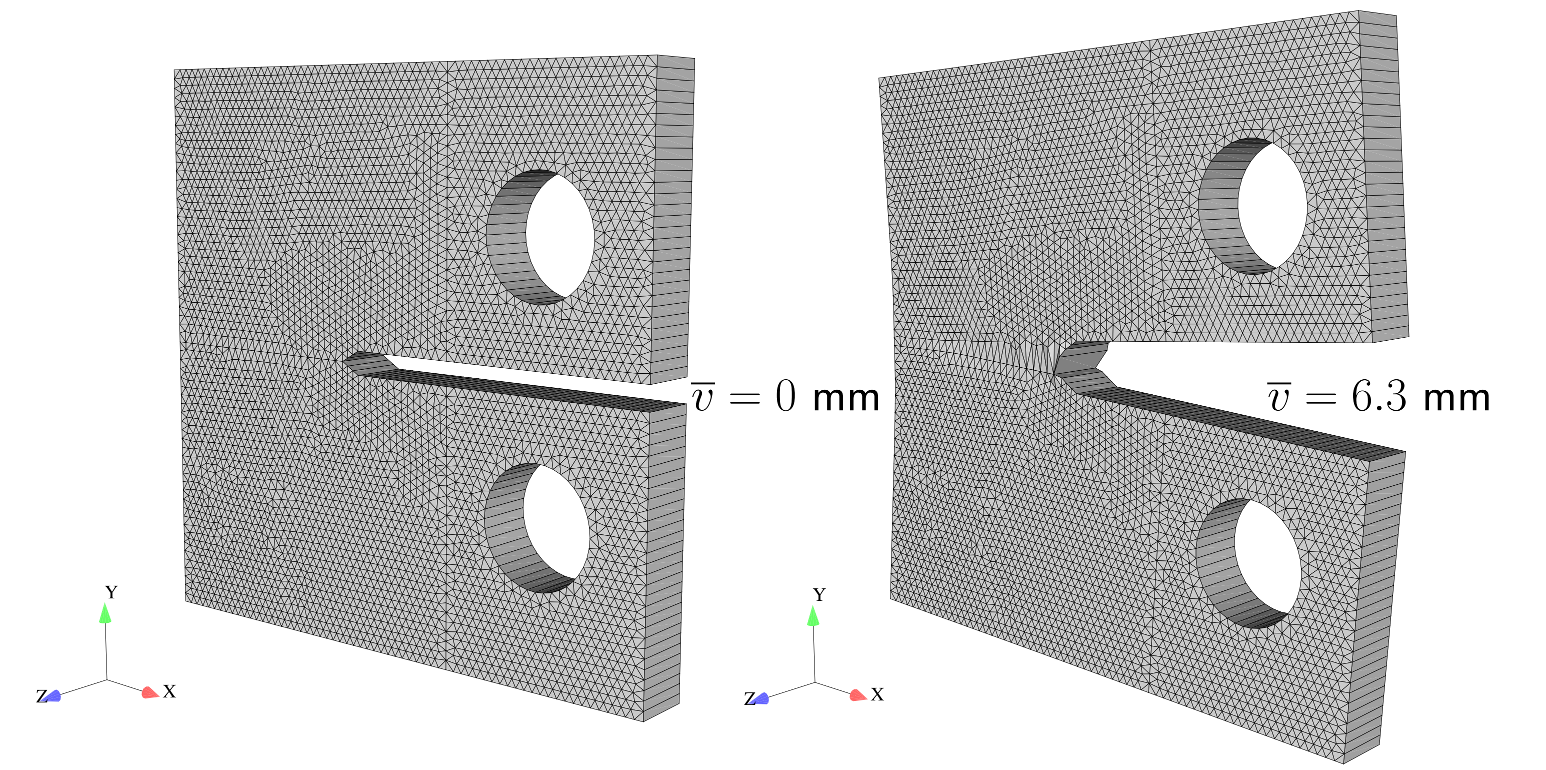
$$\int_{\Omega_{0t}} \left[\frac{1}{3}\boldsymbol{I} : \left(\mathscr{C} : \nabla\dot{\boldsymbol{u}} + \boldsymbol{\tau}_c\nabla\dot{\boldsymbol{u}}^T + \nabla\dot{\boldsymbol{u}}\boldsymbol{\tau}_c\right) - \dot{p}\right] \widetilde{\theta}\mathrm{d}\Omega_{0t} = \delta\dot{W}_{pt}$$

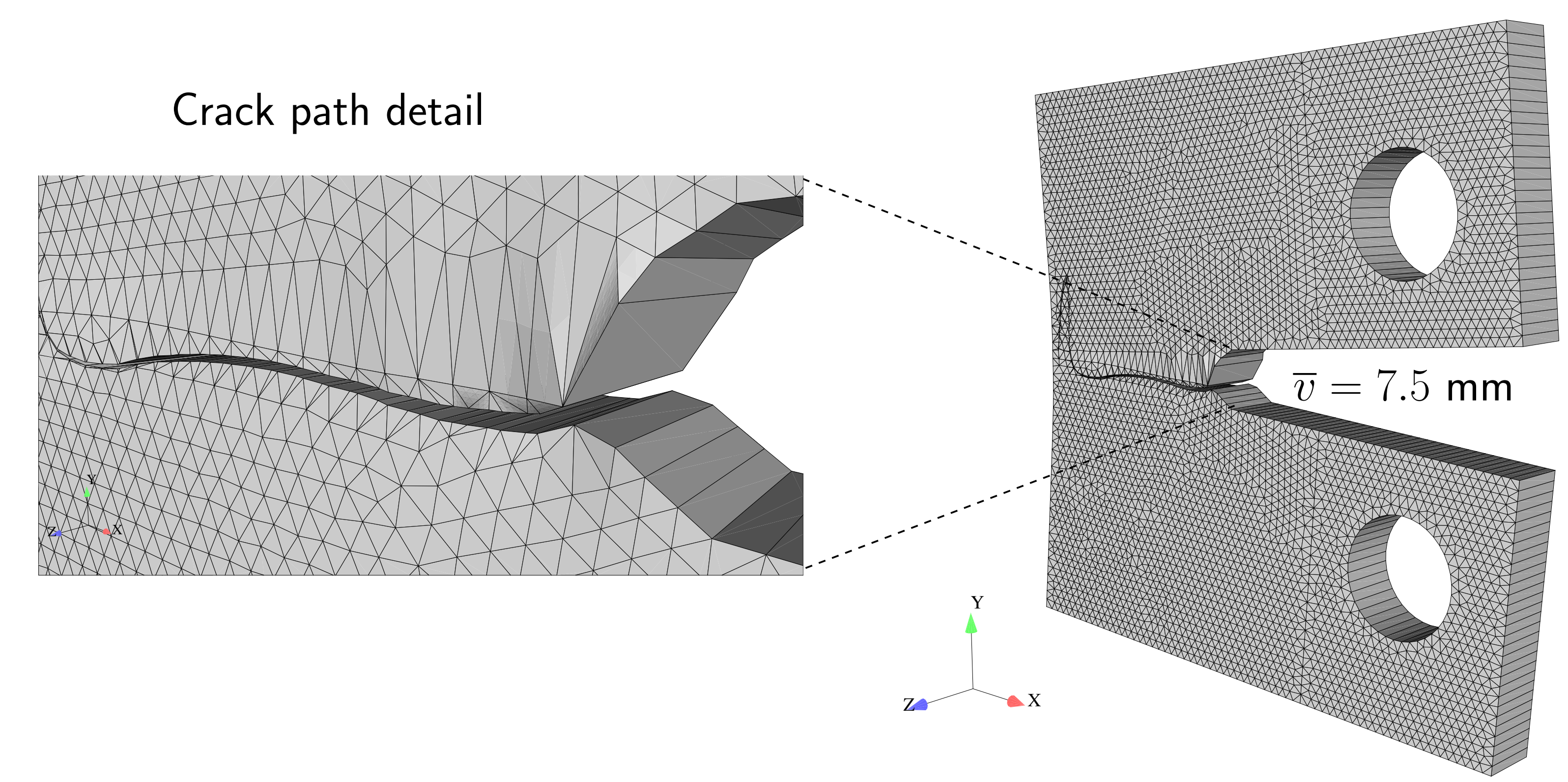
Technology: bubble displacement linear pressure

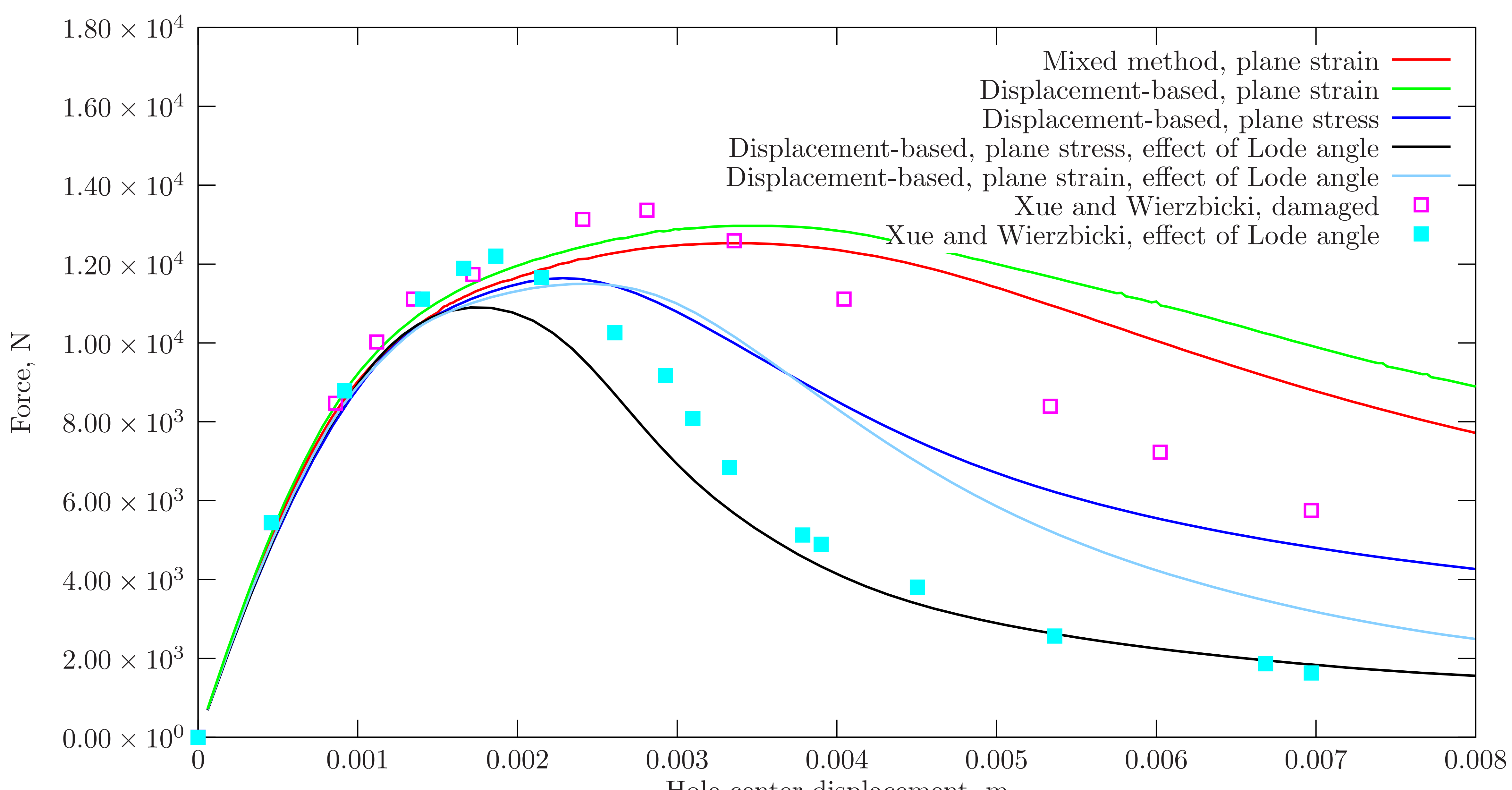
Complementarity smoothed and the compact tension test







