

PAPER • OPEN ACCESS

## A Team Allocation Decision for Aircraft Fleet Maintenance

To cite this article: D P Pereira *et al* 2021 *IOP Conf. Ser.: Mater. Sci. Eng.* **1024** 012102

View the [article online](#) for updates and enhancements.

# A Team Allocation Decision for Aircraft Fleet Maintenance

D P Pereira<sup>1</sup>, R Melicio<sup>1,2</sup> and V M F Mendes<sup>1,3,4</sup>

<sup>1</sup>ICT, Escola de Ciências e tecnologia, Departamento de Física, Universidade de Évora, Portugal

<sup>2</sup>IDMEC, Instituto Superior Técnico, Universidade de Lisboa, Portugal

<sup>3</sup>CISE, Electromechatronic Systems Research Centre, Universidade da Beira Interior, Portugal

<sup>4</sup>Dep. of Electrical Engineering and Automation, Instituto Superior de Engenharia de Lisboa, Portugal

Corresponding author: Rui Melicio; ruimelicio@gmail.com

**Abstract.** This paper is about a computer application for planning maintenance of aircraft fleet in what regards allocation of skilled technicians. The formalization for this planning is a mathematical programming problem written as a minimizing one. The decision variables are the allocation of the skilled technicians. The data are the number of available technicians, the working hours needed to accomplish maintenance, the costs due to the daily operation of facilities, and due to the fleet downtime. Although, the formalization is of a non-linear integer-programming problem, a transformation of variables from positive integer ones to Boolean ones is conceivable and allows a reformulation as a pure integer linear programming problem. Also, a relaxation of the integer variables to continuous ones allows for a relaxed formulation as a convex non-linear programming problem. A case study of two fleets illustrates both formulations: solved by the pyomo library calling the GLPK for the pure linear integer programming problem and calling solvers in Scipy library for the relaxed formulation. An inference presented about the results of the case study addresses the issue of using nonlinear programming to emulate the solution of linear integer programming.

## 1. Introduction

Airline companies must face the impact of a highly regulated and monitored air transport business settings and regulatory frameworks that impact operation, and, therefore, on the industry's efficiency due to the time required for the aircraft to stay in the hangar, i.e., the hangar downtime [1-3]. Airlines that develop a competitive advantage in aircraft maintenance, repair, and overhaul (MRO) not only have an advantage in terms of fares, but also in an enhanced rotation, shorter lead times, and reduced hangar downtime to improve aircraft availability and lower overall operating cost [4]. Hence, the importance of a support information system for planning of MRO in yielding safe and airworthy aircraft is crucial to business survival, not only by diminishing the probability of incidents, accidents, and augmenting safety but also by attaining the highest availability while maintaining work performance. MROs must develop strategies to optimize the use of the workforce, meet maintenance requirements and minimize costs. These strategies may involve reviewing the distribution of shifts and the duration of the work week [5], optimizing line maintenance task scheduling [6], better predicting the amount of maintenance unscheduled work [7], scheduling individual tasks to maximize maintenance flexibility [8] or reducing the long term number of maintenance checks by an efficient maintenance planning [9]. To take timely decisions, MRO management must have an appropriate data system [10] and customized support software systems to take decisions about the allocation of skilled technicians. This paper presents a practical programming methodology to achieve minimum total maintenance cost by a convenient allocation of technicians. Python is an object-oriented computer



language and from all software languages a convenient one, having software packages with friendly interfaces to address mathematical computational problems. The Pyomo is one of those open-source software packages used in this paper to model and solve mathematical programming problems by the calling of solvers [11]. GLPK is one of these solvers, an ANSI C freeware linear mixed-integer programming solver [12]. Another package used in this paper is SciPy Python library of routines for modeling and solving scientific problems [13-15].

## 2. Problem formulation

The setting for the mathematical programming problem of planning maintenance in this paper is the following. Let the MRO have a set  $F$  of fleets  $f$  to go under maintenance. The fleet  $f$  is a set  $K^f$  of aircraft  $k$  to go under check  $c$  belonging to a set of feasible checks  $C$ . Consider that an aircraft  $k$  of a fleet  $f$  under check  $c$  has a maintenance cost  $MC_{kc}^f$ . Besides the cost due to labor ( $MHC_{kc}^f$ ), the maintenance cost ( $MC_{kc}^f$ ) has three main parcels: the cost of materials and services needed for the check ( $MSC_{kc}^f$ ), the cost associated with facilities, and the opportunity cost associated with hangar downtime. These last two costs are from data usually given per day, respectively, for facilities ( $Fac_{kc}^f$ ) and for downtime ( $DC_{kc}^f$ ). The goal is to minimize  $MC_{kc}^f$  written as a function of the allocation of maintenance teams with the appropriate number of technicians of each skill, and subjected to labor, technical and operational constraints. The maintenance cost ( $MC_{kc}^f$ ) is a function of the time the aircraft stays in the hangar, given as follows:

$$MC_{kc}^f = MHC_{kc}^f + MSC_{kc}^f + (Fac_{kc}^f + DC_{kc}^f) t_{kc}^f \quad (1)$$

