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Quality-price choices and market configurations when location matters ☆

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ABSTRACT

This paper investigates how preexistent asymmetries in the way consumers value each firm's product influence quality-price decisions when consumers differ in quality valuation but incur a transportation cost when buying from the firm located on the periphery. We show that for a given location, the high-quality firm charges a higher price and that for given qualities, the firm located in the center charges higher prices. Regarding quality choices, we show that the firm located in the center may be able to behave as a partial coverage monopolist. Under duopoly, quality differentiation always exists, and in general, the high-quality firm may be located either in the center or on the periphery. Moreover, the qualities offered by both firms are higher when the high-quality firm is on the periphery, showing a substitutability effect between location and quality. Thus, incentivizing the high-quality firm to locate on the periphery improves overall market quality.

1. Introduction

Many vertically differentiated markets evidence preexistent asymmetries between firms, i.e., firms have *ex-ante* characteristics that distinguish them in the eyes of consumers, implying that consumers will value differently products, with the same hedonic quality, sold by two different firms. The relevant preexistent characteristic may relate to location. In the housing market, one urban land developer (ULD) builds houses at the Central Business District (CBD), whereas the other builds houses on the periphery. The lower accessibility of the houses that are not centrally located creates an asymmetry between the firm operating at the CBD and the firm operating at the periphery. In the tourism sector, a hotel located close or far from the main city attractions also creates an asymmetry that may influence hotels' quality-price choices. The preexisting characteristics, that distinguish potential competitors, may also be related to other aspects valued by consumers, such as previous brand reputation, the firm having or not having a corporate social responsibility certification or being or not being known for its sustainability practices.[1]

This paper investigates how these preexistent asymmetries between firms influence their quality and price decisions. Does the firm with a locational disadvantage try to fight its disadvantage by lowering its price and/or by increasing its quality? Does the firm with a locational advantage exploit it by increasing its price and/or by decreasing its quality? What is the impact of preexistent asymmetries on the equilibrium market configuration? To answer these questions, we model preexistent asymmetries by assuming that one firm is located at the CBD, whereas the other is located on the periphery and that consumers who buy from the firm located on the periphery incur transportation costs, where transportation costs can have a spatial interpretation (like in the housing market) or an interpretation related to the disutility incurred by buying a given quality product from a firm that lacks some other desirable characteristic (for instance, the disutility of buying from a firm without a good sustainability reputation).

The paper solves a two-stage game in a vertically differentiated market, where consumers differ in how they value quality, but for the same quality, all incur the same transportation cost when buying from the firm located on the periphery. Two potential competitors first

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[1] Buying green goods instead of brown provides the consumer with additional satisfaction associated with the warm glow effect and being portrayed as a socially worthy citizen (Mantovani et al., 2016).

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simultaneously choose the quality of their product (housing or other) and simultaneously choose prices in the second stage. We assume that quality improvement has fixed costs but also increases marginal production costs. Thus, quality improvement has cost implications for both the price-stage game and the quality-stage game. We determine the global best responses and examine all possible equilibrium candidates, providing a full characterization of the different market configurations. Moreover, we investigate whether, in the subgame perfect Nash equilibrium (SPNE), different market configurations arise endogenously (partial or full market coverage), instead of exogenously assuming one or the other.

Regarding the price-stage game, our results show that, for given quality vectors, firms compensate their locational advantage (disadvantage) by increasing (lowering) their price. However, the compensations are not necessarily symmetric and depend on the market configuration. Moreover, there are cases where the equilibrium price of one ULD is not affected by the transportation costs (for instance, when qualities are such that the ULD located at the CBD has a guaranteed monopoly or when this firm is the low-quality duopolist and there is a full coverage corner solution).

Our quality choices' results differ significantly from previous literature. The most common result in vertical product differentiation (VPD) models is the existence of two SPNEs in which firms differentiate their quality levels (one firm offers a high-quality product and the other offers a low-quality product) and the two SPNEs are symmetric to each other (i.e., the roles of the firms are reversed). Our work shows that, when transportation costs are positive, we may have a unique SPNE, two, infinite or none in pure strategies. There are two cases where a unique SPNE occurs. The first one happens when transportation costs are high and the lowest quality valuation is below a certain cutoff, in which case the firm with a locational advantage can behave as a partial coverage monopolist. This result is quite different from what happens in the absence of locational asymmetries where, under the same cost assumptions, monopoly never occurs in equilibrium (Pires et al., 2022b).[2] The second case with a unique SPNE occurs for low values of the lowest quality valuation and for intermediate transportation costs. In this unique SPNE, there is a duopoly with partial coverage and the firm with the locational advantage is the one offering lower quality, which reveals a substitutability effect between quality and a more desirable location.[3]

One of the most interesting results of our work is that, when there are two SPNEs, in one equilibrium firms offer higher qualities than in the absence of preexistent asymmetries, while in the other equilibrium the reverse happens. In the equilibrium where the high-quality firm is located at the CBD, qualities offered are lower than when the high-quality firm is located on the periphery (and lower than with nil transportation costs). This occurs because the high-quality firm compensates its locational advantage (disadvantage) by decreasing (increasing) its quality, and given the strategic complementarity of quality choices, the low-quality firm does the same. This result shows a sort of substitutability effect between location and quality in firms' strategic interaction that is driven by the high-quality firm. A similar result holds when there are more than two SPNEs. As in Pires et al. (2022b), for intermediate lowest quality valuation, there exist multiple equilibria located in the frontier between the corner and interior full coverage regions. However, with transportation costs, these multiple equilibria depend on whether the high-quality firm has a locational advantage or disadvantage, with higher qualities being offered when the high-quality firm is located on the periphery.

Moreover, there are cases in which no SPNE exists, namely when the lowest quality valuation and transportation costs are high. The fact that transportation costs are high provides a strong incentive for the firm located at the CBD to choose quality levels where it can behave as a monopolist, including choosing the same quality as the firm located on the periphery when this firm chooses intermediate quality. However, the fact that the lowest quality valuation is high gives the periphery firm an incentive to offer positive quality that differs substantially from the quality offered by the centrally located firm, so as to be able to operate. Hence, there is no SPNE in pure strategies.

Our article provides an important contribution to the VPD literature. Departing from the pioneer work of Shaked and Sutton (1982), one of the earliest studies about how product differentiation relaxes price competition, VPD is an old topic that deserves to be revisited. The literature on VPD differs on assumptions such as the type of competition, the timing of quality choices, the distribution of consumers' valuations of quality and the nature of the costs of quality improvements.[4] Regarding costs, some initial VPD models assumed nil costs (Gabszewicz and Thisse, 1979; Tirole, 1988), whereas Shaked and Sutton (1982), Lambertini (1999), and Niem (2019) assume fixed costs for quality improvement while variable costs do not change with quality. This last assumption is reasonable when producers improve quality by advertising or research and development. Other authors assume variable costs increasing with quality, like Mussa and Rosen (1978), Aoki and Prusa (1996) and Nguyen et al. (2014), and argue that higher quality requires more expensive inputs or a more specialized labor force.[5]

Within the VPD literature, our work is most related to the papers that endogenize quality choices and market configurations, instead of a priori assuming the configuration (see Wauthy, 1996; Liao, 2008 and, recently, Pires et al., 2022b). The last work shares with the current paper the assumption of variable costs increasing with quality, whereas Wauthy (1996) assumes costless quality and Liao (2008) assumes fixed quality improvement costs. However, none of these papers consider possible preexisting asymmetries between the two firms, and all assume that potential competitors start on equal footing, which is rarely the case in the real world. As already mentioned, the preexistent asymmetries between firms have strong implications for the results, compared with the case of no asymmetries (nil transportation costs). As Pires et al. (2022a) show, in the second stage game, there are some unusual equilibria, like the Nash equilibrium where the low-quality firm is a monopolist.[6] However, in our second-stage game, the firm located centrally is able to behave as a constrained monopolist when it offers the same quality as its rival.[7] Our results also have important differences with respect to Pires et al. (2022b), namely the possibility of having a unique SPNE as well as the existence of cases where there

[2] Pires et al. (2022b) provide a full characterization of the different market configurations that arise endogenously in a two-stage quality-price game.

[3] It should be noted that having a unique SPNE eliminates the coordination problems arising from the typical multiplicity of equilibria in VPD models.

[4] Recently, Jorge et al. (2022) provided a good review of the VPD literature, identifying the most popular research paths and summarizing the main results on VPD (see table of chapter 16 for a good overview of the main assumptions used in previous theoretical VPD models).

[5] Motta (1993) compares fixed and variable costs of quality improvement assumptions in the same VPD model and concludes that, in both cases, firms choose to differentiate products in the first stage to soften price competition in the second stage. Cheng (2014) also compares both types and shows that quality differentiation increases with demand uncertainty, increasing further under variable costs than under fixed costs.

[6] Pires et al. (2022a) address price competition, emphasizing the role of the heterogeneity of consumers' quality valuations as well as of the lowest quality valuation in determining equilibrium levels of market coverage (full or partial) and competition (monopoly or duopoly).

[7] One study with similarities to our second-stage game is Mantovani et al. (2016), who assume exogenous qualities. The main difference is that, in their model, environmental costs are linked to relative emissions and it is assumed that the higher quality product has higher emissions. Thus, in their model, hedonic quality and location are related in a very specific way, whereas in our model there is no such relationship. Even so, some of their results are similar to ours. For instance, both low and high-quality monopolies are possible.

is no SPNE, thereby demonstrating the impact of preexistent firms' asymmetries on quality choices and market configurations.

Our work also contributes to the study of strategic interactions in the housing market. According to DiPasquale (1999) and, more recently, Garcês et al. (2022), housing supply remains understudied and the vast majority of this literature assumes that the housing industry is perfectly competitive. However, there are a few exceptions like Arnott (1987), Baudewyns (2000), and Arnott and Igarashi (2000), who found evidence of imperfect competition in the housing market. Moreover, differences in housing quality, housing accessibility and household tastes can clearly be sources of market power and lead to strategic interactions between ULDs under imperfect competition. Our results are consistent with previous results in this field. In particular, we show that, ceteris paribus, house prices tend to fall with distance from the CBD (Edlund et al., 2016; D'Acci, 2019), but the behavior of quality is not so evident (e.g., D'Acci, 2019) as we can have a SPNE where the high-quality firm is located at the CBD and have a SPNE where the reverse happens. Our work contributes to this field of research with an emphasis on the degree of market coverage attained in an oligopolistic market, that is, our results shed light on the circumstances under which the housing market is fully or only partially covered. In an international context where affordable housing has become a challenge for many citizens and a subject of widespread protests, associated with rising house prices, falling real income levels, and worsening credit conditions, it is important to understand the conditions that facilitate housing access. Some European countries are facing a major housing problem among young generations.[8] Moreover, high-quality houses are being increasingly rated, namely the so-called smart houses, that allow energy saving and a greener and more environmentally friendly lifestyle. Our paper demonstrates that it is possible to achieve full market coverage, but it clearly requires competition on the supply side, as well as a sufficiently high level of consumers' quality valuation. Hence, programs that increase consumers' sensitivity to housing quality may contribute to the market converging or approaching a full coverage outcome. Considering that the distribution of consumers' quality valuations and willingness to pay is related with income levels, public policies that improve real income are also welcome in this respect. Furthermore, our results show that higher quality houses will be overall offered when the high-quality supplier is located on the periphery, so incentivizing high-quality constructions on the periphery (instead of the CBD) may actually improve the general quality of the housing stock.

The article is organized as follows. In the next section, we describe the model and impose necessary conditions on the two firms quality levels such that their demand is positive. This allows us to restrict our analysis of the SPNE to cases where at least one firm has positive demand. In this section, we also define some cutoff valuations that allow us to simplify the exposition. In Section 3, we study the Nash equilibrium of the price game, obtaining the equilibrium prices for the second stage analytically, assuming given quality levels. In Section 4, we characterize the SPNE. Finally, Section 5 summarizes the main implications of our results and the conclusions. An Appendix contains all proofs.

2. The model and preliminary results

2.1. The model

We modify a standard VPD model to account for possible preexistent asymmetries between the firms. There are two ULDs, indexed by $i = 1, 2$. ULD 1 stays at the CBD, whereas ULD 2 builds houses at a more peripheral location. We thus assume that ULD 1 has an *ex-ante* competitive advantage. In the first stage, each ULD simultaneously decides the quality of its houses, k_i. In the second stage of the game, each ULD simultaneously decides its housing price, p_i.

Consumer's net utility if he buys a house from urban developer i is given by:

$$U = \theta k_i - t d_i - p_i$$

Parameter θ is a taste parameter that reflects how much the consumer values quality. This parameter is uniformly distributed across the population between $\underline{\theta}$ and $\overline{\theta} = \underline{\theta} + 1$. Parameter d_i is the distance from the urban developer i's house to the CBD and $t > 0$ is the transportation cost by unit of distance. As ULD 1 is located at the CBD, $d_1 = 0$. For simplicity, we assume that $d_2 = 1$. It should be noted that when a consumer buys a house, he is also choosing his own location (where he wants to live). If we assume that jobs and shops are located in the CBD (like in the traditional monocentric city model), a consumer who buys a house in a peripheral location must move whenever he goes to work or shopping, which explains the inclusion of the transportation cost in the utility function. On the contrary, a consumer who buys a house in the CBD does not incur transportation costs.[9]

Considering the locations of the two ULDs, the net utility of the consumer with quality valuation θ is:

$$\begin{cases} U_1(\theta) = \theta k_1 - p_1 & \text{if he buys from ULD 1} \\ U_2(\theta) = \theta k_2 - t - p_2 & \text{if he buys from ULD 2} \\ U_0(\theta) = 0 & \text{if he does not buy} \end{cases}$$

Among these three options, the consumer chooses the alternative that gives him the highest net utility.[10]

In the price game, we assume that the ULDs have constant marginal production costs that depend on the quality chosen in the first stage, that is, the total production costs are given by:

$$C(q_i) = c_i q_i \quad \text{with } c_i = \frac{k_i^2}{2}$$

where c_i is the marginal production cost and q_i is the quantity of houses produced.