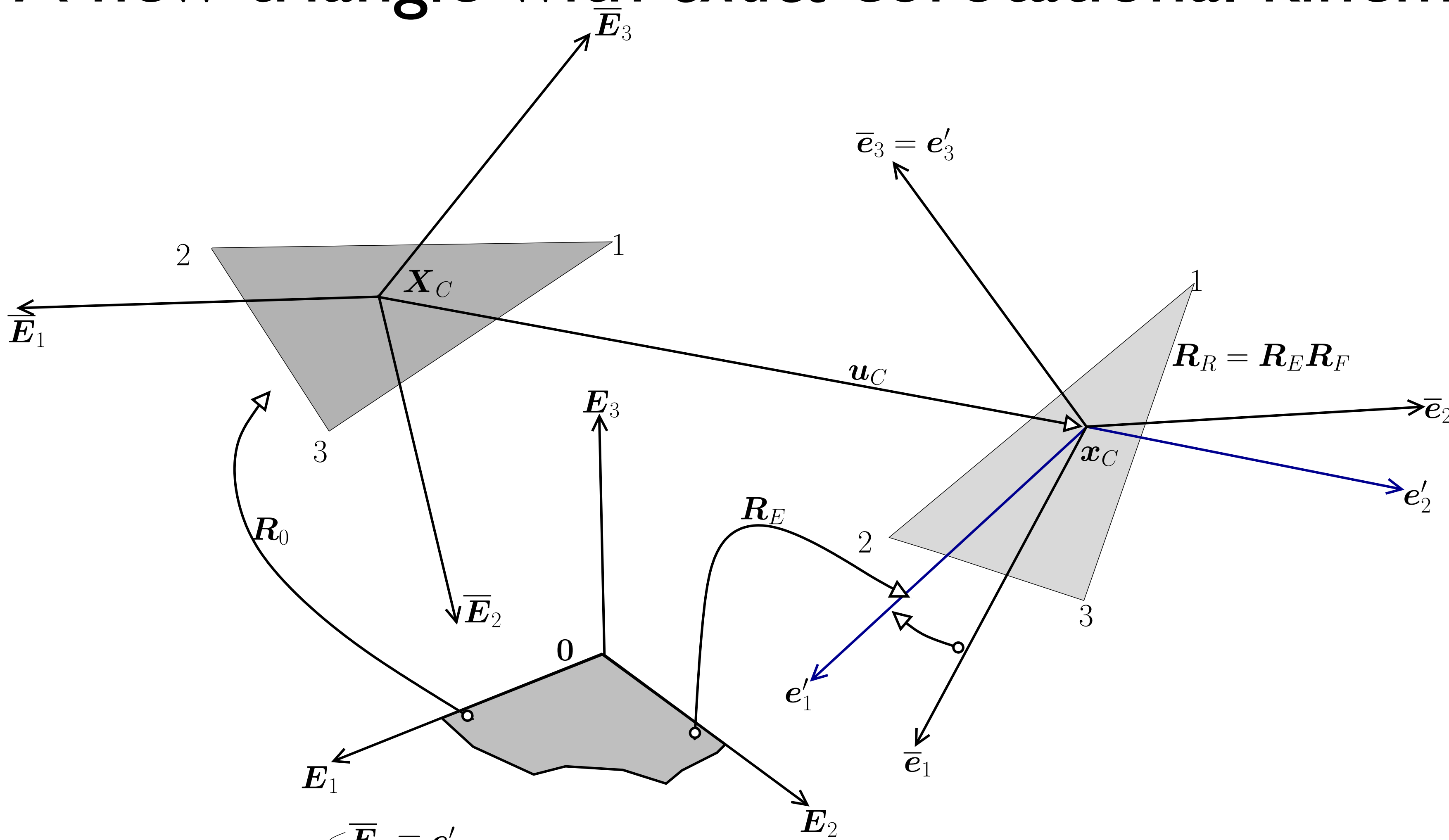


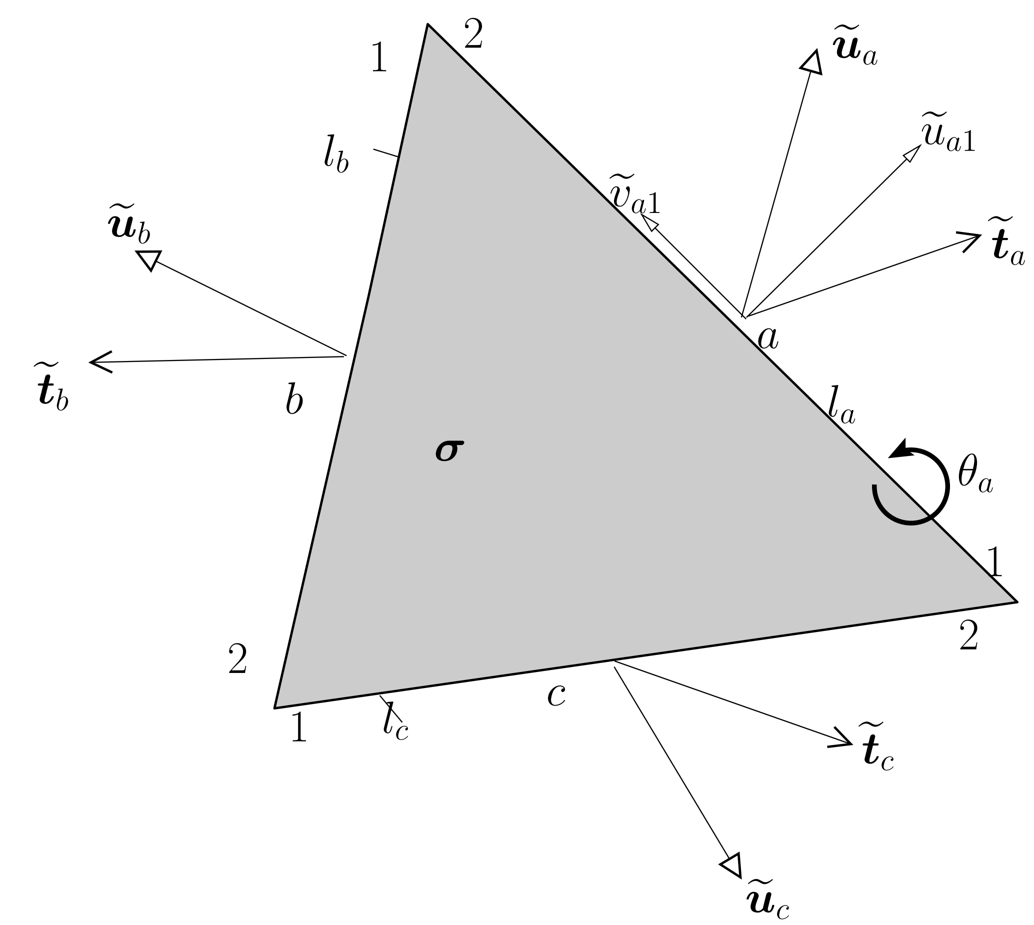


Exceptionally <u>accurate</u> results with thickness variation

Shells

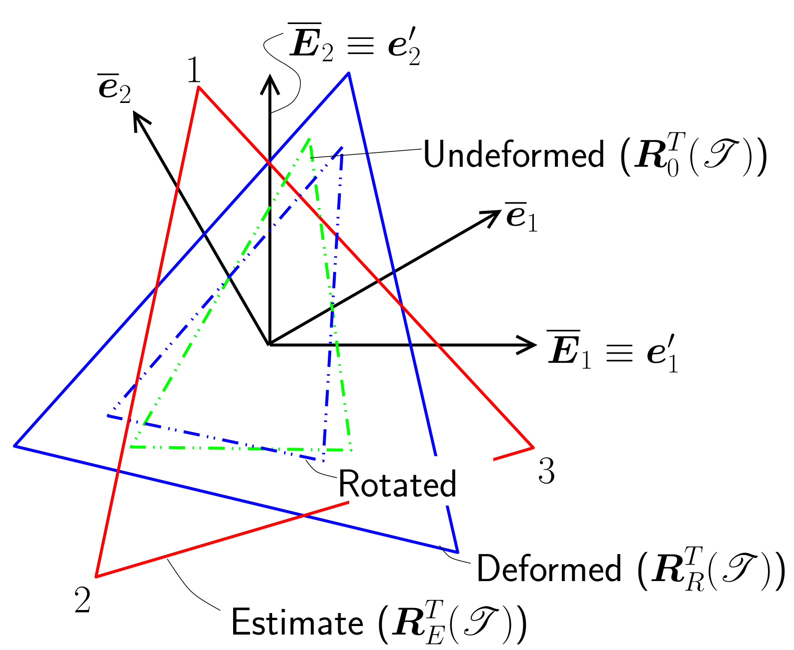
A new triangle with *exact* corotational kinematics for FeFp plasticity



