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**Output Persistence and Upside Down Electoral Business Cycles  
What Does Really Matter?**

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**Resumo:**

This note shows in what circumstances output persistence may invert the pattern of the electoral cycle when inflation expectations are of the adaptive or rational type and the government preferences are quadratic over output and inflation.

**Palavras-chave:** Electoral cycles, Output persistence

**Classificação JEL:** E23, E32, E52, E61

## 1. Introduction

Some years ago authors have started to pay attention to the consequences arising from the fact that real variables, such as unemployment or output, exhibit a degree of persistence over time (see, e.g., Jonsson, 1997; Lockwood, 1997; Svensson, 1997). A particularly interesting consequence of output persistence is that it may turn upside down the political business cycle, which, in its typical form, is associated with depressions at the beginning of the mandate followed by pre-election inflationary expansions. This consequence on the pattern of the typical political business cycle is shown to exist by Gärtner (1996) who considers a model with adaptive expectations and a linear (in output)-quadratic (in inflation) policy objective function. Furthermore, Gärtner (1999) gives also some credit to the output persistence hypothesis from an empirical point of view.

The linearity of preferences over output is far from being innocuous as it implies an independence of policy (e.g. inflation) from expectations (e.g. expected inflation), which excludes the dynamics that indeed play a crucial role when output persists over time. The objective of this note is thus to show what are the consequences of output persistence on the dynamic pattern of the electoral cycle (EC, hereafter) by considering a full-quadratic objective function. The results of the model show that the EC can assume either the typical pattern or the reverse one and that output persistence, in any case, may turn the EC upside down. Besides the electoral policy implications, these results are also decisive for the empirical detection of an EC.<sup>1</sup> As this note also considers both rational and adaptive expectations it is also possible to show in which circumstances the rational and adaptive expectations solutions coincide.

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<sup>1</sup> This means that it is not possible, in general, to *always* use the potentially observed pre-elections expansions as empirical evidence supporting the existence of an opportunistic behaviour by the government. Sadly, this mistake seems to persist in the (empirical) literature.

## 2. The model

Recently some authors have assumed an extended version of the standard aggregate supply curve  $y_t = \bar{y} + \beta(\pi_t - \pi_t^e)$ , where  $y_t$  denotes the level of output (measured in logarithms) that deviates from the natural level,  $\bar{y}$ , whenever the inflation rate,  $\pi_t$ , deviates from its expected level  $\pi_t^e$ , by considering

$$y_t = (1 - \eta)\bar{y} + \eta y_{t-1} + \delta(\pi_t - \pi_t^e), \quad (1)$$

where  $\eta$  measures the degree of output persistence.<sup>2</sup> When normalizing the natural level of output such that  $\bar{y} = 0$  the aggregate supply curve reduces to:

$$y_t = \phi y_{t-1} + \alpha(\pi_t - \pi_t^e). \quad (2)$$

Concerning the government's objective function, we make the standard assumption that the government faces a mandate divided into two periods and that the discounted loss, at a rate  $\rho$ , results from quadratic deviations of output and inflation from their desired values, which are assumed to be  $\tilde{y} > 0$  and zero, respectively. Therefore,

$$L = \frac{1}{2}(\pi_1^2 + \lambda(y_1 - \tilde{y})^2) + \rho \left( \frac{1}{2}(\pi_2^2 + \lambda(y_2 - \tilde{y})^2) \right) \quad (3)$$

represents the government's loss function, which is to be minimized using inflation during the mandate, subject to the structure of the economy given by (2).

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<sup>2</sup> This way of introducing persistence, which results in expression (1), is the most common in the literature (see, e.g., Gärtner, 1996; Jonsson, 1997; Lockwood, 1997; Svensson, 1997). Svensson (1997) justifies the existence of an autoregressive term on the Phillips curve when wage setters set nominal wages one period in advance, disregarding non-union workers' preferences for real wages and employment, and where union membership depends on previous unemployment.

### 3. The rational expectations case

The minimization of (3) subject to (2), assuming that  $\pi_t^e = E[\pi_t | I_{t-1}]$ , immediately leads to the optimal discretionary policies:

$$\pi_1 = \alpha\lambda(\tilde{y} - \phi y_0 + \phi\rho(\tilde{y} - \phi^2 y_0)(1 + \lambda\alpha^2))$$

and

$$\pi_2 = \alpha\lambda(\tilde{y} - \phi^2 y_0),$$

which, in turn, result in the output levels  $y_1 = \phi y_0$  and  $y_2 = \phi y_1$ . Consequently, in general, output will in both periods be either below or above its natural level. If output does not persist over time,  $\phi = 0$ , the constancy of output at the natural level is achieved with a constant policy,  $\pi_1 = \pi_2 = \alpha\lambda\tilde{y}$ . The inexistence of a cycle at the output level is also verifiable when  $\phi = 1$  but, in this case,  $|\pi_1| > |\pi_2|$  given that  $\pi_1 = (1 + \rho + \lambda\rho\alpha^2)\pi_2$ .