In (1),  $t_{kc}^f$  is the time duration of the check in the hangar, which depends on the workloads of the required skills and the respective number of technicians allocated;  $Fac_{kc}^f$  aggregates the costs of the daily operations and maintenance in the hangar, for example, the general support equipment, the tools, the hydraulic and pneumatic power supply, maintenance of buildings, heating and lighting;  $DC_{kc}^f$  is the daily revenue loss for having the aircraft at the hangar instead of flying paying passengers. Thus, there are maintenance costs incurred in function of the time duration of the check in the hangar. The allocation of the maintenance team is relevant for the time-dependent parcels of equation (1), i.e.  $Fac_{kc}^f t_{kc}^f$  and  $DC_{kc}^f t_{kc}^f$ , as  $t_{kc}^f$  depends on the total person-hours for the particular skill and the number of technicians allocated to the check as follows:

$$t_{kc}^f = \frac{1}{d} \sum_{s=1}^S \frac{WL_{kcs}^f}{x_{kcs}^f} \quad (2)$$

In (2),  $WL_{kcs}^f$  and  $x_{kcs}^f$  are respectively total person-hours and the number of technicians of the technical skill  $s$  required to the maintenance check  $c$ ;  $d$  is the number of daily hours of labor a technician does on a aircraft; and time  $t_{kc}^f$  is expressed in days. The  $Fac_{kc}^f$  plus  $DC_{kc}^f$  is the maintenance cost per day of work incurred by fleets  $f$  aircraft  $k$  to go under check  $c$  and the total maintenance cost  $TMC$  is as follows:

$$TMC = \frac{1}{d} \sum_{f=1}^F \sum_{k=1}^{K^f} \sum_{c=1}^{C_k} (Fac_{kc}^f + DC_{kc}^f) \sum_{s=1}^S \frac{WL_{kcs}^f}{x_{kcs}^f} \quad (3)$$

MRO companies have only access to a limited workforce, so the number of technicians must be constrained to the available technicians for the aircraft maintenance as follows:

$$\sum_{f=1}^F \sum_{k=1}^{K^f} \sum_{c=1}^{C_k} x_{kcs}^f \leq X_s \quad s \in (1, \dots, S) \quad (4)$$

In (4),  $X_s$  is the number of technicians with skill  $s$  available for the maintenance of the fleets. Also, the working space for the maintenance in the neighborhood of an aircraft has area constraint, imposing that only is possible at the same maintenance check to allocate a limited number of technicians given as follows:

$$\sum_{s=1}^S x_{kcs}^f \leq A_{kc}^f \quad f = 1, \dots, F; k = 1, \dots, K^f; c = 1, \dots, C_k, x_{kcs}^f \in \{1, \dots, X_s\} \quad (5)$$

In (4) and (5) the restriction of limited workforce with the skill  $s$  is imposed and the constraint of only a feasible number of technicians is possible to allocate to fleets  $f$  aircraft  $k$  to go under check  $c$ .

In (5),  $A_{kc}^f$  is maximum number of technicians at work in fleet  $f$  aircraft  $k$  under check  $c$ . The goal is to minimize the cost  $TMC$  (3) by choosing the aircraft check maintenance team with the appropriate number of skilled technicians. From (3) the team with the appropriate number of skilled technicians does not depend on  $d$ , i.e., the optimum decision is independent of the number of daily hours of labor on the aircraft as long as the value of  $d$  is the same at every workday. So, the formalization for this planning of maintenance is the mathematical programming problem written as follows:

$$\min \sum_{f=1}^F \sum_{k=1}^{K^f} \sum_{c=1}^{C_k} (Fac_{kc}^f + DC_{kc}^f) \sum_{s=1}^S \frac{WL_{kcs}^f}{x_{kcs}^f} \quad (6)$$

s.t.