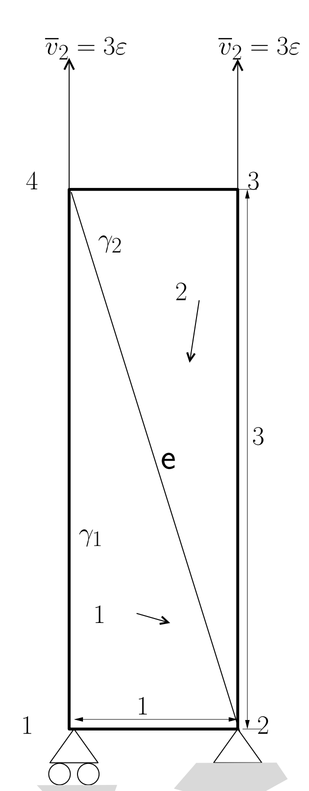


Drilling freedoms

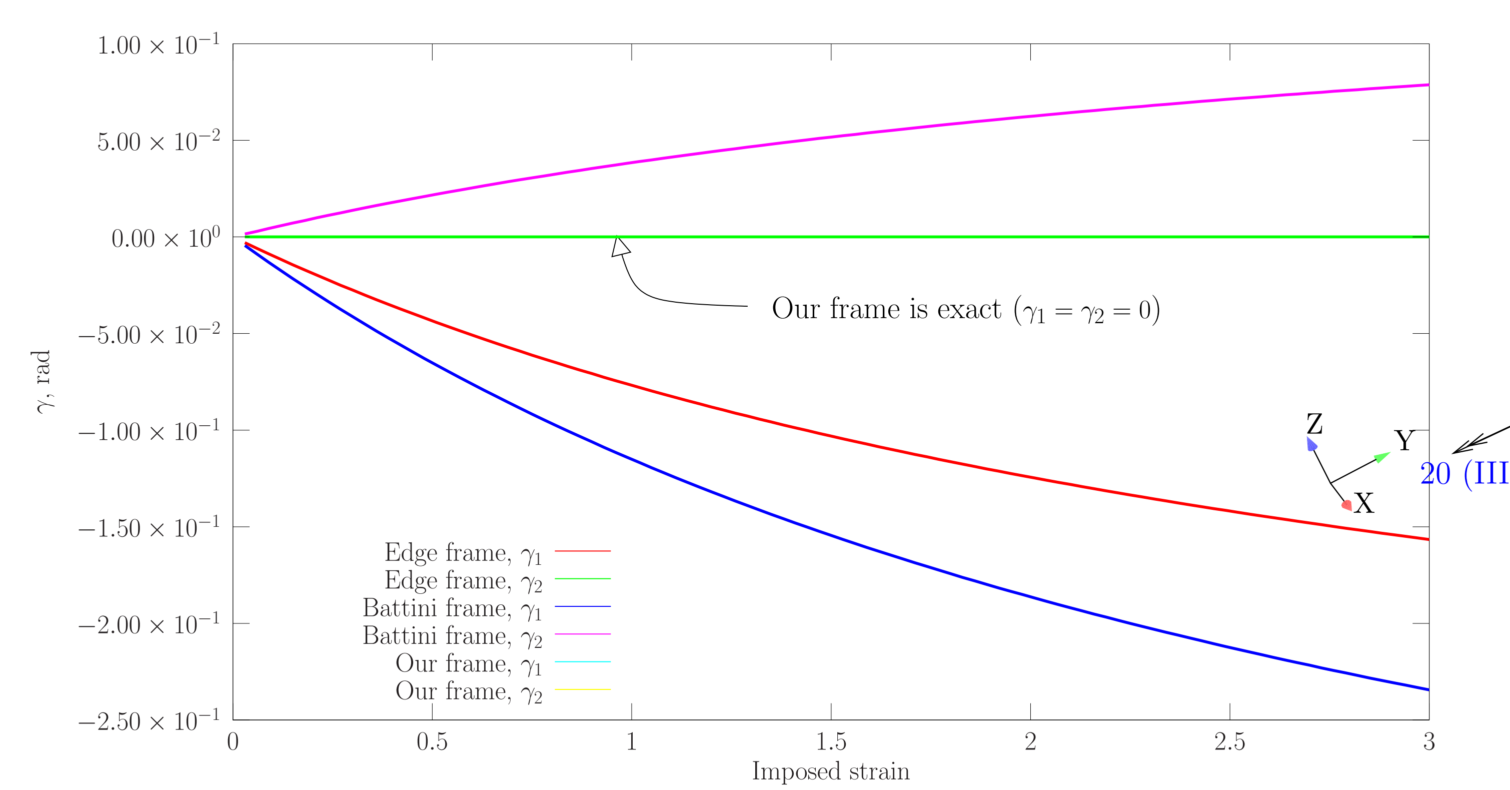
Patch and invariance tests



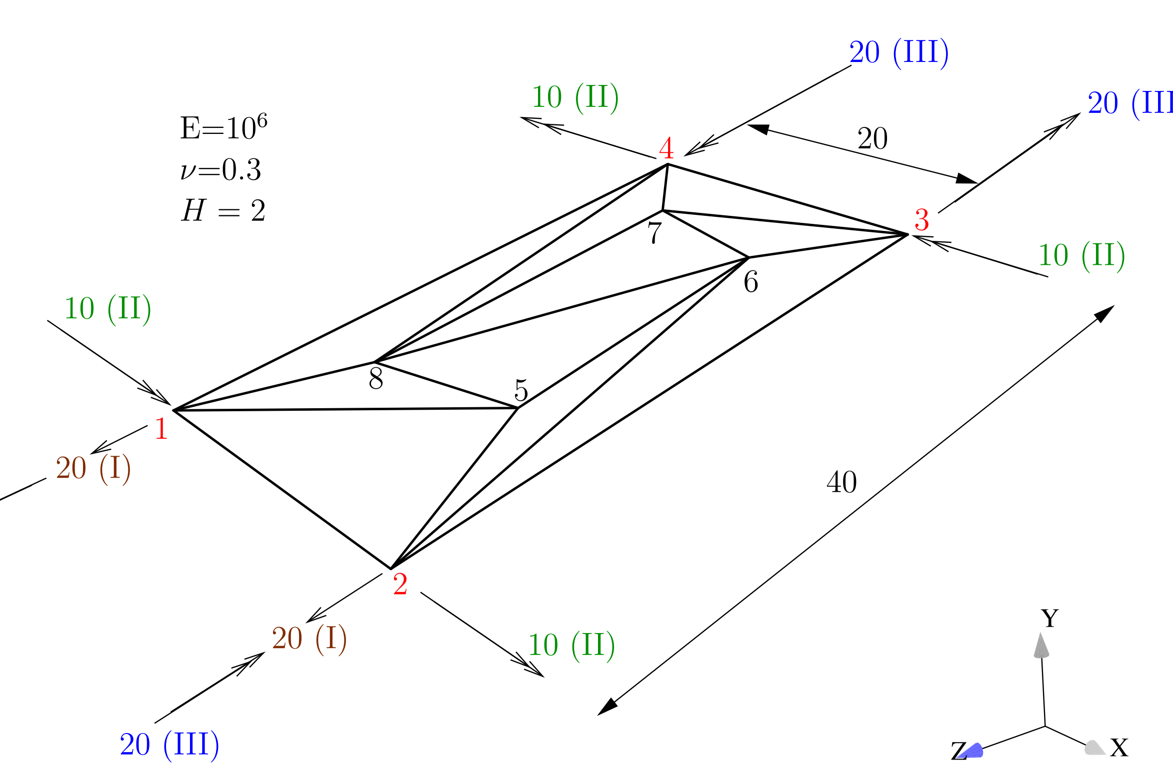
Frames



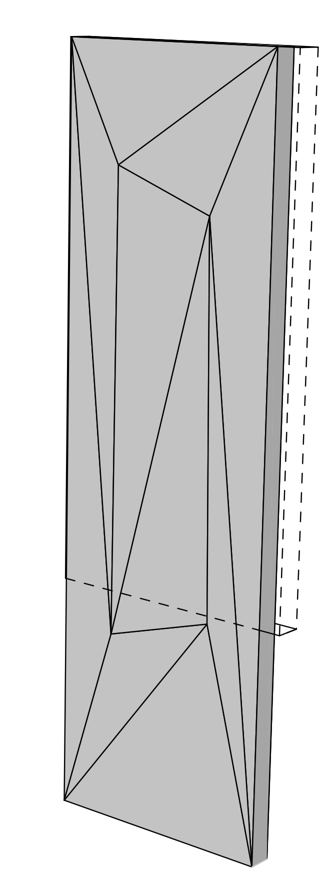
Connectivities: Element 1: 2,4,1
Element 2: 2,3,4