For intermediate degrees of output persistence,  $0 < \phi < 1$ , different kinds of policy cycles can be observed. This is the case because

$$\pi_2 - \pi_1 = \alpha\lambda\phi((1 - \phi + \rho\phi^2 + \lambda\rho\alpha^2\phi^2)y_0 - (1 + \lambda\alpha^2)\rho\tilde{y})$$

can assume both positive or negative values. Even if  $y_0 = \tilde{y}$  it is possible to observe a *typical* EC, i.e.  $\pi_2 > \pi_1$ , or an *atypical* one, i.e.  $\pi_2 < \pi_1$ . In particular, in case of  $y_0 = 0$  a reversion on the typical pattern of the EC is observed as  $\pi_2 = \alpha\lambda\tilde{y}$  whereas  $\pi_1 = \alpha\lambda\tilde{y}(1 + (1 + \lambda\alpha^2)\phi\rho)$ .

#### 4. The adaptive expectations case

Following the adaptive expectations hypothesis, the government minimizes (3) subject to (2), where  $\pi_1^e = \gamma\pi_0 + (1-\gamma)\pi_0^e$  and  $\pi_2^e = \gamma\pi_1 + (1-\gamma)\pi_1^e$ .

We restrain the analysis to the steady-state cycle, that is the situation where  $y_2 = y_0$ . In this case, it is straightforward to verify that  $y_2 = -y_1$ , i.e. output expansions and depressions, of the same magnitude, succeed over time. To put it more precisely, the expansion takes place at the end of the mandate – *typical EC pattern* – if  $\pi_2 > \pi_1$  – or at the beginning of the mandate – *atypical EC pattern* – if  $\pi_2 < \pi_1$ . In fact:

$$y_2 = -y_1 = \frac{\alpha(\pi_2 - \pi_1)}{(1+\phi)(2-\gamma)}.$$

The solution to the government's problem can be written down as  $\mathbf{A}p = q$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & \frac{(1+\lambda\alpha^2+\rho(\gamma-\phi)(\gamma-\phi-1))\lambda\gamma\alpha^2}{(1+\lambda\alpha^2)^2+\rho\lambda\alpha^2(\gamma-\phi)^2} & 0 & \frac{(1+\lambda\alpha^2+\rho(\gamma-\phi)(\gamma-\phi-1))\lambda(1-\gamma)\alpha^2}{(1+\lambda\alpha^2)^2+\rho\lambda\alpha^2(\gamma-\phi)^2} & 0 & \frac{(1+\lambda\alpha^2-\rho\phi(\gamma-\phi))\alpha\lambda\phi}{(1+\lambda\alpha^2)^2+\rho\lambda\alpha^2(\gamma-\phi)^2} \\ 0 & 1 & 0 & \frac{\lambda\alpha^2}{1+\lambda\alpha^2} & \frac{\alpha\lambda\phi}{1+\lambda\alpha^2} & 0 \\ 0 & -\gamma & 1 & \frac{\gamma-1}{\gamma-1} & 0 & 0 \\ -\gamma & 0 & \gamma-1 & 1 & 0 & 0 \\ -\alpha & 0 & \alpha & 0 & 1 & -\phi \\ 0 & -\alpha & 0 & \alpha & -\phi & 1 \end{bmatrix},$$

$$p' = [\pi_1 \quad \pi_2 \quad \pi_1^e \quad \pi_2^e \quad y_1 \quad y_2]$$

and

$$q' = \begin{bmatrix} \frac{(1 + \lambda\alpha^2 - \rho(\gamma - \phi))\alpha\lambda\tilde{y}}{(1 + \lambda\alpha^2)^2 + \rho\lambda\alpha^2(\gamma - \phi)^2} & \frac{\alpha\lambda\tilde{y}}{1 + \lambda\alpha^2} & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Simple manipulation of the optimal electoral policies shows that:

$$\pi_2 - \pi_1 = \frac{(\gamma - \phi)(1 + \phi)(2 - \gamma)\alpha\lambda\rho\tilde{y}}{(1 + \phi)(2 - \gamma) + \lambda\alpha^2(2 + \rho(\gamma - \phi))}. \quad (4)$$

Equation (4) shows that what dictates the pattern observed on the EC is simply the relationship that exists between the degree of expectations persistence and the degree of output persistence. The *typical* (resp. *atypical*) EC pattern should be observed when  $\gamma > \phi$  (resp.  $\gamma < \phi$ ).

## 5. An expected result

Plainly, when the degrees of persistence on adaptive expectations and on output are such that  $\gamma = \phi$  the optimal policies during the mandate coincide, leading to the inexistence of a cycle on output, which remains at its natural level. A simple inspection of the adaptive expectations solution for this case, i.e.  $\pi_1 = \pi_2 = \alpha\lambda\tilde{y}$ , shows that it indeed coincides with the rational expectations solution when, in this case, output does not show persistence over time. In this sense, the  $\phi = 0$  case when expectations are rational is formally equivalent to the  $\gamma = \phi$  case when expectations are adaptive.

## 6. Conclusions

Our main result is that output persistence may *indeed* invert the pattern of the electoral cycle, which can already be upside down – assuming as a reference the *typical* pattern – given that preferences are quadratic over inflation *and* output.

This result has clear policy implications, not only to the way policy instruments should be used to obtain the best electoral outcomes but also at the level of competition policies used to diminish persistence. Moreover, in terms of the empirical detection of electoral cycles, it seems crucial to understand that the observation of a *typical* pattern of the cycle over the mandate cannot be used in an isolated way, *i.e.* without a test on the level of persistence, as a signal of opportunistic behaviour by the incumbent.

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