Moreover, we assume that in the first stage of the game there is an investment cost of quality given by:

$$I(k_i) = \begin{cases} 0 & \text{if } k_i = 0 \\ F & \text{if } k_i > 0 \end{cases}$$

where F is a positive constant. This assumption is not relevant for the second stage of the game, but influences the determination of the equilibrium qualities in the first stage.

Throughout the paper we use the housing metaphor. However, we should keep in mind that the model can be applied in other settings where quality choices are taken but there exists some other characteristic valued by consumers in which firms differ when the game starts, creating and asymmetry on how much consumers are willing to pay for the same quality product sold by the two firms.

To find the SPNE, we need to find first the Nash equilibrium of the second stage of the game (the simultaneous choice of prices), then go back to the first stage and find the solution of the complete game. Before we compute the Nash equilibrium, it is useful to derive some preliminary results.

[8] In this respect, see for instance https://www.euronews.com/business/2023/10/07/europes-housing-crisis-portugal-turkey-and-luxembourg-struggle-to-find-solutions and Valderrama et al. (2023).

[9] Note that results would be similar if, assuming $d_1 = 0$, we normalized t ($t = 1$) and explored how the SPNE depends on d_2. This alternative approach would give us information about the housing pricing as a function of location and quality (hedonic housing pricing).

[10] Note that the two ULDs are not in a symmetric position unless there are no transportation costs. For positive transportation costs, if the two ULDs offer the same quality and the same price, all consumers prefer a ULD 1 house to a ULD 2 house. Thus our model can be interpreted as bidimensional vertical differentiation model, with one of the quality dimensions exogenously fixed. However, when $t = 0$ our model is similar to the traditional VPD model and replicates Pires et al. (2022b).

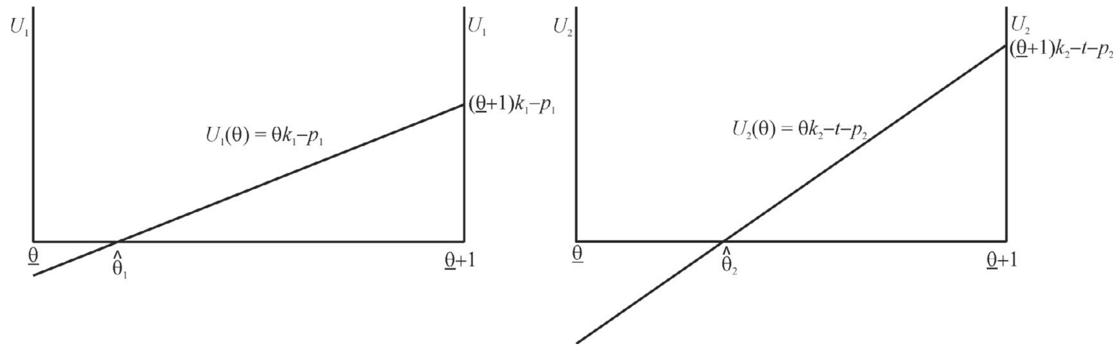

Fig. 1. Indifferent consumer between buying and not buying from ULD 1 and from ULD 2, respectively.

2.2. Some preliminary results

We start by imposing necessary conditions on the two ULDs' quality levels for their demand to be positive. This will allows us to restrict our analysis of the Nash equilibrium for vectors of qualities (k_1, k_2) where at least one of the ULD has positive demand. Moreover, to simplify the exposition, it is useful to define some cut-off valuations.

Pricing at marginal cost is the most favorable situation for consumers. In this case, if they want to buy zero, they have nil demand in all other cases.

Lemma 1. *A necessary condition for ULD 1 to have positive demand with marginal cost pricing is that $k_1 < 2(\underline{\theta}+1)$. Similarly, ULD 2 only has positive demand with marginal cost pricing if $t \leq \frac{(\underline{\theta}+1)^2}{2}$ and $\underline{k}_2 < k_2 < \overline{k}_2$ where $\underline{k}_2 = \underline{\theta} + 1 - \sqrt{(\underline{\theta}+1)^2 - 2t}$ and $\overline{k}_2 = \underline{\theta} + 1 + \sqrt{(\underline{\theta}+1)^2 - 2t}$.*

Proof. See Appendix. ■

What happens is that for $k_1 \geq 2(\underline{\theta}+1)$ and for $k_2 \geq \overline{k}_2$, the quality offered by ULD 1 and ULD 2, respectively, is too high. As marginal production costs are increasing with quality, for those levels of quality, price will be so high that even the consumer who values quality the most would prefer not to buy the house. However, when $k_2 \leq \underline{k}_2$, the quality of ULD 2 is too low. Considering its locational disadvantage, the quality of ULD 2 is so low that no consumer wants to buy a house from ULD 2, even if it charges price equal to marginal cost.

We start by determining the indifferent consumers between buying and not buying from each ULD:

Lemma 2. *Let $\hat{\theta}_1 = \frac{p_1}{k_1}$ and $\hat{\theta}_2 = \frac{p_2+t}{k_2}$. Any consumer with $\theta \geq \hat{\theta}_i$ prefers to buy from ULD$_i$ than not buy. The consumer with valuation $\theta = \hat{\theta}_i$ is the indifferent consumer between buying from ULD$_i$ and not buying.*

Proof. See Appendix. ■

Fig. 1 shows the utility of buying a house from ULD 1 (on the left) and from ULD 2 (on the right) in a case where some consumers prefer not to buy any of the houses, i.e., the indifferent consumers, $\hat{\theta}_1$ and $\hat{\theta}_2$, are at the right of $\underline{\theta}$.

Note that the slope of the utility function of buying from ULD i is equal to k_i and, therefore, for positive qualities, utility is increasing with θ. This has implications on the way the consumers choose between the two ULDs.

Lemma 3. *If the highest valuation consumer, $\underline{\theta}+1$, prefers the house of lower quality, then all the consumers prefer the house of lower quality.*

Proof. See Appendix. ■

A similar result holds when the lower valuation consumer prefers the high-quality house:

Lemma 4. *If the lowest valuation consumer, $\underline{\theta}$, prefers the house of higher quality, then all the consumers prefer the house of higher quality.*

Proof. See Appendix. ■

The proofs show that, for a given quality differential, the consumer decision depends on the difference between the total prices, $p_2 + t - p_1$, where $p_2 + t$ is the total price of firm 2 and p_1 is the total price of firm 1. Note that, in the two previous cases, only one of the ULD has positive demand.

For both firms to have positive demand, the price differential cannot be too high. Otherwise, all consumers would prefer the low-quality house. However, the price differential cannot be too low. Otherwise, all consumers would prefer the more expensive high-quality house.

Lemma 5. *If prices are such that both ULDs have positive demand, the higher quality ULD serves the higher valuation consumers, whereas the lower quality ULD serves the lower valuation consumers.*

Proof. See Appendix. ■

The previous results show that either only the high-quality ULD operates, only the low-quality ULD operates, or both ULDs operate and compete with each other. We cannot have a situation where both firms operate but are not interacting. Hence, we cannot have local monopolies.

If the two ULDs have the same quality, the utility functions of buying from the two ULDs have the same slope and either ULD 1 is strictly preferred to ULD 2 for all consumers, or the reverse, or all consumers are indifferent between buying from ULD 1 and buying from ULD 2. The next lemma determines the indifferent consumer between buying from either ULD 1 or ULD 2:

Lemma 6. *If $k_1 > k_2$, the indifferent consumer between buying from ULD 1 or buying from ULD 2 is:*

$$\tilde{\theta} = \frac{p_1 - p_2 - t}{k_1 - k_2}$$

If $k_2 > k_1$, the indifferent consumer between buying from either ULD 1 or ULD 2 is:

$$\tilde{\theta} = \frac{p_2 - p_1 + t}{k_2 - k_1} = \frac{p_1 - p_2 - t}{k_1 - k_2}$$

Consumers with $\theta > \tilde{\theta}$ prefer to buy from the high-quality ULD, whereas consumers with $\theta < \tilde{\theta}$ prefer to buy from the low-quality ULD.

Proof. See Appendix. ■

Fig. 2 illustrates this result for $k_2 > k_1$ (on the left) and $k_1 > k_2$ (on the right).

Note that the indifferent consumer can also be written as follows:

$$\tilde{\theta} = \frac{p_H - p_L}{k_H - k_L} + \frac{t\left(2I_L - 1\right)}{k_H - k_L}$$

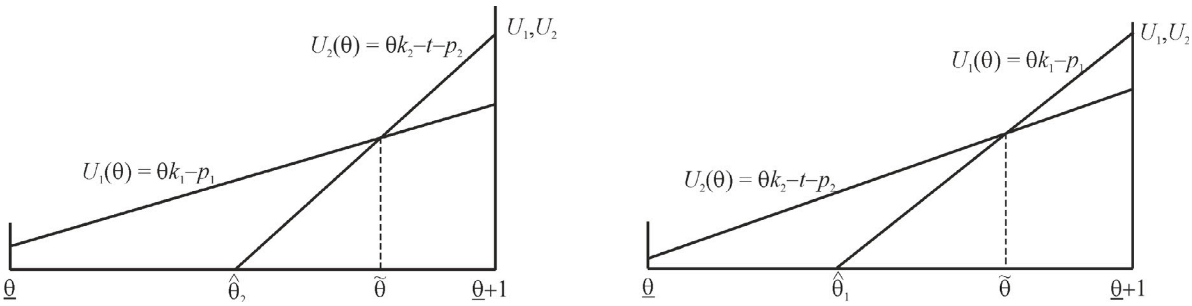

Fig. 2. Indifferent consumer between buying from ULD 1 and ULD 2 when $k_2 > k_1$ (left) and when $k_1 > k_2$ (right).

where the subscripts H an L refer to high and low quality, respectively and I_L is an indicator function that takes value 1 if ULD 1 is the low-quality firm and takes value 0 otherwise. The first term in the indifferent consumer expression is the ratio of the price differential with respect to the quality differential (the usual term in VPD models), whereas the second term captures the advantage of ULD 1 by being located in the CBD, which, for given qualities, increases its demand.

3. Nash equilibrium of the price game

Considering that any price below the marginal cost is weakly dominated by charging a price equal to marginal cost, to derive the Nash equilibrium we restrict the analysis to prices $p_i \geq c_i$. In the second-stage game, we find all the types of Nash equilibria described in Pires et al. (2022a). However, there are two additional types of Nash equilibria that can occur in our model. First, when $t > 0$, ULD 1 behaves as a constrained monopolist when $k_1 = k_2$. Recall that, in this case, consumers either all prefer firm 1, or all prefer firm 2 or all are indifferent and they will buy from the firm with lower total price (p_1 and $p_2 + t$). This creates a discontinuity in the demand functions, giving incentives to firms to charge a total price slightly lower than the rival to capture the whole demand. When $t = 0$, this leads to the well known Bertrand paradox where both firms charge a price equal to the marginal cost. When $t > 0$, firm 1 has an advantage in this pricing game, as it is able to capture the whole demand by charging a price slightly below $c_2 + t$. Hence firm 1 behaves as a constrained monopolist. Second, there are cases where ULD 1 can behave as a high-quality partial coverage monopolist, a situation that was not possible for $t = 0$.

Although we derived the Nash equilibrium for all possible (k_1, k_2) vectors and these equilibria were considered when checking for possible deviations of candidates to SPNE and in deriving the quality best responses, to simplify our exposition in this section we do not describe all these equilibria. The reason is that our focus is on the SPNE and some quality vectors can be easily ruled out as equilibrium quality choices in our two-stage game.[11]

First, quality vectors with $k_2 = k_1 > 0$ cannot be SPNE. For $t = 0$, this would correspond to a homogeneous product duopoly with price competition and thus, in equilibrium, firms would get a nil operating profit. However, then, considering the quality investment costs, firms gain by deviating and differentiating their products. For $t > 0$, ULD 1 is able to offer a higher surplus than ULD 2 and can behave as a constrained monopolist by choosing $k_1 = k_2$.[12] However, choosing $k_2 = k_1$ can never be a best response for ULD 2 as it would get a negative net profit, due to the investment costs. Hence ULD 2 gains by deviating to $k_2 = 0$ or to a sufficiently different quality where it can profitably operate.

We can also disregard as possible SPNEs cases in which one of the firms is a monopolist, whereas the other is offering a positive quality but is unable to operate, as it happens in quality combinations where the low-quality firm is a monopolist. These cases cannot be SPNEs, because the firm who is not operating has a negative net profit, so it would prefer to choose a nil quality to avoid the investment costs.

Hence, we can eliminate as possible SPNEs the cases of $k_2 = k_1$, the cases where the low-quality firm is a monopolist, and the cases where the high-quality firm is a monopolist and the low-quality firm chooses a positive quality.

In the rest of this section we derive the Nash equilibria that can be observed in the SPNE. It should also be noted that we analyze the cases of $k_1 > k_2$ and $k_2 > k_1$ as they are not symmetric.

3.1. Guaranteed monopoly

As explained in Section 2.2, no consumer will ever buy from ULD 2 if $k_2 \leq \underline{k}_2$ or $k_2 \geq \bar{k}_2$. In this case, if $0 < k_1 < 2(\underline{\theta} + 1)$ firm 1 has a guaranteed monopoly. Similarly, if $k_1 = 0$ or $k_1 \geq 2(\underline{\theta} + 1)$ and $\underline{k}_2 < k_2 < \bar{k}_2$, ULD 2 has a guaranteed monopoly.