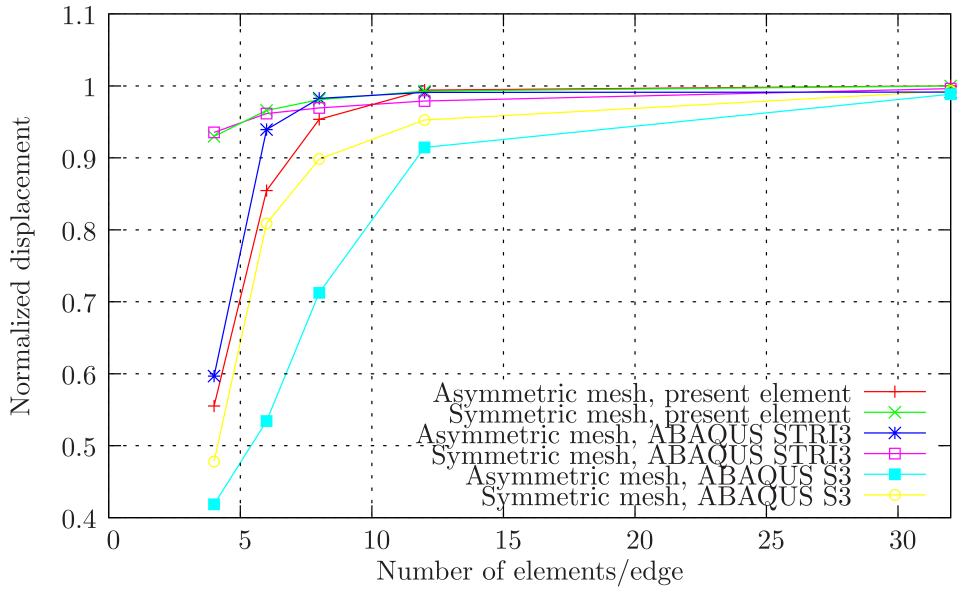
Small strain patch test

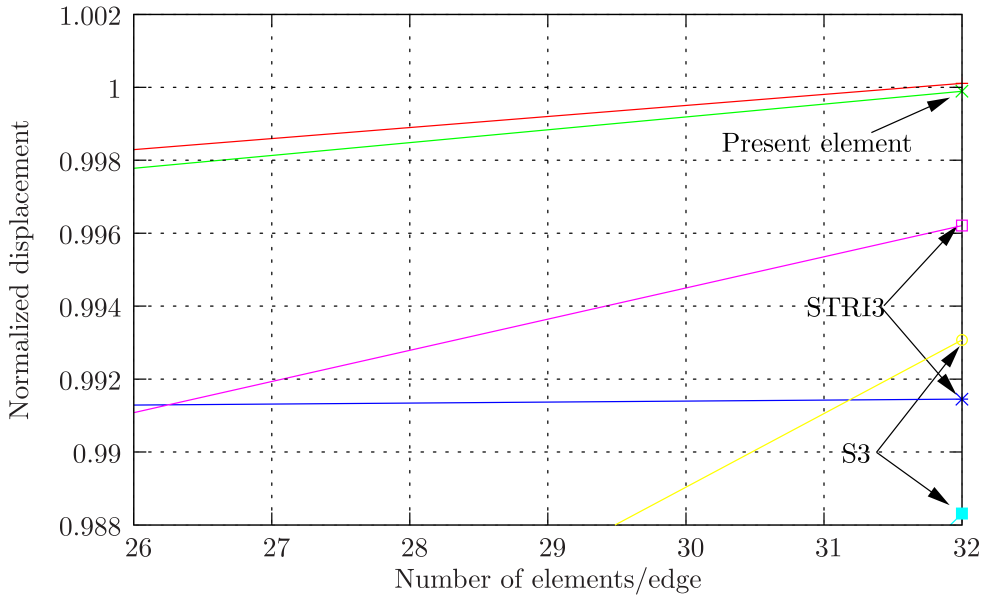


Finite strain patch test

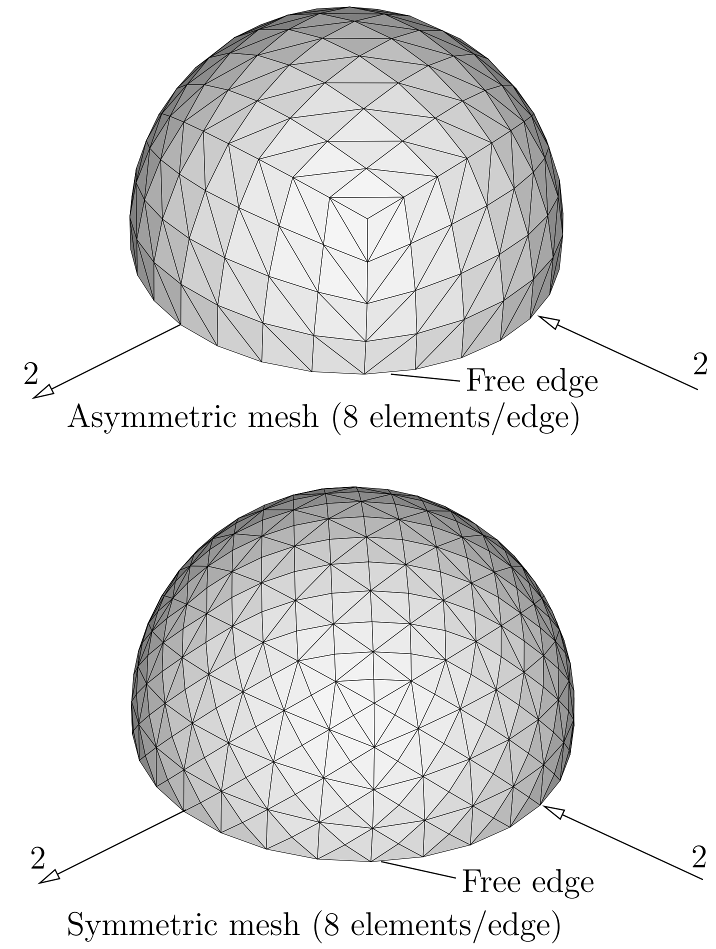


Benchmark results



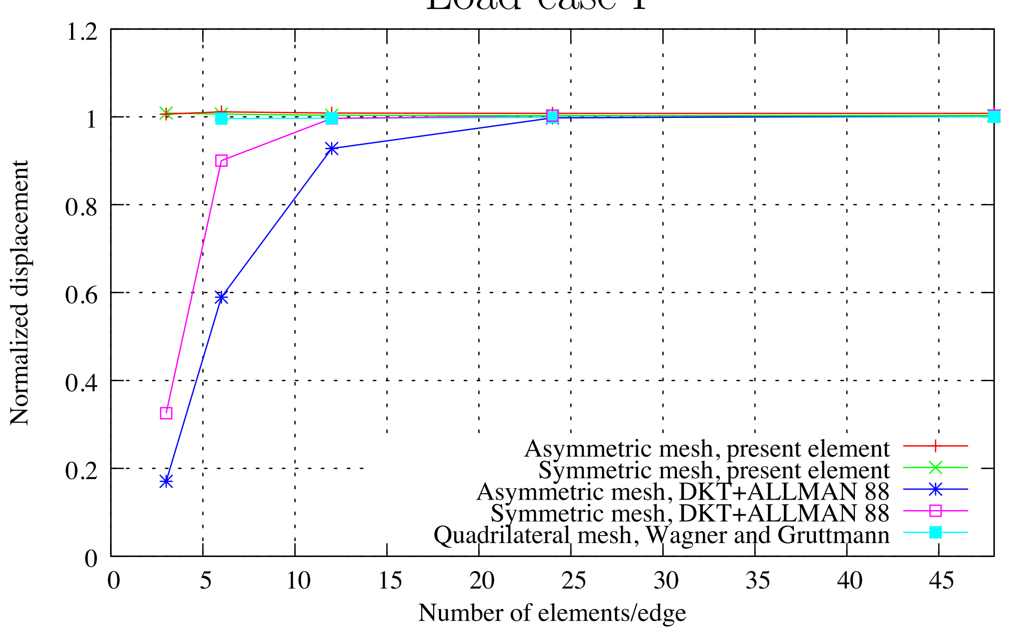


R=10 E=6.825 × 10^7
h=0.04 ν = 0.3

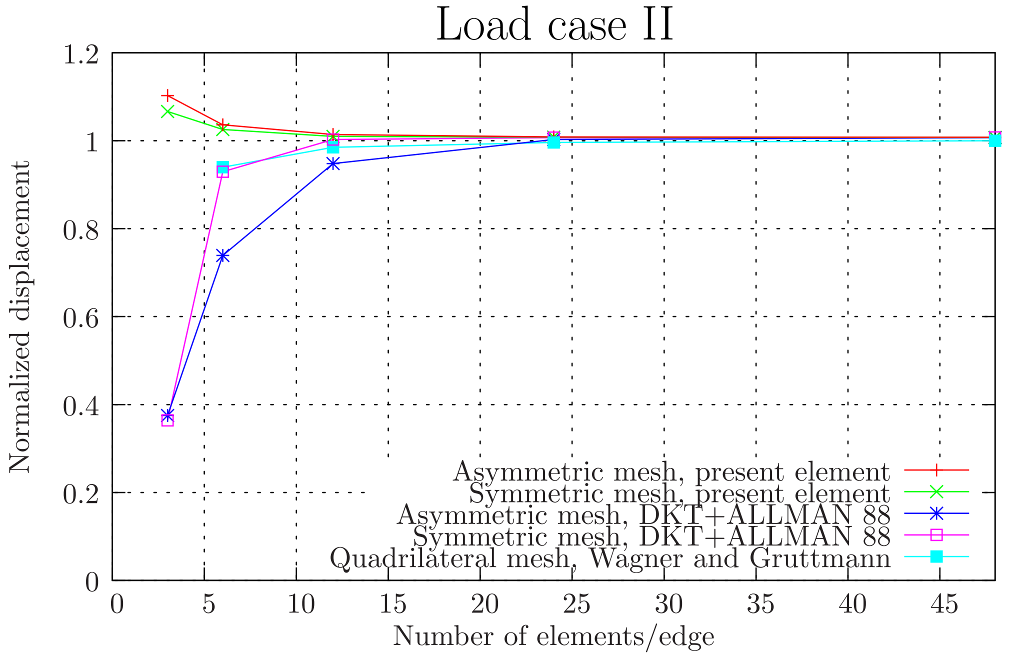


Target displacement=9.24 × 10^{-2}
(Consistent units are used)