$$x_{kcs}^f \in \{1, \dots, X_s\} \quad s \in (1, \dots, S) \quad (7)$$

$$\sum_{f=1}^F \sum_{k=1}^{K^f} \sum_{c=1}^{C_k} x_{kcs}^f \leq X_s \quad s \in (1, \dots, S) \quad (8)$$

$$\sum_{s=1}^S x_{kcs}^f \leq A_{kc}^f \quad f = 1, \dots, F; k = 1, \dots, K^f; c = 1, \dots, C_k \quad (9)$$

In the problem formulation (6) to (9), the number of technicians  $x_{kcs}^f$  are the decisions positive integer variables and the time duration of the check in the hangar for a fleet  $f$  (2) is thus a discrete dependent variable. As stated by the formulation (6) to (9), the problem has only positive integer decisions and is a non-linear integer-programming one due to the non-linearity of the objective function. But the formulation (6) to (9) admits an equivalent formulation as a pure linear integer programming problem, applying a transformation of variables. The transformation from positive integer decisions to Boolean decisions variables is as follows:

$$x_{kcs}^f = \sum_{j=1}^{X_s} j u_{kcsj}^f \quad (10)$$

$$(x_{kcs}^f)^{-1} = \sum_{j=1}^{X_s} u_{kcsj}^f / j \quad (11)$$

$$\text{with } u_{kcsj}^f \in \{0, 1\} \text{ and } \sum_{j=1}^{X_s} u_{kcsj}^f = 1$$

The inverse transformation is as follows:

$$u_{kcsj}^f = \begin{cases} 1, & \text{if } x_{kcs}^f = j \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

$$\text{with } x_{kcs}^f, j \in \{1, \dots, X_s\}$$

In (12) the number of Boolean decision variables associated with a discrete integer variable for a skill  $s$  is equal to the maximum number of skilled technicians available in that skill. So, while the number of discrete integer decisions is  $S$  per aircraft for the formulation (6) to (9), the number of Boolean decisions per aircraft using (10) is as follows:

$$N_{Bool} = \sum_{s=1}^S X_s - S \left( \sum_{f=1}^F K^f - 1 \right) \text{ and } N_{Bool}/S \leq \max \{X_s\}_{s=1, \dots, S} - \sum_{f=1}^F K^f + 1 \quad (13)$$

Substituting equations (10) and (11) in (6) to (9), the problem is as follows:

$$\min \sum_{f=1}^F \sum_{k=1}^{K^f} \sum_{c=1}^{C_k} (Fac_{kc}^f + DC_{kc}^f) \sum_{s=1}^S WL_{kcs}^f \sum_{j=1}^{X_s} u_{kcsj}^f / j \tag{14}$$

s.t.

$$\sum_{f=1}^F \sum_{k=1}^{K^f} \sum_{c=1}^{C_k} \sum_{j=1}^{X_s} j u_{kcsj}^f \leq X_s \quad s = 1, \dots, S \tag{15}$$

$$\sum_{s=1}^S \sum_{j=1}^{X_s} j u_{kcsj}^f \leq A_{kc}^f \quad f = 1, \dots, F; k = 1, \dots, K^f; c = 1, \dots, C_k \tag{16}$$

$$u_{kcsj}^f \in \{0, 1\} \quad f = 1, \dots, F; k = 1, \dots, K^f; c = 1, \dots, C_k; s = 1, \dots, S; j = 1, \dots, X_s \tag{17}$$

In (14) to (17) the objective function and the constraints are linear function of the binary decision variables. So, this formulation is a pure linear integer programming problem.

### 3. Case study

An airline company has one medium-range and one long-range aircraft to go under maintenance check. The loss of revenue per aircraft per day due to maintenance downtime is of 30 k€ and 70 k€ for medium-range and long-range, respectively. The work package of the manufacturer estimates values of 1150 person-hours for the medium-range aircraft and 2900 person-hours for the long-range aircraft checks. The person-hours by skills allocated in percentage of the estimated values, the facilities daily cost and the confines on support teams the working space for the maintenance in the neighborhood of an aircraft are in Table 1.

**Table 1.** Maintenance by skill, *Fac* daily cost, and confines on support team

Aircraft type	Systems (%)	Structures (%)	Avionics (%)	Facilities cost (k€)	Confines on support team
medium-range	60	30	10	5.0	25
long-range	40	35	25	6.5	40

The maximum number of skilled technicians available to carry out the maintenance work on the fleets are in Table 2.