The monopolist ULD may opt for charging a high price, such that the lowest valuation consumer prefers not to buy and the market is partially covered. In this case the monopolist solves the following problem:

$$\max_{p_i} \Pi_i = \left(\underline{\theta} + 1 - \hat{\theta}_i\right)(p_i - c_i) \quad \text{subject to } \hat{\theta}_i > \underline{\theta}$$

Alternatively, the monopolist may charge a lower price and fully cover the market. In this case, it solves:

$$\max_{p_i} \Pi_i = (p_i - c_i) \quad \text{subject to } \hat{\theta}_i \leq \underline{\theta}$$

The Nash equilibrium when one of the ULD has a guaranteed monopoly is as follows:

Proposition 1. *If $k_2 \leq \underline{k}_2$ or $k_2 \geq \bar{k}_2$ and $0 < k_1 < 2(\underline{\theta} + 1)$, ULD 1 can behave as a monopolist. If $\underline{\theta} \leq 1$ or $\underline{\theta} > 1$ and $k_1 > 2(\underline{\theta} - 1)$, the market is partially covered and the equilibrium prices are:*

$$p_1^* = \frac{(\underline{\theta} + 1)k_1 + \frac{k_1^2}{2}}{2} \quad \text{and} \quad p_2^* = \frac{k_2^2}{2}$$

If $\underline{\theta} > 1$ and $0 < k_1 < 2(\underline{\theta} - 1)$, the market is fully covered and the equilibrium prices are $p_1^* = \underline{\theta} k_1$ and $p_2^* = \frac{k_2^2}{2}$.

If $k_1 = 0$ or $k_1 \geq 2(\underline{\theta} + 1)$ and $\underline{k}_2 < k_2 < \bar{k}_2$, then ULD 2 can behave as a monopolist. If $\underline{\theta} \leq 1$ or $\underline{\theta} > 1$ and $\frac{k_2}{2} + \frac{t}{k_2} > \underline{\theta} - 1$, the market is partially covered and the equilibrium prices are:

$$p_1^* = \frac{k_1^2}{2} \quad \text{and} \quad p_2^* = \frac{\frac{k_2^2}{2} + k_2(\underline{\theta} + 1) - t}{2}$$

If $\underline{\theta} > 1$ and $\frac{k_2}{2} + \frac{t}{k_2} < \underline{\theta} - 1$, the market is fully covered and the equilibrium prices are $p_1^* = \frac{k_1^2}{2}$ and $p_2^* = \underline{\theta} k_2 - t$.

Proof. See Appendix. ■

[11] The analysis of the omitted cases is available from the authors upon request.

[12] The monopolist is a constrained monopolist if the constraint that requires the monopolist to offer to consumers at least the same surplus as the one offered by the rival is binding.

The intuition for this result is that when consumers quality valuation is low, $\underline{\theta} \leq 1$, the monopolist is better off by partially covering the market, as full coverage would imply a too low price (in the limit case of $\underline{\theta} = 0$, the price would have to be 0 to have full coverage). Although, when the lowest quality valuation is high, $\underline{\theta} > 1$, the monopolist is also better off by partially covering the market if the quality is high. The reason is that: for a high-quality product, the marginal production costs are also high. However, the lower valuation consumers do not want to buy the product and the market is not fully covered. On the contrary, when consumers have a high quality valuation but with lower quality levels, the monopolist fully covers the market.

The optimal price of the monopolist is increasing on its quality. It increases linearly when the market is fully covered, but it increases at an increasing rate if the market is partially covered (due to the shape of marginal costs). The optimal price of ULD 1 does not depend on t because the demand of ULD 1 when it has a guaranteed monopoly is not a function of t. Conversely, the optimal price of ULD 2, when it has a guaranteed monopoly, is decreasing with t. This last result is an immediate consequence of t having a negative impact on the firm's demand (if the market is partially covered) and a negative impact on the price that can be charged to the lowest valuation consumer (if the market is fully covered).

3.2. Interior full coverage duopoly

When both ULDs operate and fully cover the market, we already know that the high-quality ULD covers the consumers with higher valuation, whereas the low-quality ULD covers the consumers with lower valuation (Lemma 5). The profit functions for the two ULDs are given by:

$$\Pi_L = \left(\widetilde{\theta} - \underline{\theta}\right)\left(p_L - c_L\right)$$
$$\Pi_H = \left(\underline{\theta} + 1 - \widetilde{\theta}\right)\left(p_H - c_H\right)$$

Due to the locational advantage of ULD 1, the indifferent consumer, $\widetilde{\theta}$, is not symmetric when $k_1 > k_2$ and $k_2 > k_1$. Thus, the equilibrium in the price game and the condition under which the equilibrium holds depend on who is the high-quality ULD:

Proposition 2. *When houses have better quality on the periphery* ($k_2 > k_1$), $2\left(\underline{\theta} - 1\right) \leq k_2 + k_1 + \frac{2t}{k_2 - k_1} \leq 2\left(\underline{\theta} + 2\right)$ *and* $\frac{k_1}{3}\left(2\underline{\theta} + 1 - k_1\right) + \frac{k_2}{3}\left(\underline{\theta} - 1 - \frac{k_2}{2}\right) - \frac{t}{3} \geq 0$, *in equilibrium both ULD operate under full coverage and the equilibrium prices are:*

$$p_1^* = \frac{(1 - \underline{\theta})\left(k_2 - k_1\right) + k_1^2 + \frac{k_2^2}{2} + t}{3} = p_L^0 + \frac{t}{3}$$
$$p_2^* = \frac{(\underline{\theta} + 2)(k_2 - k_1) + k_2^2 + \frac{k_1^2}{2} - t}{3} = p_H^0 - \frac{t}{3}$$

where p_L^0 *and* p_H^0 *are the equilibrium prices of the low-quality and high-quality ULD when* $t = 0$.

By contrast, when houses have better quality at the CBD ($k_1 > k_2$), $2\left(\underline{\theta} - 1\right) \leq k_2 + k_1 - \frac{2t}{k_1 - k_2} \leq 2\left(\underline{\theta} + 2\right)$ *(indifferent consumer is between* $\underline{\theta}$ *and* $\underline{\theta} + 1$*) and* $\frac{k_1}{3}\left(\underline{\theta} - 1 - \frac{k_1}{2}\right) + \frac{k_2}{3}\left(2\underline{\theta} + 1 - k_2\right) - \frac{2}{3}t \geq 0$ *(lowest valuation consumer has nonnegative surplus), the equilibrium prices are:*

$$p_1^* = \frac{(\underline{\theta} + 2)(k_1 - k_2) + k_1^2 + \frac{k_2^2}{2} + t}{3} = p_H^0 + \frac{t}{3}$$
$$p_2^* = \frac{(1 - \underline{\theta})(k_1 - k_2) + k_2^2 + \frac{k_1^2}{2} - t}{3} = p_L^0 - \frac{t}{3}$$

where p_L^0 *and* p_H^0 *are the equilibrium prices of the low-quality and high-quality ULD when* $t = 0$.

Proof. See Appendix. ■

The equilibrium prices depend on the quality differential, on the marginal costs, $c_i = \frac{k_i^2}{2}$, and on t. Under full coverage, increasing t positively influences the equilibrium price of ULD 1 and negatively influences the equilibrium price of ULD 2. This happens because increasing t increases the demand of ULD 1 and decreases the demand of ULD 2. Consequently, it is optimal for ULD 1 to charge a higher price (its best response shifts to the right), whereas for ULD 2 it is optimal to decrease its price (its best response shifts down). The changes in the equilibrium prices are completely symmetric (p_1 increases $\frac{t}{3}$ while p_2 decreases $\frac{t}{3}$).

It should be highlighted that the conditions for the existence of an interior full coverage duopoly are not symmetric when $k_1 > k_2$ and $k_2 > k_1$. The condition regarding the lowest valuation consumer getting a nonnegative surplus (the last condition mentioned in the proposition as required for the result to hold) is harder to be satisfied when $t > 0$ and it is easier to be satisfied when the high-quality firm is located on the periphery.

3.3. Partial coverage duopoly

If the lowest valuation consumer gets a negative surplus at the interior full coverage equilibrium prices, the Nash equilibrium cannot be a duopoly with full coverage. With partial market coverage, the profit functions of the ULD selling the low-quality houses and high-quality houses are given, respectively, by:

$$\Pi_L = \left(\widetilde{\theta} - \widehat{\theta}_L\right)\left(p_L - c_L\right)$$
$$\Pi_H = \left(\underline{\theta} + 1 - \widetilde{\theta}\right)\left(p_H - c_H\right)$$

where both $\widehat{\theta}_L$ and $\widetilde{\theta}$ depend on who is the high-quality ULD. The equilibrium in the price game is as follows:

Proposition 3. *When houses have better quality on the periphery* ($k_2 > k_1$), $\frac{k_2\left(k_2 + \frac{k_1}{2} - 2(\underline{\theta}+1)\right)}{4k_2 - k_1} + \frac{t(2k_2 - k_1)}{(4k_2 - k_1)(k_2 - k_1)} \leq 0$ *and* $k_2 - k_1 + \frac{k_2^2}{2} + k_1 k_2 + t - 3\underline{\theta} k_2 > 0$ *both ULD operate in equilibrium and there is partial market coverage. The equilibrium prices are:*

$$p_1^* = \frac{k_1(\underline{\theta} + 1)\left(k_2 - k_1\right) + k_1^2 k_2 + \frac{k_2^2}{2} k_1}{4k_2 - k_1} + \frac{tk_1}{4k_2 - k_1} = p_L^0 + \frac{tk_1}{4k_2 - k_1}$$
$$p_2^* = \frac{2k_2(\underline{\theta} + 1)\left(k_2 - k_1\right) + \frac{k_1^2}{2} k_2 + k_2^3}{4k_2 - k_1} - \frac{t\left(2k_2 - k_1\right)}{4k_2 - k_1}$$
$$= p_H^0 - \frac{t\left(2k_2 - k_1\right)}{4k_2 - k_1}$$

where p_L^0 *and* p_H^0 *are the equilibrium prices of the low-quality and high-quality ULD when* $t = 0$.

When houses have better quality at the CBD ($k_1 > k_2$), $\frac{k_1(k_1 + 0.5k_2 - 2(\underline{\theta}+1))}{4k_1 - k_2} - \frac{tk_1}{(4k_1 - k_2)(k_1 - k_2)} \leq 0$ *and* $k_1 - k_2 + 0.5k_1^2 + k_1 k_2 + \frac{2tk_1}{k_2} - 3\underline{\theta} k_1 > 0$ *both ULD operate in equilibrium and there is partial coverage. The equilibrium prices are:*

$$p_1^* = \frac{2k_1(\underline{\theta} + 1)(k_1 - k_2) + k_1^3 + \frac{k_2^2}{2} k_1}{4k_1 - k_2} + \frac{tk_1}{4k_1 - k_2} = p_H^0 + \frac{tk_1}{4k_1 - k_2}$$
$$p_2^* = \frac{k_2(\underline{\theta} + 1)(k_1 - k_2) + k_1 k_2^2 + \frac{k_1^2}{2} k_2}{4k_1 - k_2} - \frac{t\left(2k_1 - k_2\right)}{4k_1 - k_2}$$
$$= p_L^0 - \frac{t\left(2k_1 - k_2\right)}{4k_1 - k_2}$$

Proof. See Appendix. ■

In a partial coverage duopoly, it is also true, as in the interior full coverage duopoly, that ULD 1 charges a higher equilibrium price and ULD 2 charges a lower equilibrium price than in the case of $t = 0$, but in this case, the absolute change in the price of ULD 2 is larger than ULD 1, as $2k_H - k_L > k_L$. When $t > 0$, it is easier to satisfy the condition that separates partial coverage from full coverage ($k_2 - k_1 + \frac{k_2^2}{2} + k_1 k_2 + t - 3\underline{\theta} k_2 \geq 0$ or $k_1 - k_2 + \frac{k_1^2}{2} + k_1 k_2 + \frac{2tk_1}{k_2} - 3\underline{\theta} k_1 \geq 0$); but, when $k_2 > k_1$, it is more difficult to guarantee that ULD 2 operates (as shown in the Appendix, the first condition required for this equilibrium to hold guarantees that the indifferent consumer is below $\underline{\theta}$+1, thus firm 2 operates).

3.4. Corner full coverage duopoly

We may have cases where a duopoly holds but neither interior full coverage nor partial coverage hold. This happens, for instance, if $k_2 > k_1$, $\frac{k_1}{3}\left(2\underline{\theta} + 1 - k_1\right) + \frac{k_2}{3}\left(\underline{\theta} - 1 - \frac{k_2}{2}\right) - \frac{t}{3} < 0$ and $k_2 - k_1 + \frac{k_2^2}{2} + k_1 k_2 + t - 3\underline{\theta} k_2 \leq 0$. In this case, the Nash equilibrium of the price game involves the low-quality ULD offering a nil surplus to the lowest valuation consumer and the high-quality ULD choosing the quality that is the best response to that choice of the low-quality ULD.

Proposition 4. *When houses have better quality on the periphery ($k_2 > k_1$), $\frac{k_1}{3}\left(2\underline{\theta} + 1 - k_1\right) + \frac{k_2}{3}\left(\underline{\theta} - 1 - \frac{k_2}{2}\right) - \frac{t}{3} < 0$, $k_2 - k_1 + \frac{k_2^2}{2} + k_1 k_2 + t - 3\underline{\theta} k_2 \leq 0$ and $\underline{\theta} - \frac{1}{2} \leq \frac{\underline{\theta}(k_2 - 2k_1) + \frac{k_2^2}{2} + t}{2(k_2 - k_1)} \leq \underline{\theta} + \frac{1}{2}$ in equilibrium both firms operate and there is full coverage corner solution. The equilibrium prices are:*

$$p_1^* = \underline{\theta} k_1 = p_L^0$$

$$p_2^* = \frac{k_2 - k_1 + \underline{\theta} k_2 + \frac{k_2^2}{2} - t}{2} = p_H^0 - \frac{t}{2}$$

where p_L^0 and p_H^0 are the equilibrium prices of the low-quality and high-quality ULD when $t = 0$.