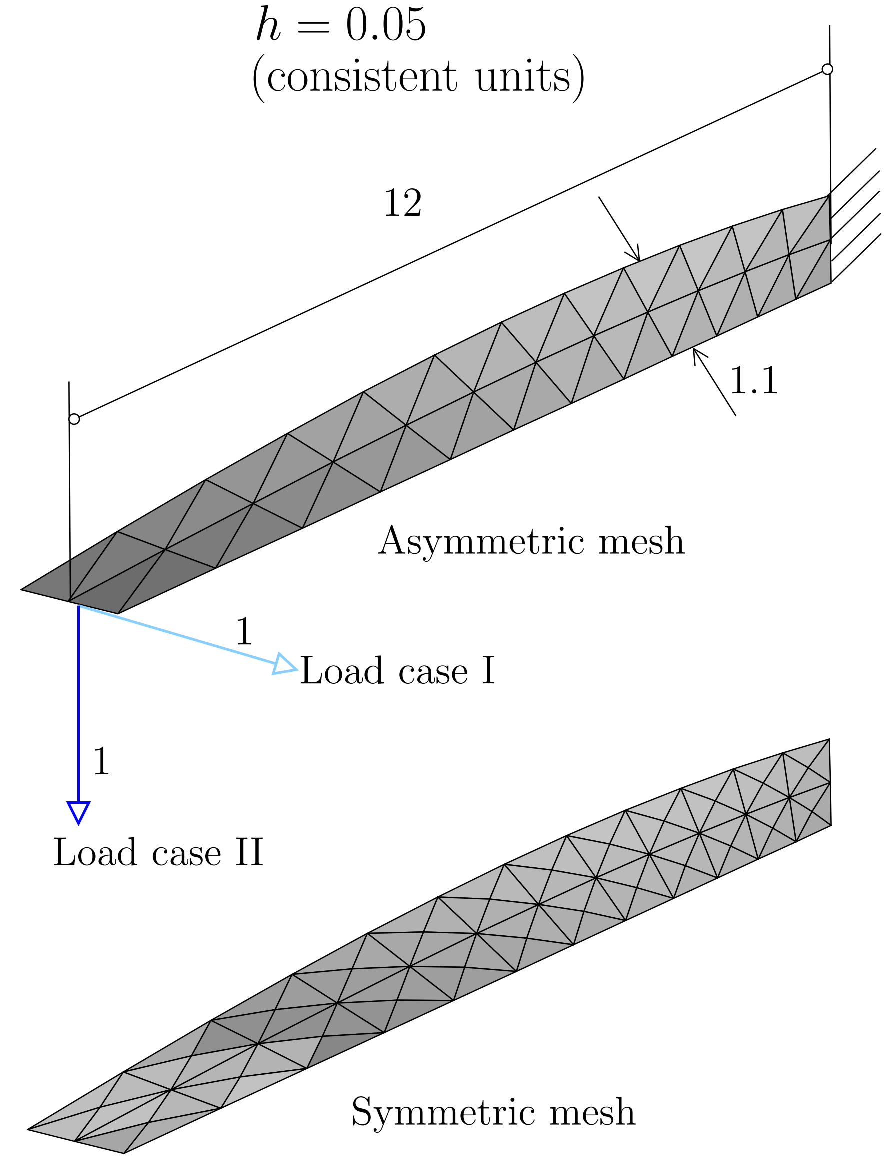
Load case I

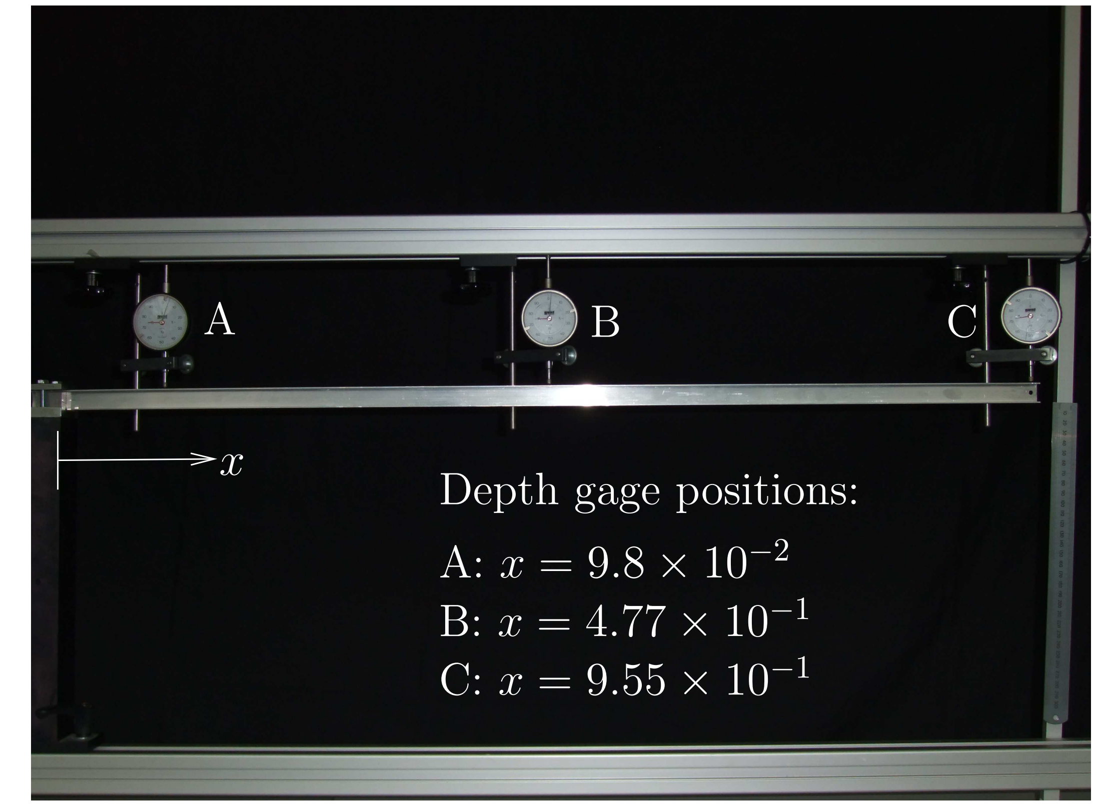


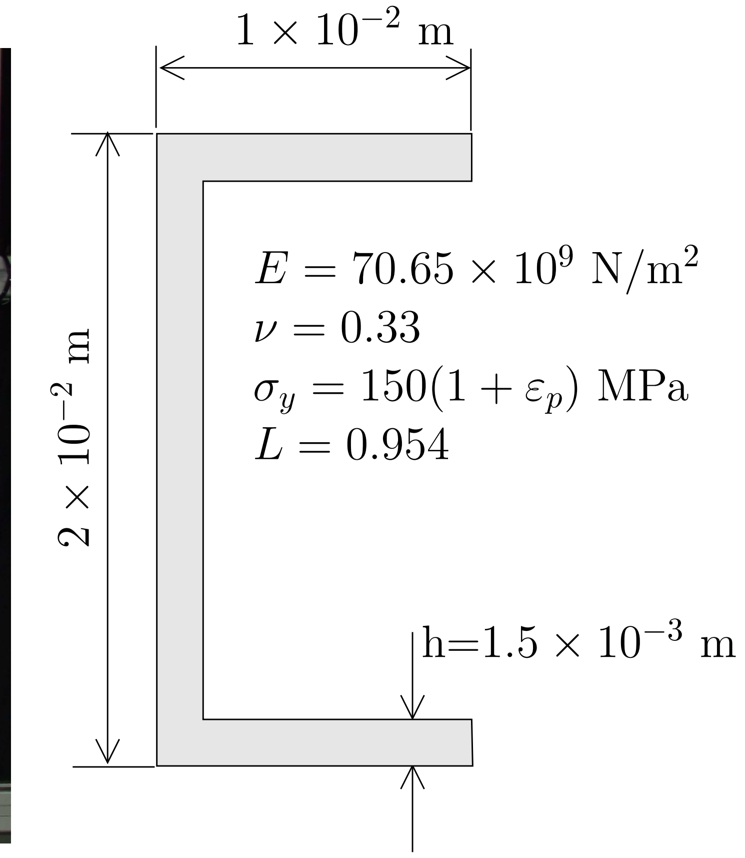
Load case II



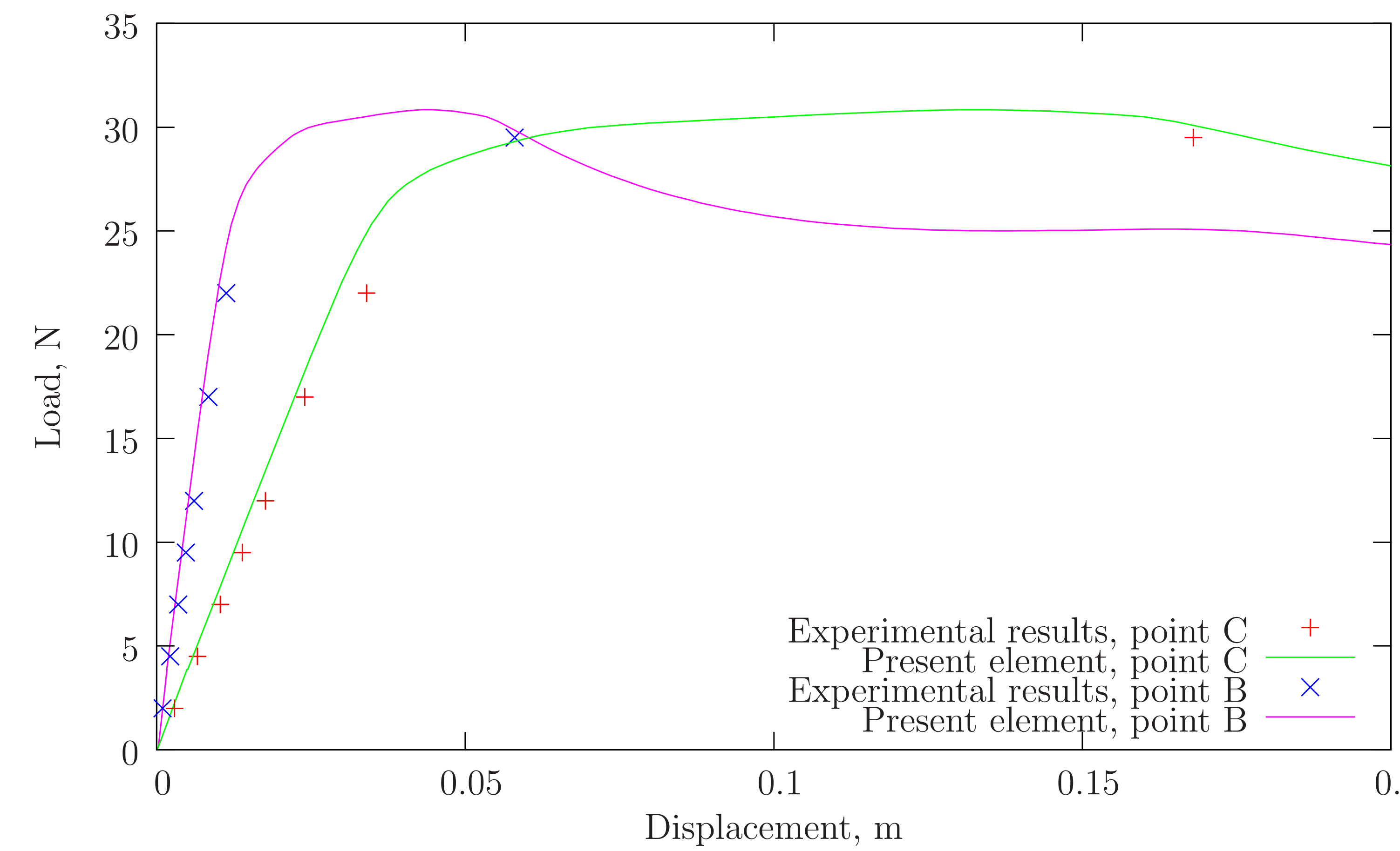
E=29 × 10^6
$\nu = 0.22$
$h = 0.05$
(consistent units)





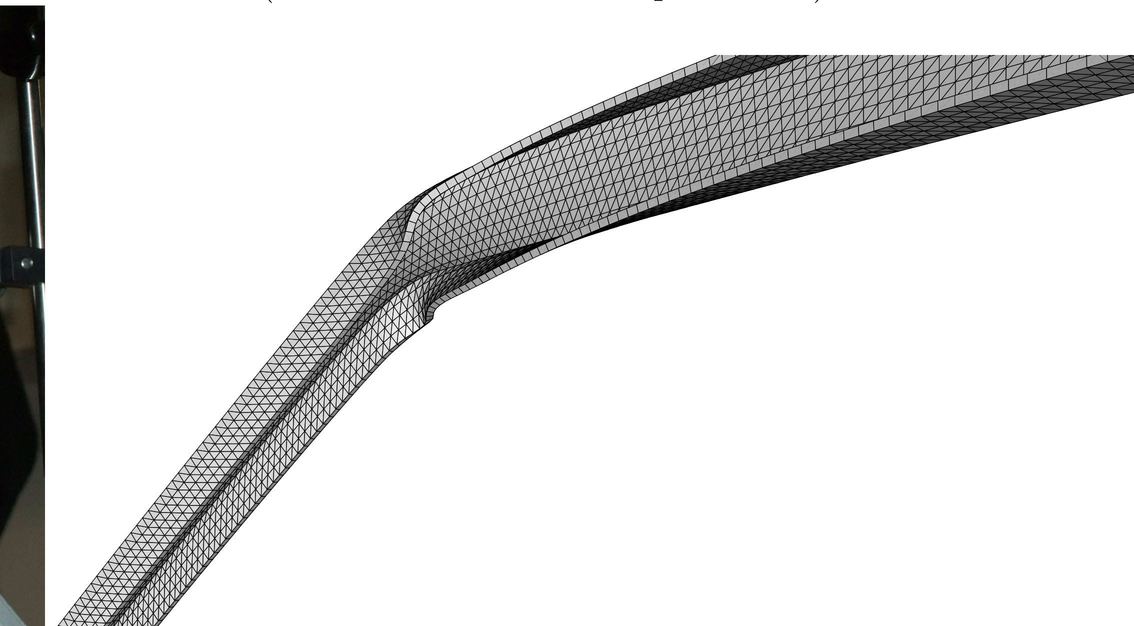


Mesh: 4 × 8 × 400 elements
