Extinction and Maturity Times for Population and Individual Growth in Random Environments

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Abstract. Let $X(t)$ be the size of a population (nr. of individuals, biomass) at time t or the size of an individual (weight, lenght, etc.) at age t. Let the relative growth rate in a deterministic environment be $\frac{1}{X(t)}\frac{dX(t)}{dt} = f(X(t))$ with an appropriate function f (typical examples are the logistic $f(x) = r(1 - x/K)$ or the Gompertz $f(x) = r\ln(K/x)$ models). In a random environment, we approximate the fluctuations by $\sigma\varepsilon(t)$, where $\sigma > 0$ is the intensity and $\varepsilon(t)$ is standard white noise.

We assume that environmental fluctuations affect the relative growth rate, obtaining the stochastic differential equation (SDE) model (1) $\frac{1}{X}\frac{dX}{dt} = f(X(t))+\sigma\varepsilon(t)$ (see Braumann (2008) and Carlos and Braumann (2006) for population growth applications and Filipe and Braumann (2007) for individual growth applications). Another possibility is that they affect a parameter like the r in the logistic or the Gompertz model, obtaining models of the type (2) $\frac{1}{X}\frac{dX}{dt} = f(X(t)) + \theta f(X(t))\varepsilon(t)$.

Let $X(0) = x$. For both model types, we study the first passage times T_b and T_a, respectively through a high $b > x$ threshold (could be the maturity size at which a bovine is sold or a pest population outbreak size at which control measures are required) and a low $a < x$ threshold (could be the extinction threshold size of a population). We study the general case and, for the particular cases of the logistic and the Gompertz models, we compute the mean and variance of T_b and T_a.

Keywords: stochastic differential equation models, population growth, individual growth, extinction time, maturity time

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