When houses have better quality at the CBD ($k_1 > k_2$), $\frac{k_1}{3}\left(\underline{\theta} - 1 - 0.5k_1\right) + \frac{k_2}{3}\left(2\underline{\theta} + 1 - k_2\right) - \frac{2}{3}t < 0$, $k_1 - k_2 + \frac{k_1^2}{2} + k_1 k_2 + \frac{2tk_1}{k_2} - 3\underline{\theta} k_1 \leq 0$ and $\underline{\theta} \leq \frac{\underline{\theta}(k_1 - 2k_2) + \frac{k_1^2}{2}}{2(k_1 - k_2)} \leq \underline{\theta} + 1$ in equilibrium both firms operate and there is full coverage corner solution. The equilibrium prices are:

$$p_1^* = \frac{\underline{\theta} k_1 + \frac{k_1^2}{2}}{2} = p_H^0$$

$$p_2^* = \underline{\theta} k_2 - t = p_L^0 - t$$

Proof. See Appendix. ■

Hence, in the corner full coverage equilibrium, the price of ULD 1 is not affected by the locational advantage, because it sets a price such that the lowest quality valuation consumer ($\underline{\theta}$) gets a nil surplus, whereas the price of ULD 2 is decreasing with t, with a larger decrease when ULD 2 is the low-quality firm as in that case, to give nil surplus to consumer $\underline{\theta}$, ULD 2 has to fully compensate the lowest valuation consumer by the loss associated with not being at the CBD.

4. Subgame perfect Nash equilibrium

From Pires et al. (2022b), we know that the equilibrium profit functions in the second stage of the game are nondifferentiable in the frontier between two market configurations, which implies that these are potential candidates to be SPNEs. Moreover, in the interior of some market configurations the analytical solutions are long and provide little intuition.

With $t > 0$, even in the simplest cases (like the interior full coverage duopoly), the analytical solutions are difficult to obtain. Hence, we developed a numerical model to solve the first stage of the game, considering the analytical solutions of equilibrium prices described in the previous section. In our numerical model, we varied $\underline{\theta}$ in the interval $[0, 2.5]$ and t in the interval $[0, 1.1]$, with increments of 0.05 for both parameters, which corresponds to 1122 parameter combinations.[13] For each parameter combination, we considered all possible qualities yielding positive demands at marginal cost pricing and considered increments of 0.005 in the choice of the qualities.[14] For each parameter combination and quality vector, the equilibrium in the price game and the corresponding profits were determined, using the results of the previous section. Based on the equilibrium profit matrices, we determined, for each k_2 (for each column of the profit matrix of ULD 1), the level of k_1 with the highest profit for ULD 1 and, similarly, for each k_1 (for each row of the profit matrix of ULD 2) we found the level of k_2 with the highest profit for ULD 2. Note that our numerical approach finds the global best responses as, for each k_j, we find the maximum profit, Π_i, considering all possible k_i. The SPNE was determined by checking when both ULD were in their best responses. Our model replicates all the analytical results in Pires et al. (2022b) when $t = 0$, as well as Wauthy (1996) results for the case of $t = 0$ and nil production cost.

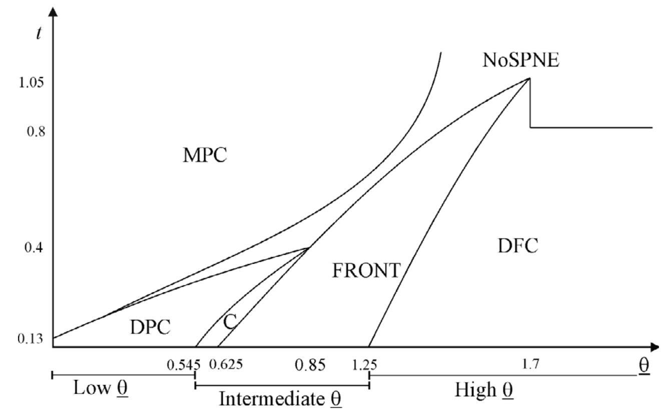

Fig. 3. Type of SPNE as a function of $\underline{\theta}$ and t.

Fig. 3 summarizes our results, regarding the type of SPNE that occurs for each combination of $\underline{\theta}$ and t. For t very small ($0 \leq t \leq 0.115$), the results are very similar to the ones obtained by Pires et al. (2022b). In the case of $t = 0$, there are four possible types of SPNE. For $\underline{\theta} \leq 0.545$, a duopoly with partial coverage (DPC, in the figure) occurs. For $0.545 < \underline{\theta} \leq 0.625$, the SPNE a full coverage duopoly with a corner solution occurs (C, in the figure). For $0.625 < \underline{\theta} < 1.25$, there are multiple SPNEs located in the frontier that separates the interior and the corner full coverage regions (FRONT, in the figure). Finally, for $\underline{\theta} \geq 1.25$, the SPNE is in the interior of the qualities region that leads to an interior full coverage duopoly in the pricing game (DFC). For $0 < t \leq 0.115$, the same equilibrium regions exist but, as shown in Fig. 3, the cutoff values of each region are increasing with t.

However, for higher t ($t \geq 0.13$) and $\underline{\theta}$ below a cutoff level, a new type of SPNE occurs, where ULD 2 chooses $k_2 = 0$ and ULD 1 is a monopolist that covers partially the market (MPC, in the figure). This result is quite logical as, for low values $\underline{\theta}$, ULD 2 is unable to offer a high enough surplus to compete with ULD 1 (due to the transportation cost).

[13] In parameter regions where there are changes in the market configuration we used smaller increments to discover the frontier between those regions more precisely. The algorithm was developed using GAUSS 22, and it is available from the authors upon request.

[14] We considered very small steps to ensure that equilibrium qualities were determine with precision. As the maximum quality that yields positive demand depends on $\underline{\theta}$, the number of (k_1, k_2) that have to be analyzed to identify the equilibrium also depends on $\underline{\theta}$. For instance, for $\underline{\theta} = 2.5$ and for each t, 1,960,000 quality vectors were analyzed.

For $t > 0.4$, we no longer have SPNE with partial coverage duopoly or SPNE with a full coverage duopoly with a corner solution in the pricing game. This result is expected because, as t increases, the asymmetry between the two ULDs is greater and it becomes more difficult for ULD 2 to compete. Moreover, as the cutoff values for the frontier and the DFC regions are increasing with t, the higher is t, the more difficult it is to have full coverage duopoly SPNE.

One aspect that needs to be highlighted is that there are parameter combinations in the transition between the monopoly region and the duopoly partial coverage (and between the monopoly and the frontier region) such that no pure strategy SPNE exists. In the case of $\underline{\theta} = 0$, the interval with no SPNE is $0.115 < t < 0.13$, which is not visible in the figure because it is very small. For higher $\underline{\theta}$, there are larger intervals of t where there is no SPNE, which are visible in the figure. Another region where there is no SPNE is for high values of $\underline{\theta}$ and t. When t is high, ULD 1 has a strong locational advantage and chooses quality levels such that it can behave as a (constrained or unconstrained) monopolist in the price game. However, the fact that $\underline{\theta}$ is high, gives ULD 2 incentives to offer a positive quality that differs from the quality of ULD 1 such that a duopoly would occur in the price game. Hence, there is no pure strategies SPNE.

This overview already provides interesting insights on the impact of t and $\underline{\theta}$ on the SPNE. However, more interesting conclusions can be derived by looking at each region and the corresponding SPNE. The equilibrium quality choices for the different parameter combinations are presented in Appendix B. Table 1 presents the equilibrium quality of ULD 1, k_1^*, when there is a unique SPNE or when there are two or more SPNEs and the firm located in the CBD is the low-quality firm. Under the same circumstances, Table 2 shows the equilibrium quality of the firm located on the periphery, k_2^*. Tables 3 and 4 present the equilibrium qualities of ULD 1 and ULD 2, respectively, when there are two or more SPNE and the firm located in the CBD is the high-quality firm.[15] To explain the equilibrium results, we will use the best responses to illustrate what happens as t and $\underline{\theta}$ change. It should be mentioned that although we represent the best responses for the specific cases of low, intermediate and high $\underline{\theta}$ (namely, $\underline{\theta} = 0.5$, $\underline{\theta} = 1$ and $\underline{\theta} = 2$), the patterns we identify are general as it is clear in the results for all combinations of $\underline{\theta}$ and t.[16]

4.1. SPNE when $\underline{\theta}$ is low

Fig. 4 illustrates the best responses in a case where $\underline{\theta}$ is such that, for $t = 0$, the SPNE quality choices lead to a partial coverage duopoly in the pricing game, i.e., $\underline{\theta} \leq 0.545$ ($\underline{\theta}$ is low). The figure shows the case of $\underline{\theta} = 0.5$, but similar figures could be obtained for other $\underline{\theta} \leq 0.545$ by choosing the appropriate t.

Let us start by analyzing the case of $t = 0$ (Fig. 4(a)). The first thing to mention is that the best responses are discontinuous. This happens for two reasons. One is that there are points where, given the quality of the rival, it is optimal to choose quality levels such that the type of market configuration in the price game changes. This happens, for instance, in the last branch of the best responses (when the rival's quality is above 2.45 where it is optimal to suddenly increase the quality and become a constrained low-quality monopolist with partial coverage, by matching the surplus offered by the other ULD). For even higher qualities of the rival, the low-quality firm can behave as an unconstrained monopolist with partial coverage, which corresponds to the vertical (horizontal) part of ULD 1 (ULD 2) best responses. The second reason for the discontinuities in the best responses is because whether it is a best response to choose a quality below or above the rival depends on the quality level of the rival. In general, the best response when the rival has a low-quality is to differentiate by choosing a higher quality. On the contrary, when the quality offered by the rival is high, the best response is to choose a lower quality. This is clear in Fig. 4(a), as the best response of ULD 1 is below the diagonal for k_2 below 0.9, but above the diagonal for $k_2 > 0.9$ and, similarly, the best response of ULD 2 is above the diagonal for $k_1 < 0.9$ and below the diagonal for $k_1 > 0.9$. This discontinuity happens in a partial coverage duopoly region, where the profit function has two local maxima but the global maximum depends on the quality of the rival. When $t = 0$ there are two SPNEs that are symmetric to each other (represented by circles in the figure). In the SPNE above the diagonal, ULD 2 has high-quality and ULD 1 has low quality. The reverse happens in the SPNE below the diagonal.

When t is positive, the firms are no longer symmetric as ULD 1 has a locational advantage. For t very small (for instance $t = 0.1$) the configuration of the best responses would not be very different from Fig. 4(a). The main difference would be that, for a small range of intermediate values of k_2, the best response of ULD 1 is to choose $k_1 = k_2$. The reason is that, with equal qualities, ULD 1 is able to match the surplus of ULD 2, by charging $p_1 < p_2 + t$ and become a constrained monopolist. For this range of intermediate values, the profit obtained by doing so is above the one obtained under duopoly partial coverage. This happens only for intermediate values, because when ULD 2 has a low quality it is profitable to differentiate with a higher quality and the reverse happens when ULD 2 has a high-quality. Thus the best response of ULD 1 would have two additional discontinuities. However, there would continue to exist two SPNEs, although not symmetric to each other. In fact, with $t > 0$, when ULD 2 is the high-quality firm the equilibrium qualities are higher than when $t = 0$. The reverse happens when ULD 1 is the high-quality firm as, in this case, the equilibrium qualities are lower than when $t = 0$. This indicates a substitutability effect. If the high-quality ULD has a locational disadvantage, it compensates the disadvantage by increasing quality. Moreover, as qualities are strategic complements, in equilibrium, the rival also increases quality. On the contrary, if the high-quality ULD has a locational advantage, it reduces the quality and, in equilibrium, both firms have lower quality. It should be noted that this feature is not specific of the case of low $\underline{\theta}$.

As t increases the locational advantage of ULD 1 becomes higher and, consequently, there are several intervals of k_2 where the best response of ULD 1 is to choose a quality level where it behaves as a (constrained or unconstrained) partial coverage monopoly, including the region where the best response of ULD 1 is to choose $k_1 = k_2$. On the contrary, the interval of k_2 where the best response of ULD 1 is to choose a quality that leads to a partial coverage duopoly, becomes smaller. This happens when $t = 0.25$ (see Fig. 4(b)). For k_2 low, the best response of ULD 1 is to choose qualities where it is a high-quality monopolist. For very low k_2, it is an unconstrained monopolist (the vertical region); for slighter higher k_2, it is a high-quality constrained monopoly (the region where the best response is negatively sloped). This implies that we have only one SPNE, where ULD 2 is the high-quality firm and there is a partial coverage duopoly. From this figure, it is easy to guess that, for slightly higher t, there is no SPNE as the partial coverage region in ULD 1 best response becomes even smaller (this happens for $t = 0.3$).

For even higher t (Fig. 4(c)), the locational advantage of ULD 1 is so high that its best response is always to choose qualities where it is a monopolist (a high-quality monopolist, when k_2 is low; and a low-quality monopolist, when k_2 is high) and a constrained monopolist choosing equal qualities, for intermediate k_2. But, in this case, there is also an important change in the best response of ULD 2. For intermediate values of k_1, it becomes a best response of ULD 2 to choose $k_2 = 0$.

[15] Whenever there are more than two SPNEs, the one shown is the one where the low-quality firm has the lowest quality (among the set of all SPNE that exist for that parameter combination).

[16] The numerical equilibrium results for the SPNE prices are available from the authors upon request.

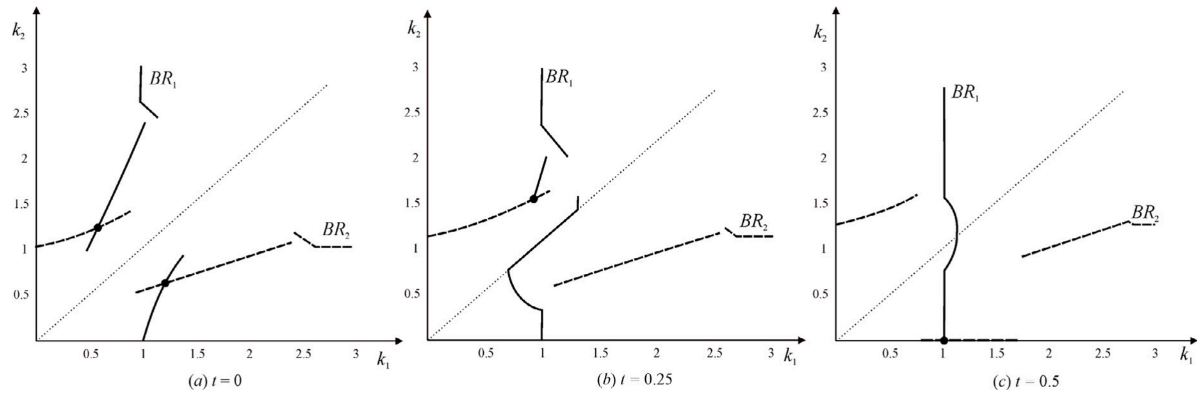

Fig. 4. Best responses for $\underline{\theta} = 0.5$.