Asymmetric arrangement is adopted

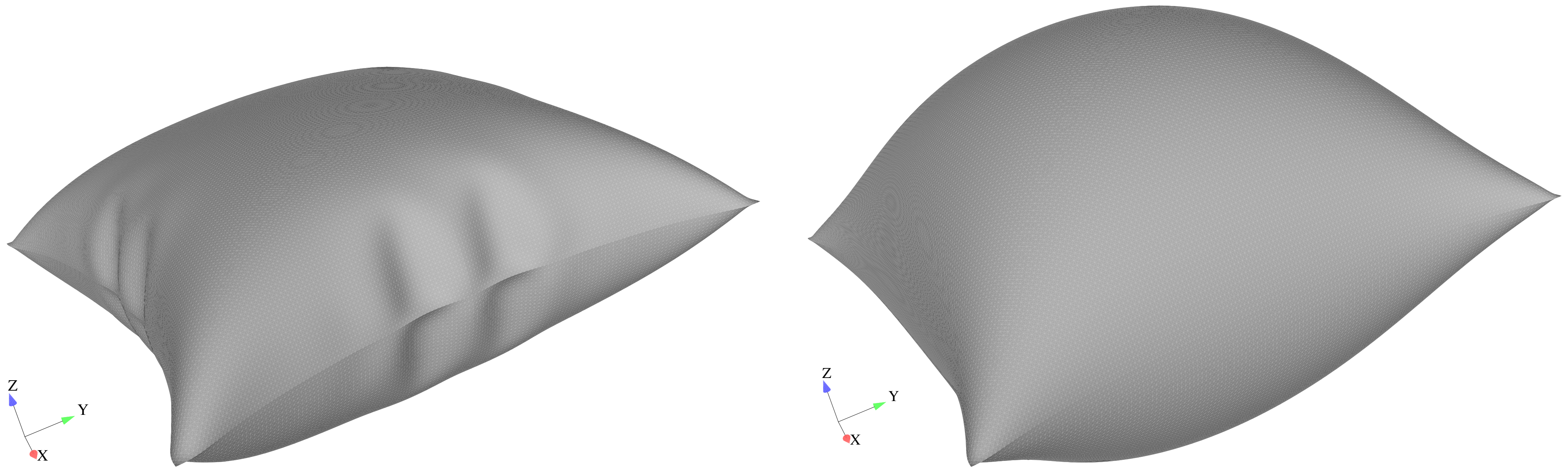


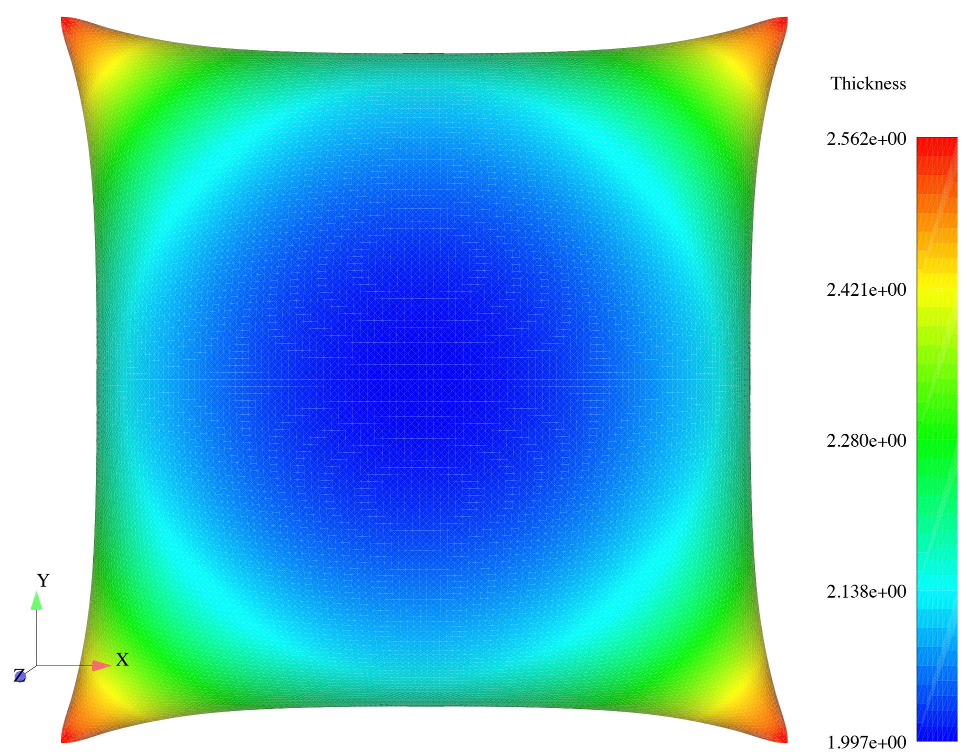
Experimentally observed elasto-plastic flange buckling

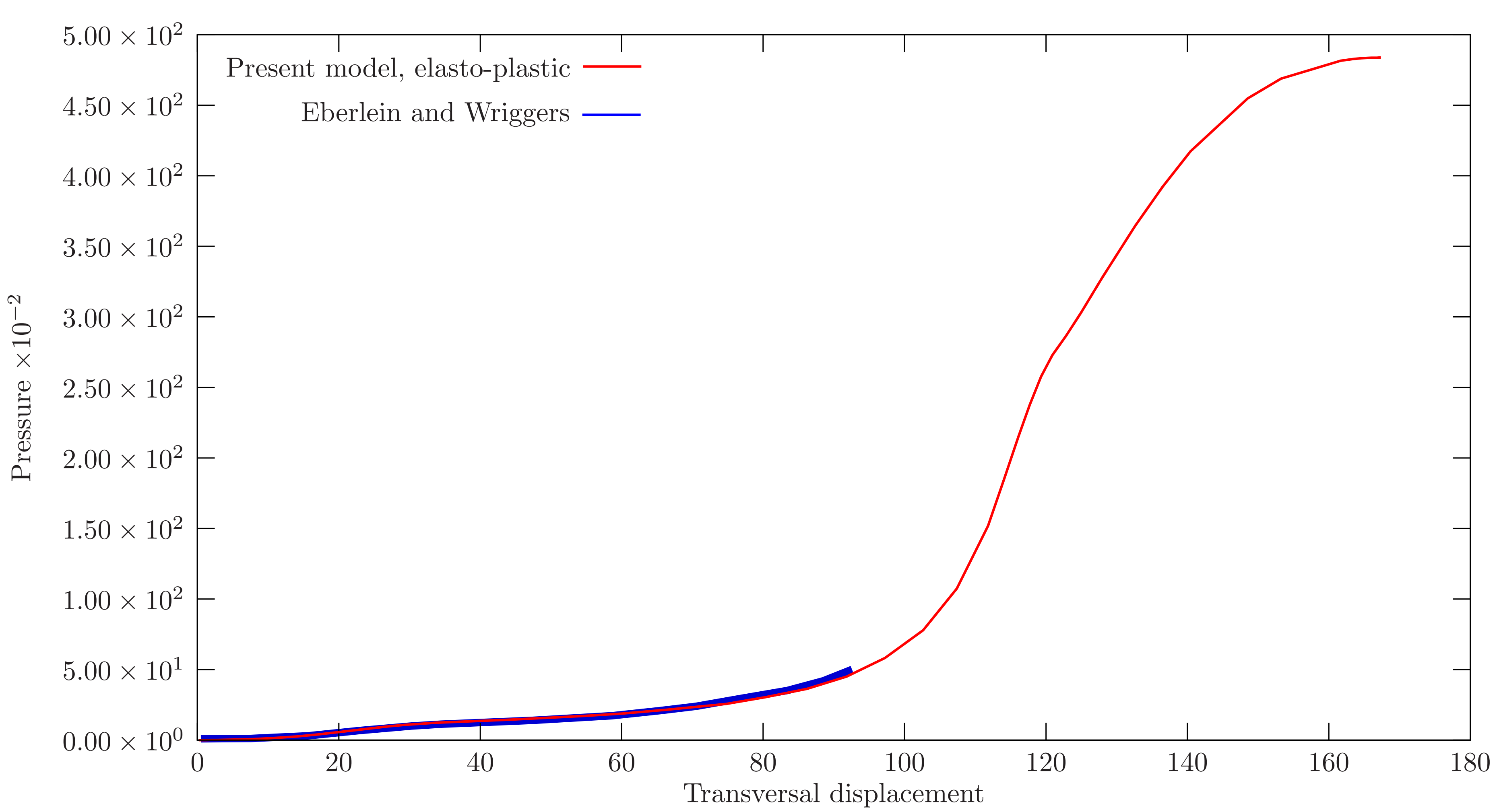
Numerically obtained elasto-plastic flange buckling
(thickness extrusion was performed)