**Table 2.** Available skilled technicians

Skills	Systems	Structures	Avionics
No. Technicians	30	15	10

The airline has two fleets:  $F=2$  and has one aircraft on each fleet under maintenance, so  $K^1 = K^2 = 1$ . The aircraft is performing the first base maintenance check  $C$ , so  $C_k=1$  and for simplicity in what follows the indices  $k$  and  $c$  having permanently the value 1 are discarded. There are three skills  $s$ , so  $S=3$  and the total number of positive integer variables is  $SF = 6$ . The total number of Boolean variables is  $(29+14+9)F = 104$ . Nevertheless, the non-linear integer-programming problem (6) to (9) is not friendly enough to solve but can be convexified with a relaxation on the constraint (7), allowing for continuous decision variables. This relaxation on the formulation (6) to (9) is the substitution of (7) by the constraint as follows:

$$1 \leq x_{kcs}^f \quad s \in (1, \dots, S) \tag{18}$$

In (18) and (8) gives an implicit formalization for the box constraints associated with the relaxed variables. So, the relaxed problem formulated by (6), (18) to (9) has a convex objective function and is a convex optimization problem with linear constraints. So, this formulation is friendly approachable to find acceptable commercial or even free-software solvers.

As mentioned, the solvers applied for this case study are the `scipy.optimize` solvers, namely, COBYLA, SLSQP, Trust-Constr. The convex optimization problem solved by these solvers with the

same point of initialization, given by  $x_1^1 = x_1^2 = 10$  and  $x_2^1 = x_2^2 = x_3^1 = x_3^2 = 5$ , gives the output shown in Table 3.

**Table 3.** Solvers output

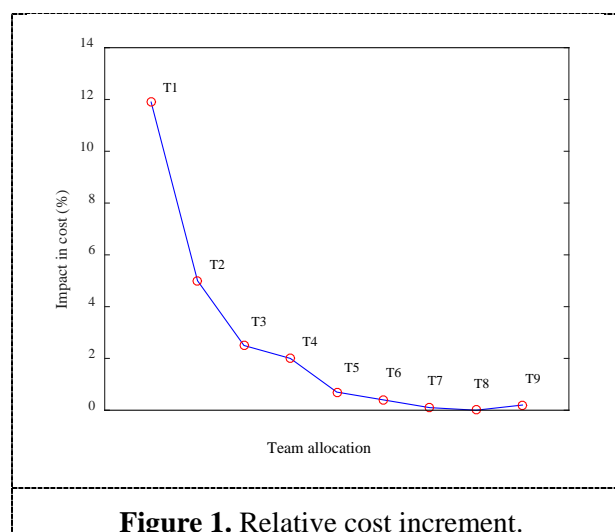
Technicians / Solvers		COBYLA	SLSQP	Trust-Constr
fleet 1	$x_1^1$	10.3	10.0	10.3
	$x_2^1$	4.2	5.0	4.2
	$x_3^1$	2.1	5.0	2.1
fleet 2	$x_1^2$	19.7	10.0	19.7
	$x_2^2$	10.8	5.0	10.8
	$x_3^2$	7.9	5.0	7.9

Table 3 shows that the solver SLSQP exited with the initial values of the variables, because more technical information about the bounds of the variables, than the required by other two solvers is necessary for processing the solution. In addition, for any solver, integer decisions are not guaranteed due to the relaxation and rounding to integers is prone to be unfeasible decisions and as expected the other two solvers run for the same final decision continuous variables. The pure linear integer programming problem (14) to (17) solved by the GLPK delivers the required integer decision for technicians allocation to the aircraft maintenance teams shown in Table 4.

**Table 4.** Teams allocation, via GLPK

Aircraft type	Systems	Structures	Avionics
medium-range	10	4	2
long-range	20	11	8

A comparison between Table 3 and Table 4 shows that the rounding of the output of the COBYLA, and Trust-Constr equal the required integer decision, but this is not always a fact even if feasibility occurs providentially after rounding. The advantage of the implementation of these two methods, relative to the GLPK, is fewer decision variables: 6 while the latter has 104. The number of decision variables per aircraft and skill for the pure linear integer programming problem (14) to (17) is limited by the inequality shown in (13). A study of the impact in the cost of not taking the optimal decisions identified by T8, but feasible ones is in Figure 1.



In Figure 1, the allocations for the study of the impact in the cost are around the optimal one in decreasing order of technician allocated to the fleet  $f = 1$  as shown in Table 5.