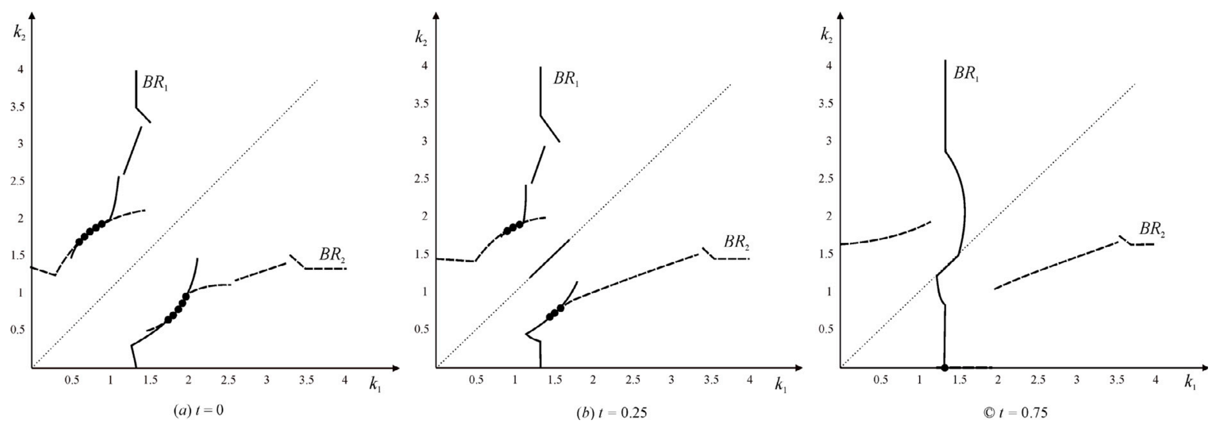

Fig. 5. Best responses for $\underline{\theta} = 1$.

Due to its locational disadvantage, ULD 2 always wants to differentiate from ULD 1. For k_1 low (high), it is profitable for ULD 2 to differentiate by choosing a higher (a lower) quality. But for intermediate k_1, due its locational disadvantage, ULD 2 cannot profitably differentiate and, hence, considering the investment cost of quality, F, the best response of ULD 2 is to choose a nil quality. Hence, for t high, in the unique SPNE, ULD 1 is a partial coverage unconstrained monopoly.

For $\underline{\theta} \leq 0.545$, the pattern is always the one described in Fig. 4. For low t, there are two SPNEs with partial coverage duopoly that are not symmetric. For intermediate values of t, there is a unique SPNE with partial coverage duopoly where ULD 2 is the high-quality firm. For high t, ULD 1 is a monopolist with partial coverage. Moreover, in the transition from the duopoly to monopoly, there is a small range of t where no SPNE in pure strategies exists.

4.2. SPNE when $\underline{\theta}$ is intermediate

In this section we describe the SPNE when $\underline{\theta}$ is intermediate ($0.545 < \underline{\theta} \leq 1.25$). For $t = 0$, this interval of $\underline{\theta}$ corresponds to the cases where the equilibrium quality choices are in the interior of the region where a corner full coverage duopoly holds in the price game or are in the frontier between the corner and interior full coverage quality regions.

Fig. 5 illustrates an intermediate $\underline{\theta}$ case, with $\underline{\theta} = 1$. When $t = 0$ (Fig. 5(*a*)) there are multiple SPNE, both for $k_2 > k_1$ and for $k_1 > k_2$. As shown in Pires et al. (2022b), in this case, the SPNE are in the frontier that separates the interior from the corner full coverage regions and there is a segment in that frontier where all the points are SPNE. In these SPNE, the profit functions of the two ULDs are not differentiable, but the left-derivative is positive and the right-derivative is negative. Thus, each firm is in its best response.

For t small (Fig. 5(*b*)), we continue to have multiple SPNEs in the frontier that separates the interior from the corner full coverage regions, but the frontier regions below and above the diagonal are no longer symmetric. The SPNEs below the diagonal correspond to quality vectors with lower qualities than in the case of $t = 0$, whereas the SPNEs above the diagonal have higher equilibrium qualities than in the case of $t = 0$. Moreover, as expected, there are several intervals of k_2 where ULD 1's best response is to choose a quality level such that it can behave as a monopolist in the price game, including the case of intermediate k_2, where the best response of ULD 1 is to choose $k_1 = k_2$.

For t high (Fig. 5(*c*)), the best responses are very similar to the ones when t is high and $\underline{\theta}$ is low (4(*c*)) and there is a unique SPNE, where ULD 2 chooses a nil quality and ULD 1 is a partial coverage monopolist. It is not represented in the figure, but there is a small set of intermediate t, in the transition from the frontier type of SPNE to the monopoly SPNE, where there is no SPNE because ULD 1 best responses are qualities in which it can behave as a monopolist, but the best response of firm 2 is to differentiate enough to be a duopolist.

It should be noted that the pattern described only occurs for $\underline{\theta} > 0.85$. For $0.545 < \underline{\theta} \leq 0.85$, there is a small region where the corner SPNE occurs (see Fig. 3).[17] For these values of $\underline{\theta}$, there is a small region of t where the SPNEs are corner full coverage equilibria, and a small region of t with partial coverage duopoly in the price game.

[17] This case is not represented in any of our best-response figures. However, the best responses when the SPNE are in the corner full coverage duopoly region are interesting as they show that, in the SPNE, the high-quality firm is on the extreme of the interior full coverage duopoly best response region while the low-quality firm is in a corner best response region, which is characterized by being negatively sloped.

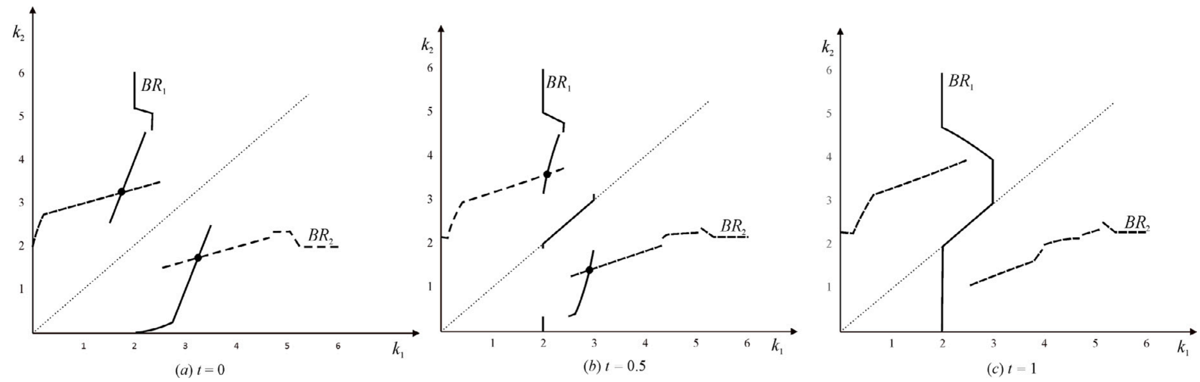

Fig. 6. Best responses for $\underline{\theta} = 2$.

4.3. SPNE when $\underline{\theta}$ is high

Fig. 6 illustrates the best responses in a particular case where $\underline{\theta}$ is high ($\underline{\theta} \geq 1.25$). In these cases, with $t = 0$ there would exist two SPNEs in the interior of the qualities regions where a full coverage duopoly with an interior solution occurs in the price game.

In this case, all the best responses imply that the market is fully covered. For $t = 0$ (Fig. 6(a)), when the rival offers a low quality, the best response is to offer a higher quality than the rival in the frontier. For slightly higher qualities of the rival, it is optimal to offer a quality higher than the rival in the interior of the duopoly full coverage. Still in that region, there is a discontinuity point where the ULD that is offering the highest quality suddenly prefers to start offering a lower quality than the rival. Finally, for higher qualities of the rival the best response is to choose a quality so that the firm becomes a low-quality (unconstrained or constrained) monopolist.

For positive but not very high t ($0 < t \leq 0.8$), the asymmetry between the two ULDs emerges. ULD 1 starts having best response regions corresponding to choices where it is a (constrained or unconstrained) monopolist and, although two SPNEs leading to a full coverage duopoly with an interior solution in the price game continue to exist, they become more and more asymmetric and the market share and profit of ULD 2 become smaller as t increases. The case of $t = 0.5$ (Fig. 6(b)) reveals well the asymmetry in the SPNE, as it is very clear that, when ULD 2 is the high-quality firm (above the diagonal), the equilibrium qualities are much higher than when ULD 1 is the high-quality firm. Comparing with the case of $t = 0$, the equilibrium qualities are higher when ULD 2 is the high-quality firm, but lower when ULD 1 is the high-quality firm. Hence, the high-quality firm always compensates its locational disadvantage or advantage by choosing a higher or a lower quality, respectively.

However, in the high $\underline{\theta}$ case and for high t, we no longer have a SPNE leading to a monopoly in the second stage game. In fact, for high $\underline{\theta}$ and high t there is no SPNE in pure strategies (Fig. 6(c)). Why does this happen? On the one hand, the fact that t is high gives ULD 1 incentives to choose quality levels where it becomes monopolist, including choosing $k_1 = k_2$, when k_2 is intermediate. But, with high $\underline{\theta}$ the best response of ULD 2 is to differentiate from ULD 1 and it is never a best response to choose $k_2 = 0$. As a consequence, we end up in a situation where there is no pure strategies SPNE.[18]

5. Conclusion

In this article, we analyzed a two-stage game between two ULDs in a VPD market, in which one of them is located at the CBD while the other one is located on the periphery. Consumers differ in the way they value the housing quality, but they all incur the same transportation cost if they buy the house on the periphery.

In the second-stage game, we obtained the same type of Nash equilibria as Pires et al. (2022a), with an additional Nash equilibrium when firms have the same quality. In this case, the firm located at the CBD behaves as a constrained monopolist by matching the surplus offered by the other firm at marginal cost pricing. We showed that, in general, the firm located at the CBD has a higher equilibrium price while the firm located on the periphery has a lower equilibrium price than with nil transportation costs, showing that, in general, firms use prices to compensate their locational advantages/disadvantage. Some cases exist, however, where the equilibrium price of the ULD located at the CBD does not depend on the transportation costs, namely when this firm has a guaranteed monopoly or when it is the low-quality firm and a duopoly with a full coverage corner solution occurs in the second-stage.

We showed that, for very low transportation costs, there are four types of SPNE, as in Pires et al. (2022b). When the lowest quality valuation is low, a duopoly with partial coverage occurs. For slightly higher quality valuations, the SPNE leads to a full coverage duopoly with a corner solution. For higher quality valuations, there are multiple SPNE in the frontier that separates the interior and the corner full coverage regions. Finally, when the lowest quality valuation is high the SPNE implies an interior full coverage duopoly. However, with positive transportation costs, the firm with the locational advantage may be able to behave as a monopolist, there are parameter regions where a unique duopoly SPNE arises in equilibrium and, finally, there are cases where no SPNE exists in pure strategies. This last result happens when the quality valuation and transportation costs are both high. In this case, the firm located at the center has incentive to explore its locational advantage and choose quality levels where it can behave as a monopolist, whereas the firm located on the periphery wants to operate and offer a quality that differs substantially from the rival, to overcome the locational disadvantage. Hence, there is no SPNE in pure strategies.

A very interesting pattern observed in all duopoly SPNEs is that there is a substitutability effect between quality and location, which is driven by the high-quality firm. When there is a unique SPNE, the firm located in the CBD is the low-quality firm. When there are two or more SPNEs, it all depends on whether the high-quality firm has a locational advantage or disadvantage. In the equilibrium where the high-quality firm is located at the CBD (on the periphery), in equilibrium both firms offer lower (higher) qualities than in the absence of preexistent asymmetries. The reason is that the high-quality firm compensates its locational advantage (disadvantage) by decreasing (increasing) its quality and, considering the strategic complementarity of quality choices, the low-quality firm does the same. Thus, equilibrium qualities may be both lower or both higher than in the absence of preexistent asymmetries, depending on which of the multiple SPNEs occurs.

[18] There may exist mixed strategies SPNE, however we do not analyze them in the current paper.

Our paper strengthens the VPD literature and determines the impact of preexistent firms' asymmetries on quality-price mix and market configuration. The lack of endogenization of the firms' locations may be pointed as a limitation of our work. Besides overcoming this limitation, future avenues for research should include sequential quality decisions and explore other sources of asymmetry such as differences in the structure of firms' costs.

Declaration of competing interest

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Data availability

No data was used for the research described in the article.

Appendix A. Proofs

Proof of Lemma 1. The consumer with the highest quality valuation, $\theta = \underline{\theta}+1$, only has positive net utility if he buys from ULD 1 at a price equal to its marginal cost, $p_1 = c_1$, if

$$U_1(\underline{\theta}+1) = (\underline{\theta}+1)k_1 - \frac{k_1^2}{2} > 0 \quad \Leftrightarrow \quad k_1 < 2(\underline{\theta}+1).$$

Thus, if k_1 is equal or greater than $2(\underline{\theta}+1)$, ULD 1 has zero demand even if it charges a price equal to its marginal cost. Similarly, a necessary condition for ULD 2 to have a positive demand is:

$$(\underline{\theta}+1)k_2 - t - \frac{k_2^2}{2} > 0.$$

The maximum value of the surplus of the highest valuation consumer is $\frac{1}{2}\left(\underline{\theta}+1\right)^2 - t$. Thus, the previous condition can only hold for $t < \frac{1}{2}\left(\underline{\theta}+1\right)^2$. In addition, the expression only is positive between the roots of the quadratic equation. Thus ULD 2 only has positive demand for prices above marginal cost if:

$$\underline{k}_2 < k_2 < \overline{k}_2 \text{ where } (\underline{\theta}+1) - \underline{k}_2 = \sqrt{(\underline{\theta}+1)^2 - 2t} \text{ and } \overline{k}_2 = (\underline{\theta}+1) + \sqrt{(\underline{\theta}+1)^2 - 2t}. \quad \blacksquare$$

Proof of Lemma 2. Consumer with valuation θ prefers to buy from ULD 1 than not buy if and only if:

$$U_1(\theta) = \theta k_1 - p_1 \geq 0 \quad \Leftrightarrow \quad \theta \geq \frac{p_1}{k_1} = \hat{\theta}_1.$$

The consumer $\hat{\theta}_1$ is indifferent between buying from ULD 1 or not buying at all. Thus, all consumers with $\theta > \hat{\theta}_1$ strictly prefer to buy from ULD 1 than not to buy, whereas all consumers with $\theta < \hat{\theta}_1$ prefer not buy than to buy from ULD 1.