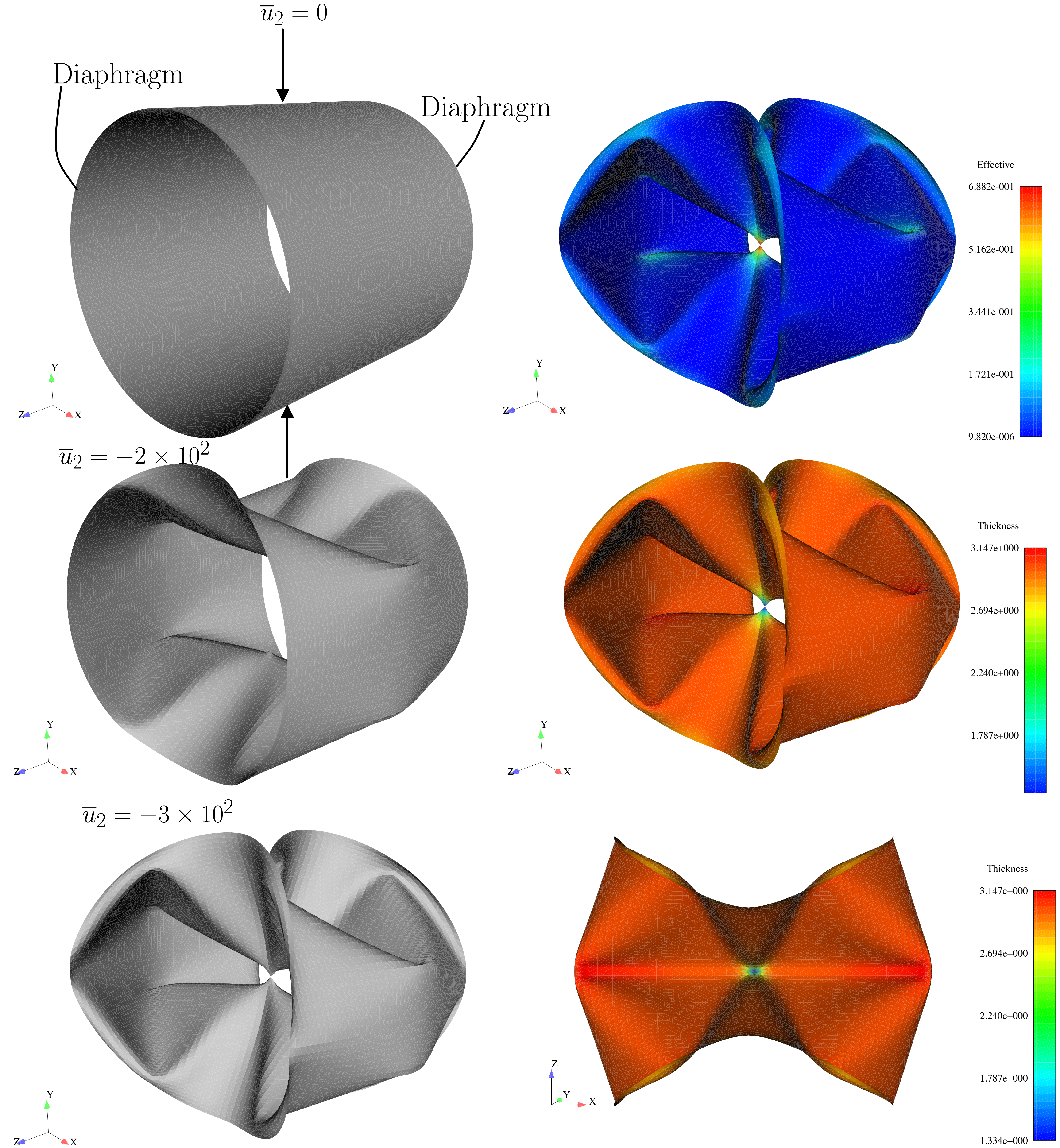


Much larger deformations than what was reported previously in the literature

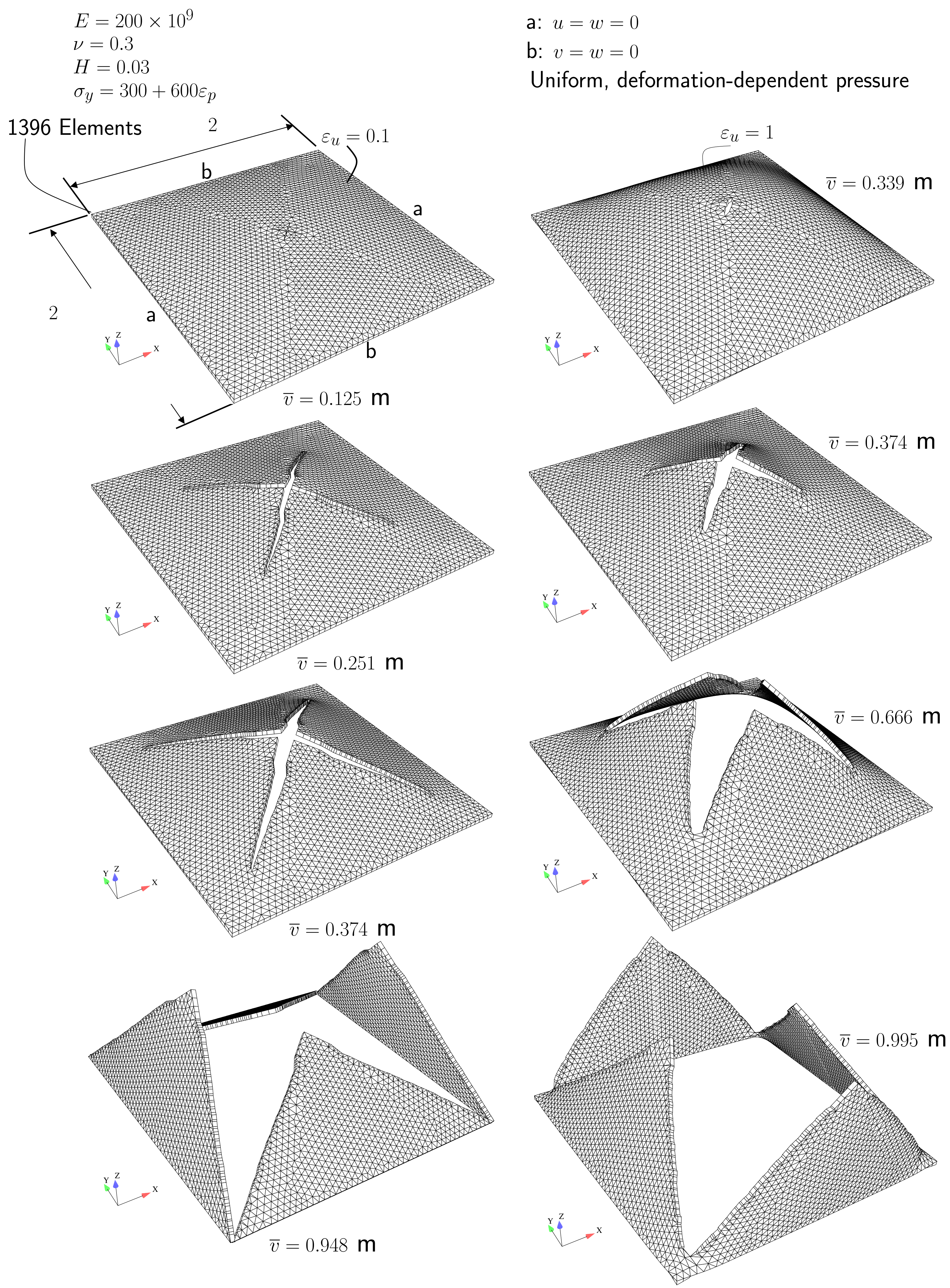








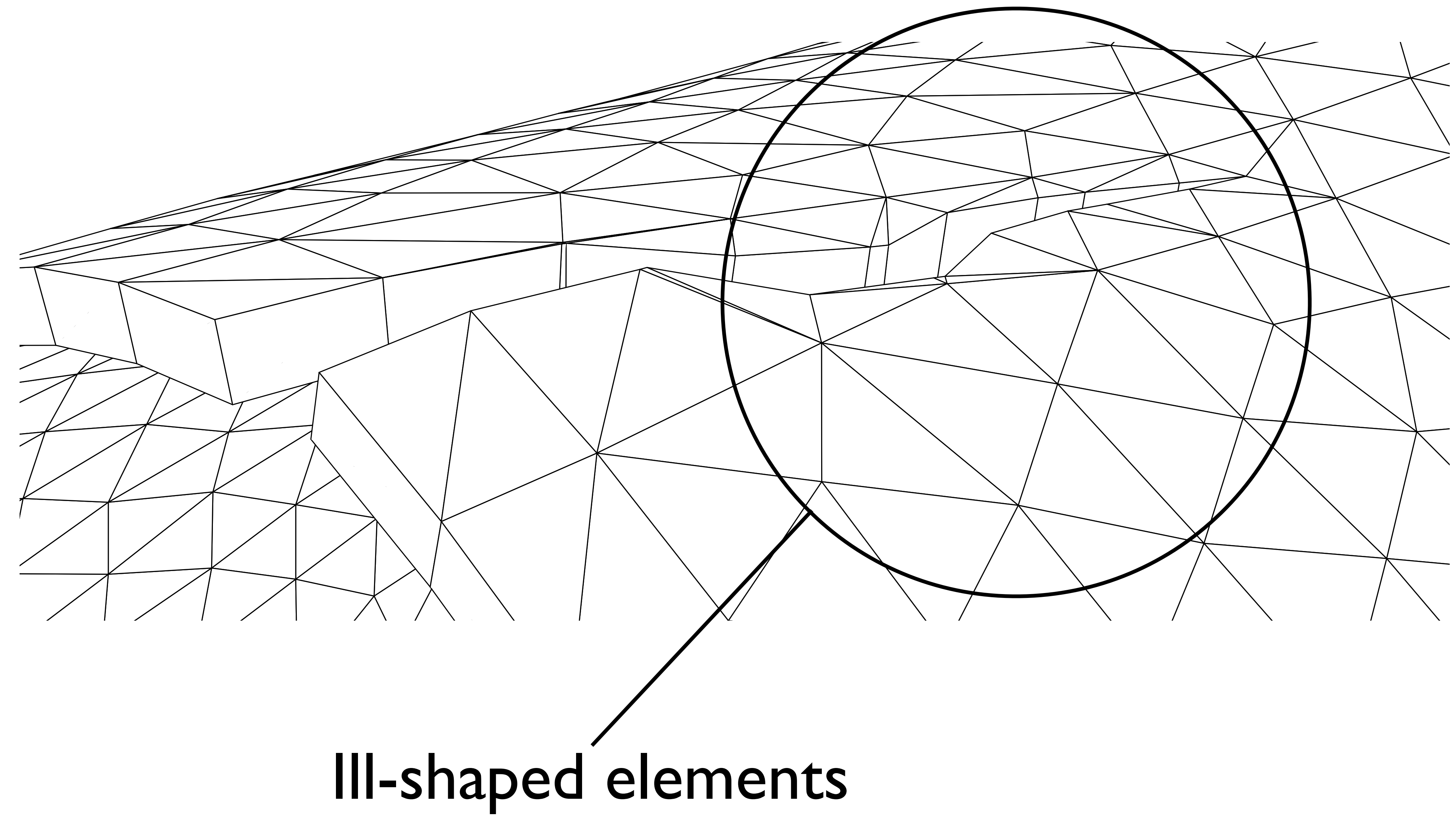
And *plate* fracture



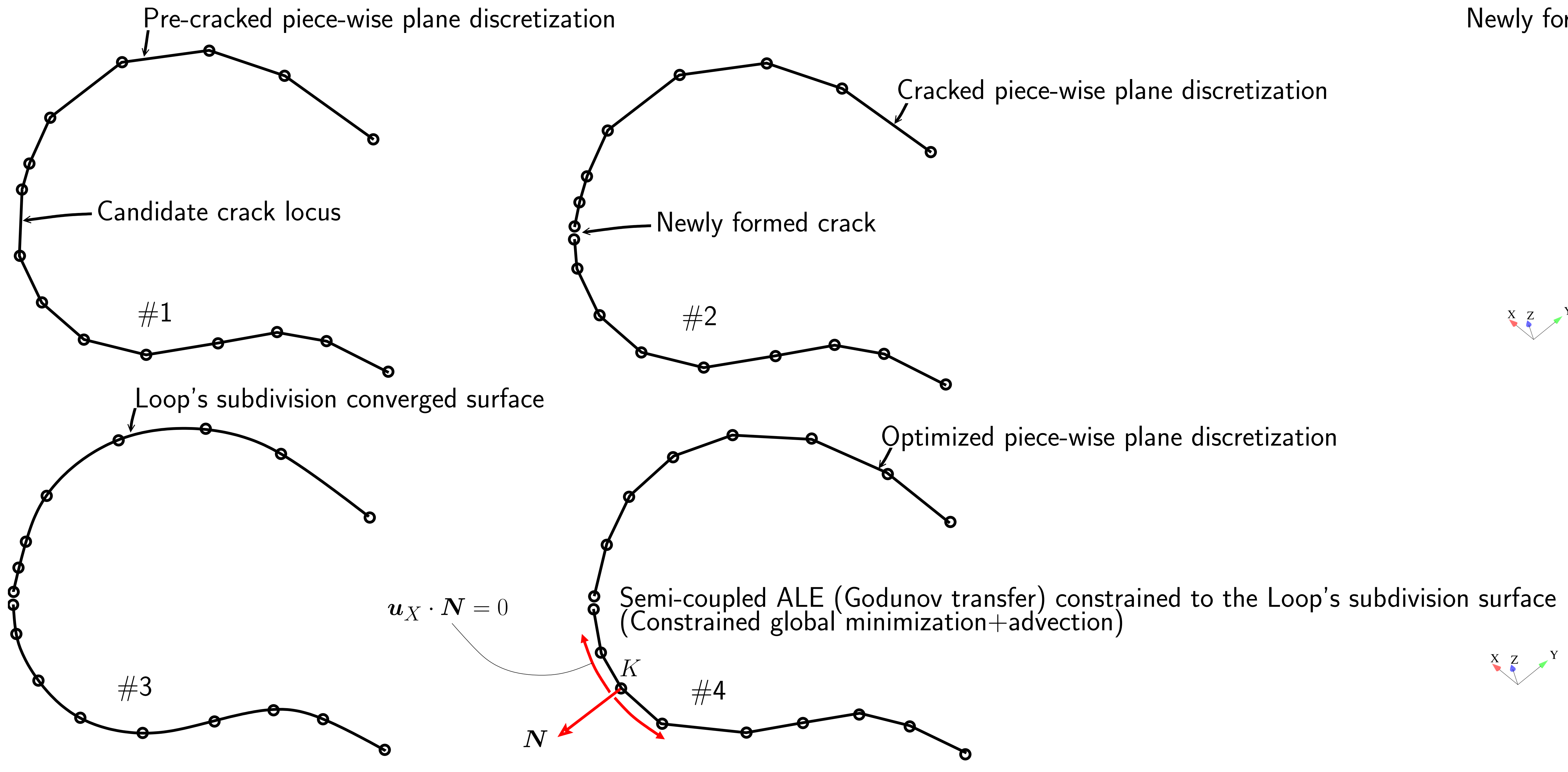
However, for shells the strategy must be updated due to:

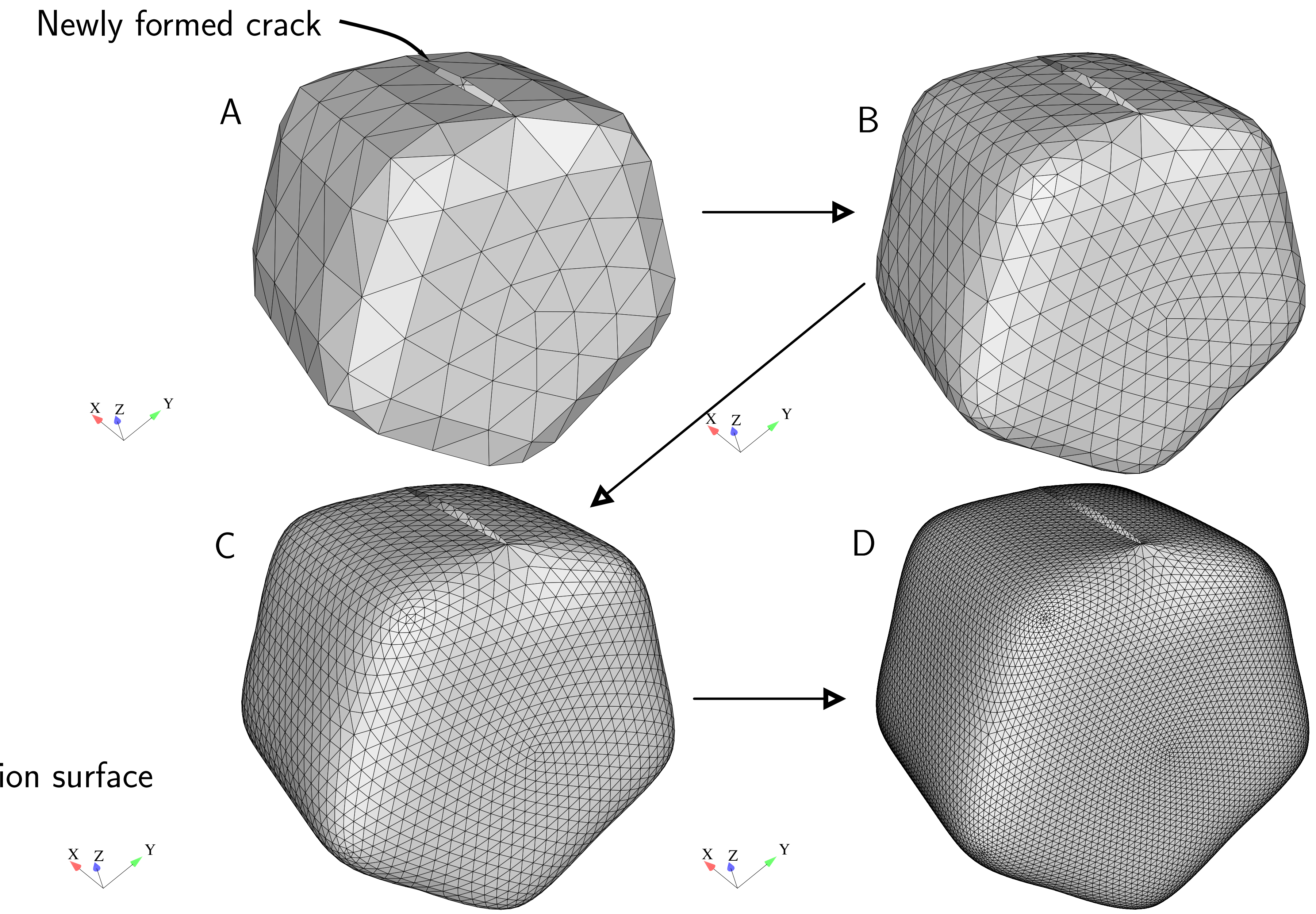
- Non-coplanarity of nodes
- Unknown shape of may surfaces

Ill-shaped elements naturally occur when the crack advances:



Our solution to shell fracture



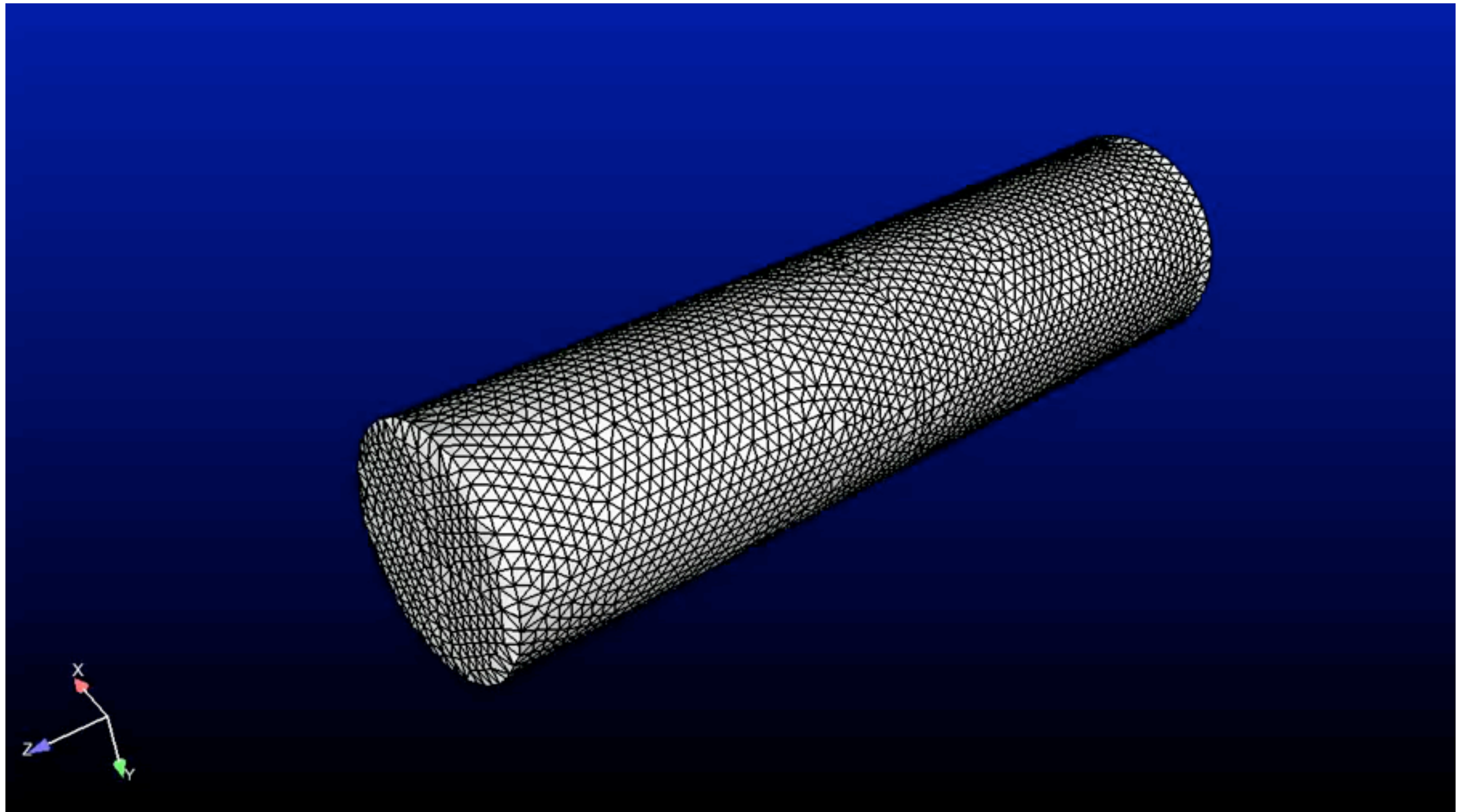


Geometrical operations are all performed in level D

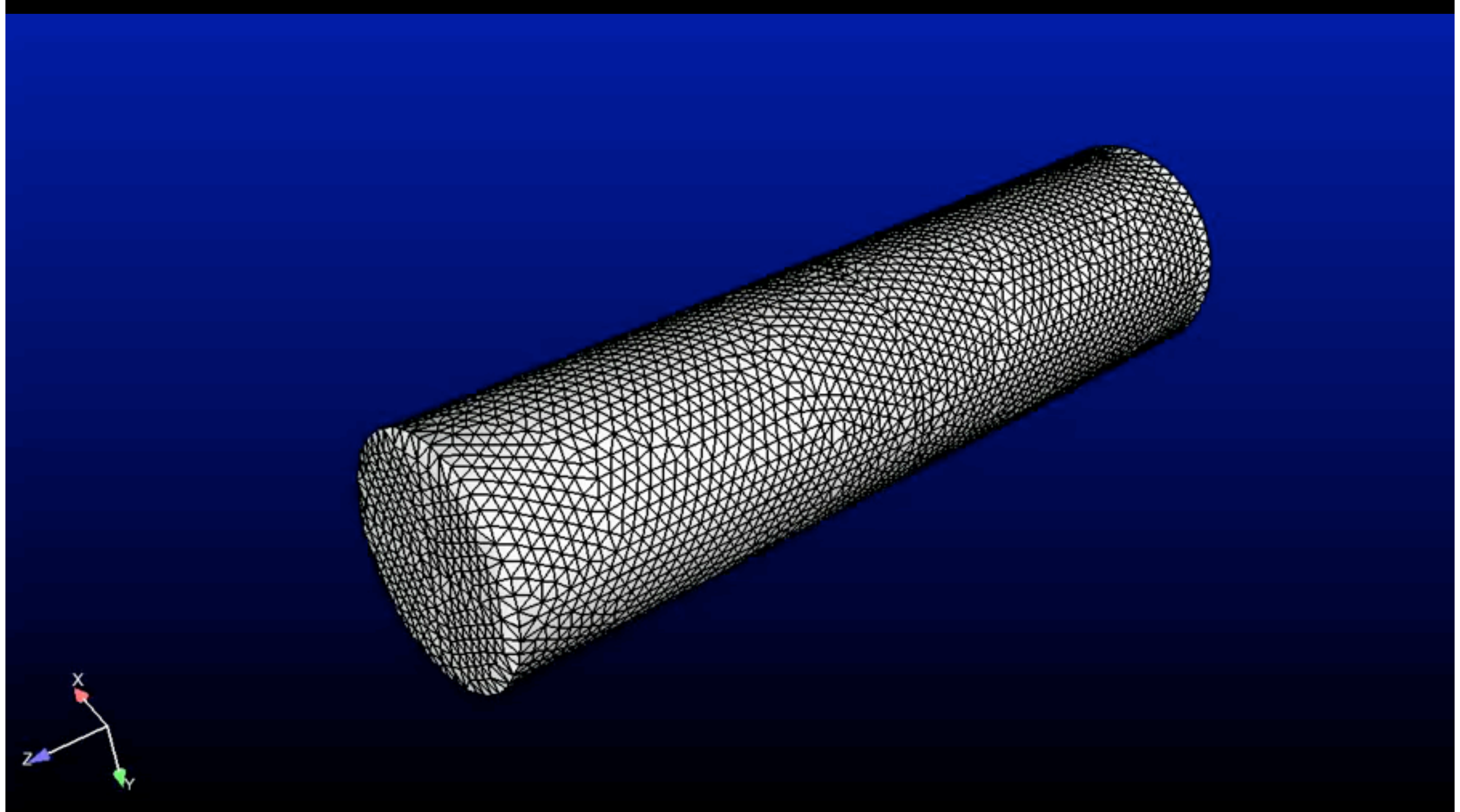
Hard to code but also very effective

The cylinder movies

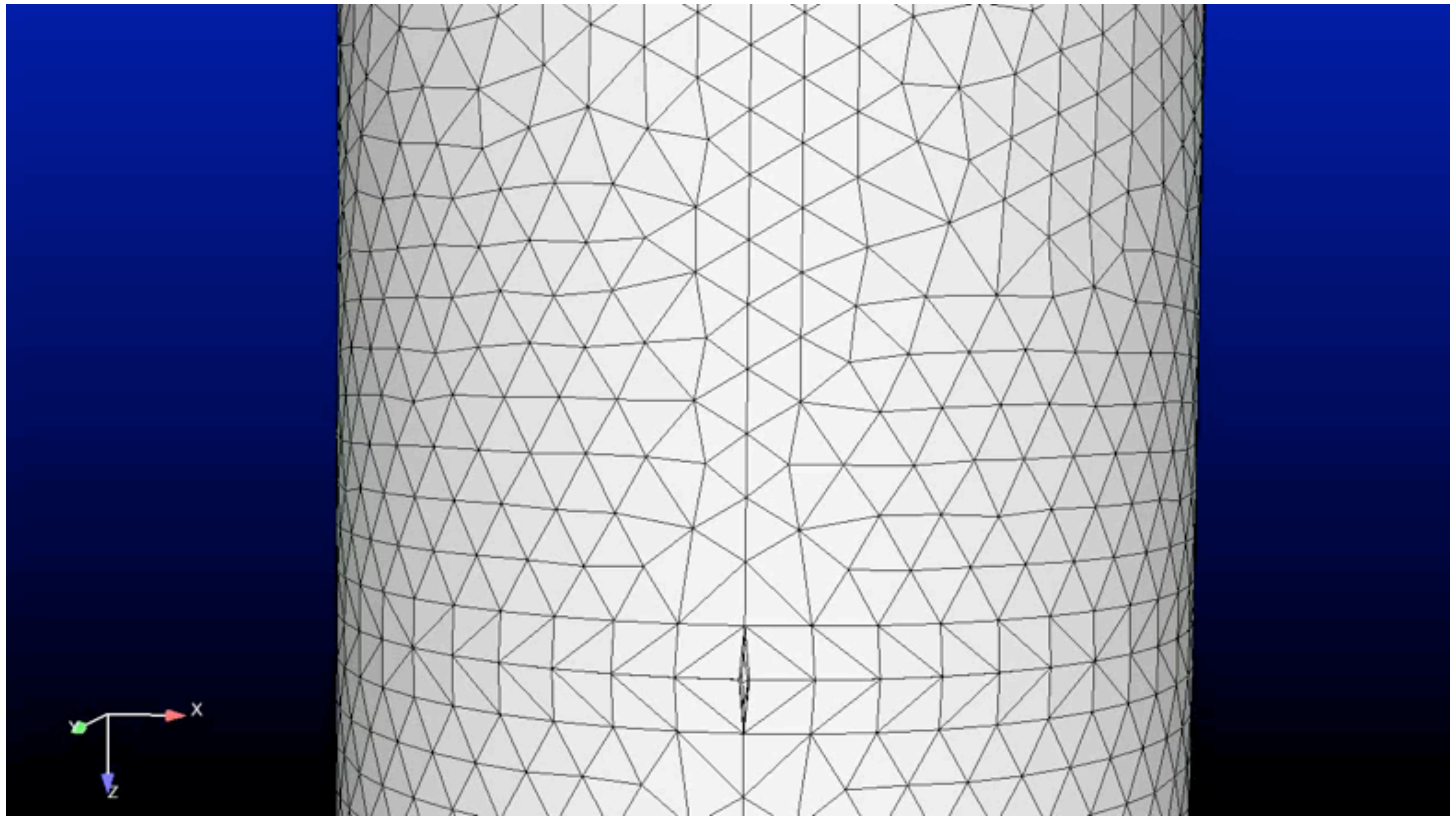
With our new ALE approach



Pristine

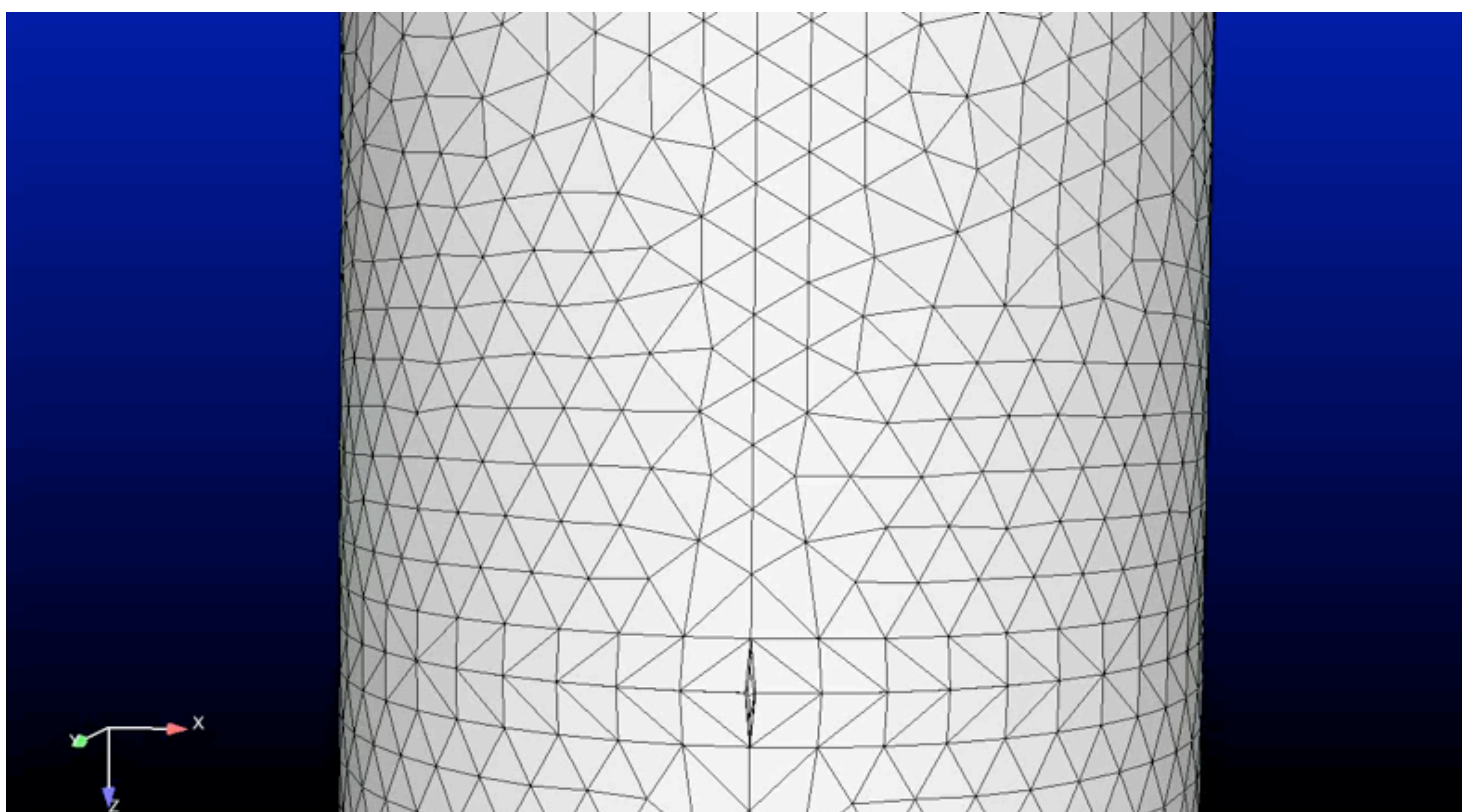


With our new ALE approach



Pristine

In detail, the effect of *geometrical* elements combined with *structural* elements



With our new ALE approach

Conclusions

- We have alternative approaches to model fracture in a large variety of situations which is based on simple ideas carefully implemented and tested. No enrichment or enhancement approaches are adopted.
- Return mapping techniques are avoided for elasto-plasticity integration.
- Our shell element has been the *best we tested in 14 years of research.*
- A simple Godunov-based ALE approach results very effective in all tests we performed so far.
- The geometrical elements ensure the mesh has a good quality, regardless of the number of cracks.
- For fully 3D problems with *multiple cracks* our tests indicate that a FULL remeshing may be less error prone than tip remeshing.
- With software like ACEGEN, the developer can concentrate on ideas instead of lengthy calculations

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