**Table 5.** T1 to T9 feasible technician allocations

Allocations			T1	T2	T3	T4	T5	T6	T7	T8	T9
Technicians	fleet 1	$x^1_1$	14	14	13	12	12	11	11	10	9
		$x^1_2$	7	6	5	5	5	5	4	4	4
		$x^1_3$	4	3	3	3	2	2	2	2	2
	fleet 2	$x^2_1$	16	16	17	18	18	19	19	20	21
		$x^2_2$	8	9	10	10	10	10	11	11	11
		$x^2_3$	6	7	7	7	8	8	8	8	8

In Figure 1, in near neighborhoods of the optimal decision, allocation T8, given by the GLPK, the increment in cost: changing only one system technician among the fleets, increases cost about 0.2%; changing one system and one structure technicians among the fleets, increases cost about 0.4%; changing two system and one structure technicians among the fleets, increases cost about 0.7%; changing two system, one structure and one avionics technicians among the fleets, increases cost about 2%; changing three system, one structure and one avionics technicians among the fleets, increases cost about 2.5%; changing four system, two structure and two avionics technicians among the fleets, increases cost about 5 %; changing four system, three structure and two avionics technicians among the fleets, increases cost about 11.9%. If a feasible decision can be achieved by rounding the continuous optimal solution, the change by one or two technicians is small. Thus, as a first approximation the problem convexification is acceptable and as the objective function is a convex function and the feasible set is a polyhedron, the convexification is more desirable.

The following simulations are for the assessment of the change in the allocations of skilled technicians in function of significant changes in the above work package. The alternative work packages for maintenance in what regard the workload and the skills distribution are in Table 6.

**Table 6.** Work packages

Work packages		Workload (h)	Systems (%)	Structures (%)	Avionics (%)
1	medium-range	4600	35	45	20
	long-range	2900	40	35	25
2	medium-range	8200	30	55	15
	long-range	2900	40	35	25
3	medium-range	4600	35	45	20
	long-range	6100	35	45	20
4	medium-range	8200	30	55	15
	long-range	6100	35	45	20
5	medium-range	8200	30	55	15
	long-range	12300	25	55	20

Table 7 shows the results for the alternative work packages using the linear programming solver GLPK and comparing with the solutions obtained by the three non-linear solvers (COBYLA, SLSQP and Trust-Constr). Where A= Alternative Work package.

Table 7 shows that SLSQP for scenarios 1, 3 and 5 exited with the initial values, and for scenarios 2 and 4, the output does not satisfy the constraints of the problem. SLSQP Scenario 2 provides a non-

integer value of 32.3 systems technicians which by rounding does not satisfy the maximum of 30 available systems technicians. SLSQP provides a non-integer value of 16.03 structures technicians which by rounding does not satisfy the maximum of 15 available structures technicians. SLSQP provides a non-integer value of 11.6 avionics technicians which by rounding does not respect the maximum of 10 available structures technicians. Also for scenario 4 a non feasible output is found by SLSQP. Both COBYLA and Trust-Constr provide non integer solutions, but for these five scenarios the round of the solution equals the exact solution. Even though feasible decisions are not guaranteed due to the relaxation and rounding to integers, these alternative work packages show that the allocation of skilled technicians does not change significantly when the work package has significant changes.

**Table 7.** Teams allocation, via GLPK and non-linear solvers (Cobyala, SLSQP, Trust-Constr)

A	Linear Prog Solver			Non-linear Programming Solvers								
	GLPK			COBYLA			SLSQP			Trust-Constr		
	Sys	Str	Avi	Sys	Str	Avi	Sys	Str	Avi	Sys	Str	Avi
1	13	7	4	13.30	7.37	4.32	10.00	5.00	5.00	13.30	7.37	4.32
	17	8	6	16.70	7.63	5.68	10.00	5.00	5.00	16.70	7.63	5.68
2	12	9	4	12.00	8.52	4.48	14.77	8.58	5.79	12.00	8.52	4.48
	18	6	6	18.00	6.48	5.52	17.56	7.45	5.79	18.00	6.48	5.52
3	11	6	4	11.10	5.55	3.70	10.00	5.00	5.00	11.10	5.55	3.70
	19	9	6	18.90	9.45	6.30	10.00	5.00	5.00	18.90	9.45	6.30
4	13	7	4	12.62	6.97	4.04	15.99	7.94	5.32	12.62	6.97	4.04
	17	8	6	17.38	8.03	5.96	18.06	8.57	6.83	17.38	8.03	5.96
5	11	5	3	11.31	5.34	3.24	10.00	5.00	5.00	11.31	5.34	3.24
	19	10	7	18.69	9.66	6.76	10.00	5.00	5.00	18.69	9.66	6.76