Similarly, one can find the consumers who prefer to buy from ULD 2 than not buying, by solving:

$$U_2(\theta) = \theta k_2 - t - p_2 \geq 0 \quad \Leftrightarrow \quad \theta \geq \frac{p_2+t}{k_2} = \hat{\theta}_2$$

Again, the consumers with valuations above $\hat{\theta}_2$ strictly prefer to buy from ULD 2 than not buying. $\blacksquare$

Proof of Lemma 3. Assuming $k_2 > k_1$, the difference in utilities for consumer θ, $U_1(\theta) - U_2(\theta)$, is positive if $\left(\theta k_1 - p_1\right) - \left(\theta k_2 - t - p_2\right) > 0$, or equivalently, $p_2 + t - p_1 > \theta\left(k_2 - k_1\right)$. Since the right-hand side of the previous expression is increasing with θ, that implies that, if the condition holds for $\left(\underline{\theta}+1\right)$, then it holds for any $\theta < \underline{\theta}+1$. A similar proof holds in the case of $k_1 > k_2$. $\blacksquare$

Proof of Lemma 4. Assuming $k_2 > k_1$, the difference in utilities for consumer θ, $U_2(\theta) - U_1(\theta)$, is positive if $\left(\theta k_2 - t - p_2\right) - \left(\theta k_1 - p_1\right) > 0$, or equivalently, if $p_2 + t - p_1 < \theta\left(k_2 - k_1\right)$. Since the right-hand side of the previous expression is increasing with θ, that implies that, if the condition holds for $\underline{\theta}$, then it holds for any $\theta > \underline{\theta}$. A similar proof holds in the case of $k_1 > k_2$. $\blacksquare$

Proof of Lemma 5. Assume that $k_2 > k_1$, then, from Lemma 3, we know that ULD 2 can only have positive demand if the highest valuation consumer prefers the higher quality house. Moreover, from Lemma 4, we know that ULD 1 can only have positive demand if the lowest valuation consumer prefers the lower quality house. This implies that $U_2(\theta)-U_1(\theta) = \left(\theta k_2 - t - p_2\right)-\left(\theta k_1 - p_1\right)$ is negative at $\underline{\theta}$ but positive at $\underline{\theta}+1$. Since the function is continuous in θ, there exists an intermediate value of θ, θ^*, where $\left(\theta k_2 - t - p_2\right) - \left(\theta k_1 - p_1\right) = 0$. Moreover, since $U_2 - U_1$ is increasing in θ, then all consumers to the right of θ^* prefer to buy the house from ULD 2, whereas all consumers to the left of θ^* prefer to buy from ULD 1. A similar proof holds in the case of $k_1 > k_2$. $\blacksquare$

Proof of Lemma 6. When $k_2 > k_1$ consumers prefer to buy from ULD 1 than from ULD 2 if:

$$\theta k_1 - p_1 \geq \theta k_2 - t - p_2 \quad \Leftrightarrow \quad \theta \leq \frac{p_2 - p_1 + t}{k_2 - k_1} \equiv \tilde{\theta}$$

The consumers $\theta > \tilde{\theta}$ strictly prefer to buy from ULD 2, whereas the consumers $\theta < \tilde{\theta}$ strictly prefer to buy from ULD 1. Therefore, the higher valuation consumers buy from the higher quality ULD while the lower valuation consumers buy from the lower quality ULD.

Similarly, when $k_1 > k_2$ consumers prefer to buy from ULD 1 than from ULD 2 if:

$$\theta k_1 - p_1 \geq \theta k_2 - t - p_2 \quad \Leftrightarrow \quad \theta \geq \frac{p_1 - p_2 - t}{k_1 - k_2} \equiv \tilde{\theta}$$

The consumers $\theta > \tilde{\theta}$ strictly prefer to buy from ULD 1, whereas the consumers $\theta < \tilde{\theta}$ strictly prefer to buy from ULD 2. Therefore, the higher valuation consumers buy from the higher quality ULD, while the lower valuation consumers buy from the lower quality ULD. $\blacksquare$

Proof of Proposition 1. From Lemma 1, we already know that if $k_2 < (\underline{\theta}+1) - \sqrt{(\underline{\theta}+1)^2 - 2t}$ or $k_2 > (\underline{\theta}+1) + \sqrt{(\underline{\theta}+1)^2 - 2t}$ and $0 < k_1 < 2(\underline{\theta}+1)$, ULD 2 has nil demand and ULD 1 has positive demand. Thus, ULD 1 behaves as a monopolist. With partial coverage, the monopolist ULD 1 solves the following problem:

$$\max_{p_1} \Pi_1 = \left(\underline{\theta}+1 - \frac{p_1}{k_1}\right)(p_1 - c_1) \quad \text{subject to} \quad p_1 > \underline{\theta} k_1$$

If we solve the unconstrained problem, the first-order condition is

$$\frac{d\Pi_1}{dp_1} = -\frac{p_1 - c_1}{k_1} + \underline{\theta} + 1 - \frac{p_1}{k_1} = 0$$

Note that $\frac{d^2\Pi_1}{dp_1^2} < 0$, thus second-order condition is satisfied. Solving the first-order condition with respect to p_1 and substituting $c_1 = \frac{k_1^2}{2}$ we obtain:

$$p_1^* = \frac{\frac{k_1^2}{2} + k_1(\underline{\theta}+1)}{2}$$

In order for partial coverage to hold:

$$U_1(\underline{\theta}) < 0 \quad \Leftrightarrow \quad \frac{\frac{k_1^2}{2} + k_1(\underline{\theta}+1)}{2} > \underline{\theta} k_1 \quad \Leftrightarrow \quad k_1\left(k_1 + 2(1-\underline{\theta})\right) > 0$$

which holds for every $k_1 > 0$ when $\underline{\theta} \leq 1$ and it also holds for $\underline{\theta} > 1$ and $k_1 > 2(\underline{\theta}-1)$. However, if $\underline{\theta} > 1$ and $k_1 \leq 2(\underline{\theta}-1)$ the previous solution no longer holds. In this case, ULD 1 fully covers the market and solves:

$$\max_{p_1} \Pi_1 = (p_1 - c_1) \quad \text{subject to} \quad p_1 \leq \underline{\theta} k_1$$

As the profit function increases linearly with p_1, it is optimal to charge the highest price possible. That is, with full coverage, the optimal price is $p_1^* = \underline{\theta} k_1$.

From Lemma 1, we know that for $k_1 = 0$ or $k_1 \geq 2(\underline{\theta}+1)$ and $(\underline{\theta}+1) - \sqrt{(\underline{\theta}+1)^2 - 2t} < k_2 < (\underline{\theta}+1) + \sqrt{(\underline{\theta}+1)^2 - 2t}$, ULD 1 has nil demand and ULD 2 has positive demand. If the market is partially covered, ULD 2 solves the following problem:

$$\max_{p_2} \Pi_2 = \left(\underline{\theta} + 1 - \frac{p_2 + t}{k_2}\right)(p_2 - c_2) \quad \text{subject to} \quad p_2 > \underline{\theta} k_2 - t$$

If we solve this problem ignoring the constraint, the first-order condition is:

$$\frac{d\Pi_2}{dp_2} = -\frac{p_2 - c_2}{k_2} + \underline{\theta} + 1 - \frac{p_2 + t}{k_2} = 0$$

Note that $\frac{d^2\Pi_2}{dp_2^2} < 0$, thus second-order condition is satisfied. Then, solving the first-order condition with respect to p_2 and substituting $c_2 = \frac{k_2^2}{2}$ we get:

$$p_2^* = \frac{\frac{k_2^2}{2} + k_2(\underline{\theta}+1) - t}{2}$$

In order for this to be the solution, i.e., the market is partially covered, the lowest valuation consumer has to get a negative surplus:

$$U_2(\underline{\theta}) < 0 \;\Leftrightarrow\; \underline{\theta} k_2 - t - \frac{\frac{k_2^2}{2} + k_2(\underline{\theta}+1) - t}{2} < 0 \;\Leftrightarrow\; \frac{1}{2}k_2\left(\underline{\theta} - 1 - \frac{k_2}{2} - \frac{t}{k_2}\right) < 0$$

which is always true for $\underline{\theta} \leq 1$. For $\underline{\theta} > 1$, the condition holds as long as $\frac{k_2}{2} + \frac{t}{k_2} > \underline{\theta} - 1$. If $\frac{k_2}{2} + \frac{t}{k_2} < \underline{\theta} - 1$, the lowest valuation consumer gets a positive surplus with the previous price, hence the market is be fully covered. In that case, the correct optimization problem is:

$$\max_{p_2} \Pi_2 = (p_2 - c_2) \quad \text{subject to} \quad p_2 \leq \underline{\theta} k_2 - t$$

As the profit function increases linearly with p_2, the optimal price is the highest price that guarantees full coverage, thus $p_2^* = \underline{\theta} k_2 - t$. ■

Proof of Proposition 2. When $k_2 > k_1$, the indifferent consumer is given by:

$$\widetilde{\theta} = \frac{p_2 - p_1 + t}{k_2 - k_1}$$

Hence, if both firms operate and the market is fully covered, the profit functions for the two ULDs are given by:

$$\Pi_1 = \left(\frac{p_2 - p_1 + t}{k_2 - k_1} - \underline{\theta}\right)(p_1 - c_1)$$
$$\Pi_2 = \left(\underline{\theta} + 1 - \frac{p_2 - p_1 + t}{k_2 - k_1}\right)(p_2 - c_2)$$

The first-order conditions of the two ULDs profit maximization problems are:

$$\frac{\partial \Pi_1}{\partial p_1} = -\frac{p_1 - c_1}{k_2 - k_1} + \left(\frac{p_2 - p_1 + t}{k_2 - k_1} - \underline{\theta}\right) = 0$$
$$\frac{\partial \Pi_2}{\partial p_2} = -\frac{p_2 - c_2}{k_2 - k_1} + \left(\underline{\theta} + 1 - \frac{p_2 - p_1 + t}{k_2 - k_1}\right) = 0$$

Note that $\frac{d^2\Pi_1}{dp_1^2} < 0$ and $\frac{d^2\Pi_2}{dp_2^2} < 0$, thus the second-order conditions of the two maximization problems are satisfied. Solving the previous system with respect to p_1 and p_2, we obtain the equilibrium prices:

$$p_1^* = \frac{(1 - \underline{\theta})(k_2 - k_1) + 2c_1 + c_2 + t}{3}$$
$$p_2^* = \frac{(\underline{\theta} + 2)(k_2 - k_1) + 2c_2 + c_1 - t}{3}$$

Substituting c_i by $\frac{k_i^2}{2}$ we get the equilibrium prices in the proposition. In equilibrium, the indifferent consumer is given by:

$$\widetilde{\theta}^* = \frac{p_2^* - p_1^* + t}{k_2 - k_1} = \frac{1 + 2\underline{\theta}}{3} + \frac{k_2 + k_1}{6} + \frac{t}{3(k_2 - k_1)}$$

For this to be an equilibrium, both firms must have positive demand, i.e.:

$$\underline{\theta} \leq \widetilde{\theta}^* \leq \underline{\theta} + 1 \quad \Leftrightarrow \quad 2(\underline{\theta} - 1) \leq k_2 + k_1 + \frac{2t}{k_2 - k_1} \leq 2(\underline{\theta} + 2)$$

Moreover, the market must be fully covered, i.e., the lowest quality valuation, $\underline{\theta}$, must have a nonnegative net utility if buying from ULD 1 at price p_1^*.:

$$U_1(\underline{\theta}) = \underline{\theta} k_1 - p_1^* \geq 0 \quad \Leftrightarrow \quad \frac{k_1}{3}(2\underline{\theta} + 1 - k_1) + \frac{k_2}{3}\left(\underline{\theta} - 1 - \frac{k_2}{2}\right) - \frac{t}{3} \geq 0$$

Note that as t increases this condition is more difficult to be satisfied.

Similarly, if $k_1 > k_2$ the indifferent consumer is given by:

$$\widetilde{\theta} = \frac{p_1 - p_2 - t}{k_1 - k_2}$$

Hence, the profit functions for the two ULDs are given by the following expressions:

$$\Pi_1 = \left(\underline{\theta} + 1 - \frac{p_1 - p_2 - t}{k_1 - k_2}\right)(p_1 - c_1)$$
$$\Pi_2 = \left(\frac{p_1 - p_2 - t}{k_1 - k_2} - \underline{\theta}\right)(p_2 - c_2)$$

The first-order conditions of the profit maximization problems are:

$$\frac{\partial \Pi_1}{\partial p_1} = -\frac{p_1 - c_1}{k_1 - k_2} + \underline{\theta} + 1 - \frac{p_1 - p_2 - t}{k_1 - k_2} = 0$$
$$\frac{\partial \Pi_2}{\partial p_2} = -\frac{p_2 - c_2}{k_1 - k_2} + \frac{p_1 - p_2 - t}{k_1 - k_2} - \underline{\theta} = 0$$

Note that $\frac{d^2\Pi_1}{dp_1^2} < 0$ and $\frac{d^2\Pi_2}{dp_2^2} < 0$, thus the second-order conditions of the two ULDs are satisfied. Solving this system of equations with respect to p_1 and p_2 we obtain the equilibrium prices:

$$p_1^* = \frac{(\underline{\theta} + 2)(k_1 - k_2) + 2c_1 + c_2 + t}{3}$$
$$p_2^* = \frac{(1 - \underline{\theta})(k_1 - k_2) + 2c_2 + c_1 - t}{3}$$

The equilibrium indifferent consumer is given by:

$$\widetilde{\theta}^* = \frac{1 + 2\underline{\theta}}{3} + \frac{k_2 + k_1}{6} - \frac{t}{3(k_1 - k_2)}$$

Again, for this to be an equilibrium, we must have $\underline{\theta} \leq \widetilde{\theta}^* \leq \underline{\theta} + 1$, which implies that

$$2(\underline{\theta} - 1) \leq k_2 + k_1 - \frac{2t}{k_1 - k_2} \leq 2(\underline{\theta} + 2)$$

and the consumer with lowest quality valuation, $\underline{\theta}$, has to have a nonnegative surplus:

$$U_2(\underline{\theta}) = \underline{\theta} k_2 - t - p_2^* \geq 0 \quad \Leftrightarrow \quad \frac{k_1}{3}\left(\underline{\theta} - 1 - \frac{k_1}{2}\right) + \frac{k_2}{3}(2\underline{\theta} + 1 - k_2) - \frac{2}{3}t \geq 0.$$ ■

Proof of Proposition 3. When $k_2 > k_1$, if both firms operate, $\widetilde{\theta}$ and $\widehat{\theta}$ are given by:

$$\widetilde{\theta} = \frac{p_2 - p_1 + t}{k_2 - k_1} \quad \text{and} \quad \widehat{\theta} = \frac{p_1}{k_1}$$

Hence, the profit functions for the two ULDs are given by:

$$\Pi_1 = \left(\frac{p_2 - p_1 + t}{k_2 - k_1} - \frac{p_1}{k_1}\right)(p_1 - c_1)$$
$$\Pi_2 = \left(\underline{\theta} + 1 - \frac{p_2 - p_1 + t}{k_2 - k_1}\right)(p_2 - c_2)$$

The first-order conditions of the two ULDs profit maximization problems are:

$$\frac{\partial \Pi_1}{\partial p_1} = -\frac{p_1-c_1}{k_2-k_1}-\frac{p_1-c_1}{k_1}+\frac{p_2-p_1+t}{k_2-k_1}-\frac{p_1}{k_1}=0$$
$$\frac{\partial \Pi_2}{\partial p_2} = -\frac{p_2-c_2}{k_2-k_1}+\underline{\theta}+1-\frac{p_2-p_1+t}{k_2-k_1}=0$$

Note that $\frac{d^2\Pi_1}{dp_1^2}<0$ and $\frac{d^2\Pi_2}{dp_2^2}<0$, thus the second-order conditions of the two ULDs are satisfied. Solving the system of first-order conditions with respect to p_1 and p_2, we obtain the equilibrium prices:

$$p_1^* = \frac{k_1(\underline{\theta}+1)\left(k_2-k_1\right)+2c_1k_2+c_2k_1+tk_1}{4k_2-k_1}$$
$$p_2^* = \frac{2k_2(\underline{\theta}+1)\left(k_2-k_1\right)+c_1k_2+2c_2k_2-t\left(2k_2-k_1\right)}{4k_2-k_1}$$

Substituting c_i by $\frac{k_i^2}{2}$ we obtain the equilibrium prices in the proposition. For this solution to hold, the lowest valuation consumer (that buys from ULD 1 because $k_1<k_2$) must have negative utility (as otherwise the market would be fully covered):

$$U_1(\underline{\theta})<0 \quad\Leftrightarrow\quad \underline{\theta}k_1-p_1^*<0 \Leftrightarrow \frac{k_1}{4k_2-k_1}\left(3\underline{\theta}k_2-\left(k_2-k_1\right)-\frac{k_2^2}{2}-k_1k_2-t\right)<0$$

which is equivalent to

$$k_2-k_1+\frac{k_2^2}{2}+k_1k_2+t-3\underline{\theta}k_2>0$$

In equilibrium, the indifferent consumer is given by:

$$\widetilde{\theta}^* = \frac{(\underline{\theta}+1)\left(2k_2-k_1\right)}{4k_2-k_1}+\frac{0.5k_2\left(k_1+2k_2\right)}{4k_2-k_1}+\frac{t\left(2k_2-k_1\right)}{\left(4k_2-k_1\right)\left(k_2-k_1\right)}$$

For this to be a Nash equilibrium it must be that $\hat{\theta}_1^*\leq\widetilde{\theta}^*$ (as otherwise all consumers would buy from ULD 2), which is equivalent to:

$$\left(\underline{\theta}+1\right)+\frac{k_2-k_1}{2}+\frac{t}{k_2-k_1}\geq 0$$

which is always true (as we are assuming $k_2>k_1$). In addition, to guarantee that ULD 2 operates (otherwise all consumers would buy from ULD 1, which would be a monopolist):

$$\widetilde{\theta}^*\leq\underline{\theta}+1 \quad\Leftrightarrow\quad \frac{k_2\left(k_2+0.5k_1-2\left(\underline{\theta}+1\right)\right)}{4k_2-k_1}+\frac{t\left(2k_2-k_1\right)}{\left(4k_2-k_1\right)\left(k_2-k_1\right)}\leq 0$$

When $k_1>k_2$ and both ULDs operate in equilibrium and there is partial coverage, $\widetilde{\theta}$ and $\widehat{\theta}$ are given by:

$$\widetilde{\theta}=\frac{p_1-p_2-t}{k_1-k_2} \quad\text{and}\quad \widehat{\theta}=\frac{p_2+t}{k_2}$$

And the profit functions are:

$$\Pi_1 = \left(\underline{\theta}+1-\frac{p_1-p_2-t}{k_1-k_2}\right)(p_1-c_1)$$
$$\Pi_2 = \left(\frac{p_1-p_2-t}{k_1-k_2}-\frac{p_2+t}{k_2}\right)(p_2-c_2)$$

The first-order conditions are given by:

$$\frac{\partial \Pi_1}{\partial p_1} = -\frac{p_1-c_1}{k_1-k_2}+\underline{\theta}+1-\frac{p_1-p_2-t}{k_1-k_2}=0$$
$$\frac{\partial \Pi_2}{\partial p_2} = -\frac{p_2-c_2}{k_1-k_2}-\frac{p_2-c_2}{k_2}+\frac{p_1-p_2-t}{k_1-k_2}-\frac{p_2+t}{k_2}=0$$

Note that $\frac{d^2\Pi_1}{dp_1^2}<0$ and $\frac{d^2\Pi_2}{dp_2^2}<0$, thus the second-order conditions of the two ULDs are satisfied. After solving the first-order conditions with respect to p_1 and p_2 we get the following equilibrium prices:

$$p_1^* = \frac{2k_1(\underline{\theta}+1)(k_1-k_2)+k_1\left(2c_1+c_2+t\right)}{4k_1-k_2}$$
$$p_2^* = \frac{k_2(\underline{\theta}+1)(k_1-k_2)+2k_1c_2+k_2c_1-t\left(2k_1-k_2\right)}{4k_1-k_2}$$

Substituting c_i for $\frac{k_i^2}{2}$ we get the equilibrium prices in the proposition.

For this solution to hold, the lowest valuation consumer has to get a negative surplus buying from ULD 2, that is:

$$U_2(\underline{\theta})<0 \quad\Leftrightarrow\quad \underline{\theta}k_2-t-p_2^*<0 \quad\Leftrightarrow\quad k_1-k_2+\frac{k_1^2}{2}+k_1k_2+\frac{2tk_1}{k_2}-3\underline{\theta}k_1>0$$

In equilibrium, the indifferent consumer is given by:

$$\widetilde{\theta}^* = \frac{\left(2k_1-k_2\right)\left(\underline{\theta}+1\right)}{4k_1-k_2}+\frac{k_1\left(k_1+0.5k_2\right)}{4k_1-k_2}-\frac{tk_1}{\left(4k_1-k_2\right)\left(k_1-k_2\right)}$$

For this to be a solution, we have to have a duopoly. Thus:

$$\widetilde{\theta}^*\leq\underline{\theta}+1 \quad\Leftrightarrow\quad \frac{k_1\left(k_1+\frac{k_2}{2}-2\left(\underline{\theta}+1\right)\right)}{4k_1-k_2}-\frac{tk_1}{\left(4k_1-k_2\right)\left(k_1-k_2\right)}\leq 0. \quad ■$$

Proof of Proposition 4. If $k_1<k_2$ and $\frac{k_1}{3}\left(2\underline{\theta}+1-k_1\right)+\frac{k_2}{3}\left(\underline{\theta}-1-\frac{k_2}{2}\right)-\frac{t}{3}<0$, $k_2-k_1+\frac{k_2^2}{2}+k_1k_2+t-3\underline{\theta}k_2\leq 0$ the equilibrium is neither an interior full coverage nor a partial coverage duopoly. In this case, if we consider the full coverage problem, ULD 1 constraint of offering a nonnegative surplus to the lowest valuation consumer becomes binding and thus the profit maximizing price is $p_1^*=\underline{\theta}k_1$, that is, the best response does not depend on p_2. On the other hand, the first-order condition of ULD 2 is:

$$\frac{\partial \Pi_2}{\partial p_2}=-\frac{p_2-c_2}{k_2-k_1}+\left(\underline{\theta}+1-\frac{p_2-p_1+t}{k_2-k_1}\right)=0$$

Note that $\frac{d^2\Pi_2}{dp_2^2}<0$, thus the second-order condition is satisfied. Solving the system of best responses and substituting the marginal costs $c_i=\frac{k_i^2}{2}$ we get the equilibrium prices.

In this solution, we already assumed that the lowest valuation consumer has nil utility. However, in order for this to be the equilibrium, we still have to check if $\underline{\theta}\leq\widetilde{\theta}^{(}\leq\underline{\theta}+1$ (so that both firms operate). In equilibrium, the indifferent consumer is:

$$\widetilde{\theta}^*=\frac{p_2^*-p_1^*+t}{k_2-k_1}=\frac{1}{2}+\frac{\underline{\theta}\left(k_2-2k_1\right)+\frac{k_2^2}{2}+t}{2\left(k_2-k_1\right)}$$

Hence, the corner full coverage duopoly only holds if:

$$\underline{\theta}\leq\widetilde{\theta}(p_1^*,p_2^*)\leq\underline{\theta}+1 \quad\Leftrightarrow\quad \underline{\theta}-\frac{1}{2}\leq\frac{\underline{\theta}\left(k_2-2k_1\right)+\frac{k_2^2}{2}+t}{2\left(k_2-k_1\right)}\leq\underline{\theta}+\frac{1}{2}.$$

If $k_2<k_1$, $\frac{k_1}{3}\left(\underline{\theta}-1-\frac{k_1}{2}\right)+\frac{k_2}{3}\left(2\underline{\theta}+1-k_2\right)-\frac{2}{3}t<0$, and $k_1-k_2+\frac{k_1^2}{2}+k_1k_2+\frac{2tk_1}{k_2}-3\underline{\theta}k_1\leq 0$, if we consider the full coverage problem, ULD 2 constraint of offering a nonnegative surplus to the lowest valuation consumer becomes binding and thus its profit maximizing price is $p_2^*=\underline{\theta}k_2-t$. On the other hand, the first-order condition of ULD 1 is:

$$\frac{\partial \Pi_1}{\partial p_1}=-\frac{p_1-c_1}{k_1-k_2}+\underline{\theta}+1-\frac{p_1-p_2-t}{k_1-k_2}=0$$

Note that $\frac{d^2\Pi_1}{dp_1^2}<0$, thus the second-order condition is satisfied. Solving the system of best responses and substituting the marginal costs $c_i=\frac{k_i^2}{2}$ we get the equilibrium prices.

In order for this to be the equilibrium, we still have to check if $\underline{\theta}\leq\widetilde{\theta}^*\leq\underline{\theta}+1$. In equilibrium, the indifferent consumer is:

$$\widetilde{\theta}^*=\frac{p_1^*-p_2^*-t}{k_1-k_2}=\frac{\underline{\theta}\left(k_1-2k_2\right)+\frac{k_1^2}{2}}{2\left(k_1-k_2\right)}$$

Thus, for the corner Nash equilibrium to hold we must have:

$$\underline{\theta}\leq\widetilde{\theta}^*\leq\underline{\theta}+1 \quad\Leftrightarrow\quad \underline{\theta}\leq\frac{\underline{\theta}\left(k_1-2k_2\right)+\frac{k_1^2}{2}}{2\left(k_1-k_2\right)}\leq\underline{\theta}+1. \quad ■$$

Appendix B. Numerical results

See Tables 1–4.