#### 4. Conclusion

A survey of the literature agrees in the fact that an adequate team allocation for aircraft fleet maintenance has an impact in reducing the cost of MRO for airlines due to the reduction of the costs incurred in the operation of facilities, and in fleet downtime. So, planning maintenance of aircraft fleets in what regards team allocation must be carefully managed, taking into consideration the available technicians per skills, the working hours needed to accomplish maintenance, the costs due to the daily operation of facilities, and to the fleet downtime. The planning maintenance of aircraft fleets in the context of taking the best decisions is an optimization problem formulated as the minimization of the costs incurred, having integer decision variables given by the number of technicians allocated to work for the maintenance of aircraft. The proposed planning maintenance has a formulation based in a non-linear integer-programming problem which is not friendly enough in what regards the available commercial solver or even free-software ones. But this formulation admits a reformulation as a pure linear integer programming problem, using a transformation of the decision variables to Boolean ones. Also, the non-linear integer-programming problem allows a convexification by relaxing the decision variables to be continuous ones, giving behavior that can be easily solved by commercial or even free-software solvers. Although, the relaxation has the advantage of having fewer decision variables than the pure linear integer programming problem, the optimum decision given by the relaxation is not necessarily a feasible and an integer one. Thus, rounding to an integer one has to be carried out. But this rounding tends to deliver an unfeasible decision, and even if the rounding is feasible is not possible to faithfully guarantee the optimal solution. The simulation of allocations with a significant



change in the work package does not change significantly the output, i.e., the sensitivity of team allocation is low.

### Acknowledgment

This work is funded through Foundation for Science and Technology (FCT) under the ICT (Institute of Earth Sciences) project UIDB/04683/2020; Portuguese Funds through the Foundation for Science and Technology (FCT) under the LAETA project UIDB/50022/2020; Portuguese Foundation for Science and Technology (FCT) under Project UID/EMS/00151/2019.

### References

- [1] Arjomandi A and Seufert J H 2014 *Economic Modelling* **41** 133-144.
- [2] Santos L F F M and Melicio R 2019 *Intern. Review of Aerospace Engineering* **12(1)** 35-45.
- [3] Dias N G, Santos L F F M and Melicio R 2019 *Proc. 9th EASN International Conference on Innovation in Aviation & Space* **304(06001)** 1-7.
- [4] Alamdari F E and Morrell P 1997 *Journal of Air Transport Management* **3(2)** 53-66.
- [5] Alfares, H K 1999 *Aircraft maintenance workforce scheduling A case study* *Journal of Quality in Maintenance Engineering* **5(2)** 78-88.
- [6] Beliën, J, Demeulemeester E, De Bruecker P, Van den Bergh J and Cardoen B 2013 *Integrated staffing and scheduling for an aircraft line maintenance problem* *Computers & Operations Research* **40(4)** 1023-1033.
- [7] Dinis D, Barbosa-Póvoa A and Teixeira, Â P 2019 *A supporting framework for maintenance capacity planning and scheduling: Development and application in the aircraft MRO industry* *International Journal of Production Economics*, **218** 1-15.
- [8] Senturk C, Kavsaoglu, M S and Nikbay M 2010 *Optimization of aircraft utilization by reducing scheduled maintenance downtime* In 10th AIAA Aviation Technology, Integration, and Operations Conference 9143.
- [9] Deng Q, Santos B F and Curran R 2020 *A practical dynamic programming based methodology for aircraft maintenance check scheduling optimization* *European Journal of Operational Research* **281(2)** 256-273.
- [10] Samaranayake P and Kiridena S 2012 *Aircraft maintenance planning and scheduling: an integrated framework* *Journal of Quality in Maintenance Engineering* **18(4)** 432-453.
- [11] Hart W E, Watson J and Woodruff D L 2011 *Math. Progr. Computation* **3(3)** 219-260.
- [12] Meindl B and Templ M 2012 *Analysis of commercial and free and open source solvers for linear optimization problems* (Vienna: Vienna University of technology) pp 1-13
- [13] Virtanen P, Gommers R, Oliphant T E and et al. 2020 *Natuer Methods Computat.* **17** 261-272.
- [14] Oliphant T E 2006 *A guide to NumPy* (USA: Trelgol Publishing)
- [15] Makhorin A 2012 *GNU linear programming kit* (Boston: Free Software Foundation).