Table 1
Equilibrium quality of the firm located in the center as a function of t and $\underline{\theta}$ when under duopoly the high-quality firm is located on the periphery (no value means that there is no SPNE).

	t													
		0.00	0.05	0.10	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.05	1.15
	0.00	0.400	0.490	0.590	0.665	0.665	0.665	0.665	0.665	0.665	0.665	0.665	0.665	0.665	0.665
	0.05	0.420	0.505	0.595	0.700	0.700	0.700	0.700	0.700	0.700	0.700	0.700	0.700	0.700	0.700
	0.10	0.440	0.520	0.610		0.735	0.735	0.735	0.735	0.735	0.735	0.735	0.735	0.735	0.735
	0.15	0.460	0.535	0.620	0.710	0.765	0.765	0.765	0.765	0.765	0.765	0.765	0.765	0.765	0.765
	0.20	0.475	0.550	0.630	0.715	0.800	0.800	0.800	0.800	0.800	0.800	0.800	0.800	0.800	0.800
	0.25	0.500	0.570	0.645	0.725	0.835	0.835	0.835	0.835	0.835	0.835	0.835	0.835	0.835	0.835
	0.30	0.520	0.585	0.660	0.735	0.865	0.865	0.865	0.865	0.865	0.865	0.865	0.865	0.865	0.865
	0.35	0.540	0.600	0.670	0.745		0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900
	0.40	0.555	0.620	0.685	0.755		0.935	0.935	0.935	0.935	0.935	0.935	0.935	0.935	0.935
	0.45	0.575	0.640	0.700	0.770		0.965	0.965	0.965	0.965	0.965	0.965	0.965	0.965	0.965
	0.50	0.513	0.655	0.720	0.780	0.920	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.55	0.510	0.675	0.735	0.795	0.925		1.035	1.035	1.035	1.035	1.035	1.035	1.035	1.035
	0.60	0.505	0.595	0.750	0.805	0.935		1.065	1.065	1.065	1.065	1.065	1.065	1.065	1.065
	0.65	0.485	0.585	0.670	0.820	0.945		1.100	1.100	1.100	1.100	1.100	1.100	1.100	1.100
	0.70	0.475	0.525	0.640		0.950		1.135	1.135	1.135	1.135	1.135	1.135	1.135	1.135
	0.75	0.465	0.490	0.590	0.770	0.960	1.085		1.165	1.165	1.165	1.165	1.165	1.165	1.165
	0.80	0.465	0.500	0.510	0.695		1.090		1.200	1.200	1.200	1.200	1.200	1.200	1.200
$\underline{\theta}$	0.85	0.450	0.485	0.510	0.645		1.100		1.235	1.235	1.235	1.235	1.235	1.235	1.235
	0.90	0.440	0.475	0.490	0.670	0.930				1.265	1.265	1.265	1.265	1.265	1.265
	0.95	0.450	0.495	0.525	0.570	0.860				1.300	1.300	1.300	1.300	1.300	1.300
	1.00	0.545	0.580	0.705	0.660	0.810	1.045			1.335	1.335	1.335	1.335	1.335	1.335
	1.05	0.640	0.670	0.710	0.740	0.810	0.975	1.245			1.365	1.365	1.365	1.365	1.365
	1.10	0.735	0.760	0.785	0.815	0.880	0.955	1.140				1.400	1.400	1.400	1.400
	1.15	0.825	0.845	0.870	0.900	0.955	1.025	1.100	1.315			1.435	1.435	1.435	1.435
	1.20	0.910	0.930	0.955	0.980	1.030	1.090	1.160	1.250				1.465	1.465	1.465
	1.25	1.000	1.020	1.040	1.060	1.105	1.160	1.225	1.305	1.420				1.500	1.500
	1.30	1.050	1.080	1.115	1.140	1.185	1.235	1.295	1.355	1.445	1.640			1.535	1.535
	1.40	1.150	1.180	1.215	1.250	1.315	1.380	1.430	1.485	1.550	1.625	1.760			
	1.50	1.250	1.280	1.315	1.350	1.415	1.480	1.550	1.615	1.670	1.735	1.810	1.915		
	1.60	1.350	1.380	1.415	1.450	1.515	1.580	1.650	1.715	1.785	1.850	1.910	1.980		
	1.70	1.450	1.480	1.515	1.550	1.615	1.680	1.750	1.815	1.885	1.950	2.015	2.080	2.155	
	1.80	1.550	1.580	1.615	1.650	1.715	1.780	1.850	1.915	1.985	2.050				
	1.90	1.650	1.680	1.715	1.750	1.815	1.880	1.950	2.015	2.085	2.150				
	2.00	1.750	1.780	1.815	1.850	1.915	1.980	2.050	2.115	2.185	2.250				
	2.10	1.850	1.880	1.915	1.950	2.015	2.080	2.150	2.215	2.285	2.350				
	2.20	1.950	1.980	2.015	2.050	2.115	2.180	2.250	2.315	2.385	2.450				
	2.30	2.050	2.080	2.115	2.150	2.215	2.280	2.350	2.415	2.485	2.550				
	2.40	2.150	2.180	2.215	2.250	2.315	2.380	2.450	2.515	2.585	2.650				
	2.50	2.250	2.280	2.315	2.350	2.415	2.480	2.550	2.615	2.685	2.750				

Note: Due to space limits we do not show all results. Increments for t and $\underline{\theta}$ are 0.05 for low values and 0.1 for high ones.

Table 2
Equilibrium quality of the firm located on the periphery as a function of t and $\underline{\theta}$ when under duopoly the high-quality firm is located on the periphery (no value means that there is no SPNE).

t		0.00	0.05	0.10	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.05	1.15
	0.00	0.820	0.920	1.020	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.05	0.860	0.955	1.045	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.10	0.900	0.990	1.080		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.15	0.945	1.025	1.115	1.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.20	0.980	1.065	1.145	1.230	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.25	1.025	1.105	1.180	1.260	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.30	1.065	1.140	1.215	1.290	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.35	1.105	1.175	1.250	1.325		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.40	1.145	1.215	1.285	1.355		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.45	1.185	1.255	1.320	1.390		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.50	1.230	1.295	1.360	1.425	1.560	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.55	0.998	1.335	1.395	1.460	1.590		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.60	0.995	1.090	1.435	1.490	1.620		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.65	1.025	1.080	1.175	1.530	1.650		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.70	1.085	1.100	1.165		1.680		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.75	1.150	1.140	1.205	1.310	1.710	1.830		0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.80	1.225	1.230	1.205	1.345		1.860		0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\underline{\theta}$	0.85	1.280	1.290	1.285	1.390		1.890		0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.90	1.345	1.355	1.340	1.500	1.630				0.000	0.000	0.000	0.000	0.000	0.000
	0.95	1.435	1.460	1.465	1.485	1.685				0.000	0.000	0.000	0.000	0.000	0.000
	1.00	1.635	1.645	1.740	1.670	1.745	1.840			0.000	0.000	0.000	0.000	0.000	0.000
	1.05	1.825	1.825	1.835	1.835	1.840	1.905	1.980			0.000	0.000	0.000	0.000	0.000
	1.10	2.005	2.000	1.995	1.995	1.995	1.995	2.050				0.000	0.000	0.000	0.000
	1.15	2.175	2.165	2.160	2.160	2.150	2.145	2.140	2.185			0.000	0.000	0.000	0.000
	1.20	2.335	2.325	2.320	2.315	2.300	2.290	2.280	2.275				0.000	0.000	0.000
	1.25	2.500	2.490	2.480	2.470	2.450	2.435	2.425	2.415	2.410				0.000	0.000
	1.30	2.550	2.580	2.615	2.620	2.605	2.585	2.570	2.550	2.540	2.540			0.000	0.000
	1.40	2.650	2.680	2.715	2.750	2.815	2.875	2.855	2.835	2.815	2.790	2.775			
	1.50	2.750	2.780	2.815	2.850	2.915	2.980	3.050	3.115	3.090	3.070	3.045	3.015		
	1.60	2.850	2.880	2.915	2.950	3.015	3.080	3.150	3.215	3.285	3.345	3.320	3.290		
	1.70	2.950	2.980	3.015	3.050	3.115	3.180	3.250	3.315	3.385	3.450	3.515	3.570	3.540	
	1.80	3.050	3.080	3.115	3.150	3.215	3.280	3.350	3.415	3.485	3.550				
	1.90	3.150	3.180	3.215	3.250	3.315	3.380	3.450	3.515	3.585	3.650				
	2.00	3.250	3.280	3.315	3.350	3.415	3.480	3.550	3.615	3.685	3.750				
	2.10	3.350	3.380	3.415	3.450	3.515	3.580	3.650	3.715	3.785	3.850				
	2.20	3.450	3.480	3.515	3.550	3.615	3.680	3.750	3.815	3.885	3.950				
	2.30	3.550	3.580	3.615	3.650	3.715	3.780	3.850	3.915	3.985	4.050				
	2.40	3.650	3.680	3.715	3.750	3.815	3.880	3.950	4.015	4.085	4.150				
	2.50	3.750	3.780	3.815	3.850	3.915	3.980	4.050	4.115	4.185	4.250				

Note: Due to space limits we do not show all results. Increments for t and $\underline{\theta}$ are 0.05 for low values and 0.1 for high values.

Table 3
Quality of the firm located in the center as a function of t and $\underline{\theta}$ in the second SPNE where the high-quality firm is located in the center (when it exists).

t		0.00	0.05	0.10	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.05	1.15
	0.00	0.820	0.765												
	0.05	0.860	0.805												
	0.10	0.900	0.850	0.780											
	0.15	0.945	0.895	0.830											
	0.20	0.980	0.940	0.875											
	0.25	1.025	0.980	0.925											
	0.30	1.065	1.025	0.970	0.905										
	0.35	1.105	1.070	1.020	0.955										
	0.40	1.145	1.110	1.065	1.005										
	0.45	1.185	1.155	1.110	1.055										
	0.50	1.230	1.195	1.155	1.105										
	0.55	0.998	1.240	1.200	1.155										
	0.60	0.995	1.280	1.245	1.200										
	0.65	1.025			1.245										
	0.70	1.085	1.080			1.190									
	0.75	1.150	1.140			1.240									
	0.80	1.225	1.205	1.195											
$\underline{\theta}$	0.85	1.280	1.280	1.260	1.255										
	0.90	1.345	1.335	1.325	1.315										
	0.95	1.435	1.400	1.390	1.380	1.380									
	1.00	1.635	1.475	1.455	1.455	1.430									
	1.05	1.825	1.675	1.540	1.515	1.500	1.505								
	1.10	2.005	1.940	1.740	1.575	1.560	1.555								
	1.15	2.175	2.070	1.950	1.800	1.625	1.610	1.605							
	1.20	2.335	2.250	2.150	2.025	1.690	1.675	1.660							
	1.25	2.500	2.415	2.330	2.225	1.955	1.730	1.720	1.720						
	1.30	2.550	2.515	2.480	2.415	2.205	1.850	1.780	1.780						
	1.40	2.650	2.615	2.580	2.550	2.480	2.400	2.105	1.890	1.885					
	1.50	2.750	2.715	2.680	2.650	2.580	2.515	2.450	2.380	1.995					
	1.60	2.850	2.815	2.780	2.750	2.680	2.615	2.550	2.485	2.415	2.330				
	1.70	2.950	2.915	2.880	2.850	2.780	2.715	2.650	2.585	2.515	2.450				
	1.80	3.050	3.015	2.980	2.950	2.880	2.815	2.750	2.685	2.615	2.550				
	1.90	3.150	3.115	3.080	3.050	2.980	2.915	2.850	2.785	2.715	2.650				
	2.00	3.250	3.215	3.180	3.150	3.080	3.015	2.950	2.885	2.815	2.750				
	2.10	3.350	3.315	3.280	3.250	3.180	3.115	3.050	2.985	2.915	2.850				
	2.20	3.450	3.415	3.380	3.350	3.280	3.215	3.150	3.085	3.015	2.950				
	2.30	3.550	3.515	3.480	3.450	3.380	3.315	3.250	3.185	3.115	3.050				
	2.40	3.650	3.615	3.580	3.550	3.480	3.415	3.350	3.285	3.215	3.150				
	2.50	3.750	3.715	3.680	3.650	3.580	3.515	3.450	3.385	3.315	3.250				

Note: Due to space limits we do not show all results. Increments for t and $\underline{\theta}$ are 0.05 for low values and 0.1 for high ones.

Table 4
Quality of the firm located on the periphery as a function of t and $\underline{\theta}$ in the second SPNE where the high-quality firm is located in the center (when it exists).

t															
		0.00	0.05	0.10	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.05	1.15
	0.00	0.400	0.395												
	0.05	0.420	0.415												
	0.10	0.440	0.435	0.415											
	0.15	0.460	0.455	0.440											
	0.20	0.475	0.480	0.460											
	0.25	0.500	0.495	0.485											
	0.30	0.520	0.520	0.505	0.485										
	0.35	0.540	0.540	0.530	0.510										
	0.40	0.555	0.560	0.550	0.535										
	0.45	0.575	0.580	0.575	0.560										
	0.50	0.600	0.600	0.595	0.580										
	0.55	0.513	0.620	0.615	0.605										
	0.60	0.505	0.640	0.635	0.625										
	0.65	0.485			0.650										
	0.70	0.475	0.540			0.640									
	0.75	0.465	0.520			0.660									
	0.80	0.465	0.510	0.565											
$\underline{\theta}$	0.85	0.450	0.505	0.550	0.610										
	0.90	0.440	0.485	0.535	0.585										
	0.95	0.450	0.475	0.520	0.565	0.685									
	1.00	0.545	0.470	0.505	0.555	0.645									
	1.05	0.640	0.560	0.505	0.535	0.625	0.745								
	1.10	0.735	0.725	0.590	0.515	0.600	0.700								
	1.15	0.825	0.765	0.695	0.615	0.580	0.665	0.770							
	1.20	0.910	0.860	0.805	0.730	0.560	0.640	0.730							
	1.25	1.000	0.950	0.900	0.835	0.685	0.610	0.695	0.795						
	1.30	1.050	1.015	0.980	0.935	0.815	0.625	0.665	0.755						
	1.40	1.150	1.115	1.080	1.050	0.980	0.910	0.745	0.685	0.765					
	1.50	1.250	1.215	1.180	1.150	1.080	1.015	0.950	0.880	0.695					
	1.60	1.350	1.315	1.280	1.250	1.180	1.115	1.050	0.985	0.915	0.840				
	1.70	1.450	1.415	1.380	1.350	1.280	1.215	1.150	1.085	1.015	0.950				
	1.80	1.550	1.515	1.480	1.450	1.380	1.315	1.250	1.185	1.115	1.050				
	1.90	1.650	1.615	1.580	1.550	1.480	1.415	1.350	1.285	1.215	1.150				
	2.00	1.750	1.715	1.680	1.650	1.580	1.515	1.450	1.385	1.315	1.250				
	2.10	1.850	1.815	1.780	1.750	1.680	1.615	1.550	1.485	1.415	1.350				
	2.20	1.950	1.915	1.880	1.850	1.780	1.715	1.650	1.585	1.515	1.450				
	2.30	2.050	2.015	1.980	1.950	1.880	1.815	1.750	1.685	1.615	1.550				
	2.40	2.150	2.115	2.080	2.050	1.980	1.915	1.850	1.785	1.715	1.650				
	2.50	2.250	2.215	2.180	2.150	2.080	2.015	1.950	1.885	1.815	1.750				

Note: Due to space limits we do not show all results. Increments for t and $\underline{\theta}$ are 0.05 for low values and 0.1 for high ones.

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