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Subjective *Versus* Objective Economic Measures
A fuzzy logic exercise *

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Abstract/Resumo:
It is rather evident that there is much more (statistical) information about objective aggregates, such as inflation, output or unemployment than that concerning subjective aggregates, such as well-being, satisfaction, confidence or even expectations. Due to its characteristics, fuzzy logic can and should indeed be used to understand how some of those subjective measures can be approximated by objective ones.
This task is accomplished in the paper by the use of Portuguese data on consumer confidence – the subjective economic measure – and on the unemployment rate – the objective economic measure –. The results clearly indicate that to be a worthwhile exercise as the clear importance of unemployment on confidence is only revealed by the fuzzy logic approximation.

Palavras-chave/Keyword: Confidence, Fuzzy Logic, Objective Measures, Subjective Measures

Classificação JEL/JEL Classification: C10, C82, E32

1 Introduction and Motivation

In terms of the availability of data concerning economic aggregates, a simple comparison between *objective* measures, such as inflation, unemployment or output, and *subjective* ones, such as satisfaction, well-being, expectations or confidence, immediately shows an overwhelming amount of data for *objective* phenomena and a disappointing lack of data for *subjective* phenomena. For instance, concerning Portugal, a recent search on the OECD data basis, revealed the existence of around 143 monthly series measuring *objective* aggregates and, at most, 6 monthly series measuring *subjective* aggregates.

Due to incomplete information, economic agents may indicate to be more or less confident given a vague perception of the economic situation which, indeed, is generally measured by a reasonably amount of objective measures. As clearly pointed out in Santero and Westerlund (1996), confidence is a concept which cannot be defined precisely. This means that, when looking at the economic situation, even if all the information provided by those objective measures could be fully exploited, agents may still base their judgements on subjective criteria as, for instance, '*high*' or '*large*', '*normal*' or '*mean*' and '*low*' or '*small*' values for those objective variables. If this is the case, a qualitative approach, such as that given by *fuzzy logic*, rather than a quantitative approach may be much more appropriate. In fact, if one assumes that agents do not possess the ability to acquire, retain and process all the information needed to make crisp decisions as, for example, to *sharply* classify observed unemployment rates as high, normal or low, and feel less or more confident because of that, then fuzzy logic is a natural way of dealing with this kind of situation in which the source of imprecision is the absence of crisp or rigid defined criteria of class memberships due to states of incomplete or imperfect knowledge.

Since the seminal paper of Zadeh (1965), fuzzy set theory has gone through a tremendous growth, both in theoretical and applied fields.[1] Given that the main concern of fuzzy set theory is to capture *approximate* rather than *exact* forms of reasoning, and this also characterises many economic situations, such as forming intrinsically subjective measures of confidence, well-being, satisfaction, *etc.*, it is surprising that so few applications of fuzzy logic have been made in economics. Three exceptions are Bagnoli and Smith (1998), Draeseke and Giles (1999), and West and Linster (2003). To be more precise, fuzzy logic allows 'intermediate' values to be defined between conventional or crisp evaluations like *yes/no*, or *true/false*. The vagueness or subjectivity of concepts which it is believed characterise human thought is, thus, easily taken into account by fuzzy logic.

The objectives of this rather simple exercise on fuzzy logic are thus, on the one

[1] Just as a curiosity, it is interesting to note that there is a MATLAB toolbox entirely dedicated to fuzzy logic.

hand, to show how an objective measure, such as the unemployment rate, can be used to understand the tendency registered by a subjective measure, such as the consumer confidence indicator and, on the other hand to verify how the *retrospective* use of the objective measure can, in fact, lead to an approximation of a subjective measure which is intrinsically *prospective*, as it is the case with confidence.[2] To achieve these objectives, Portuguese data for both measures will be considered.[3] In graphical terms, the situation can be visualised as follows.

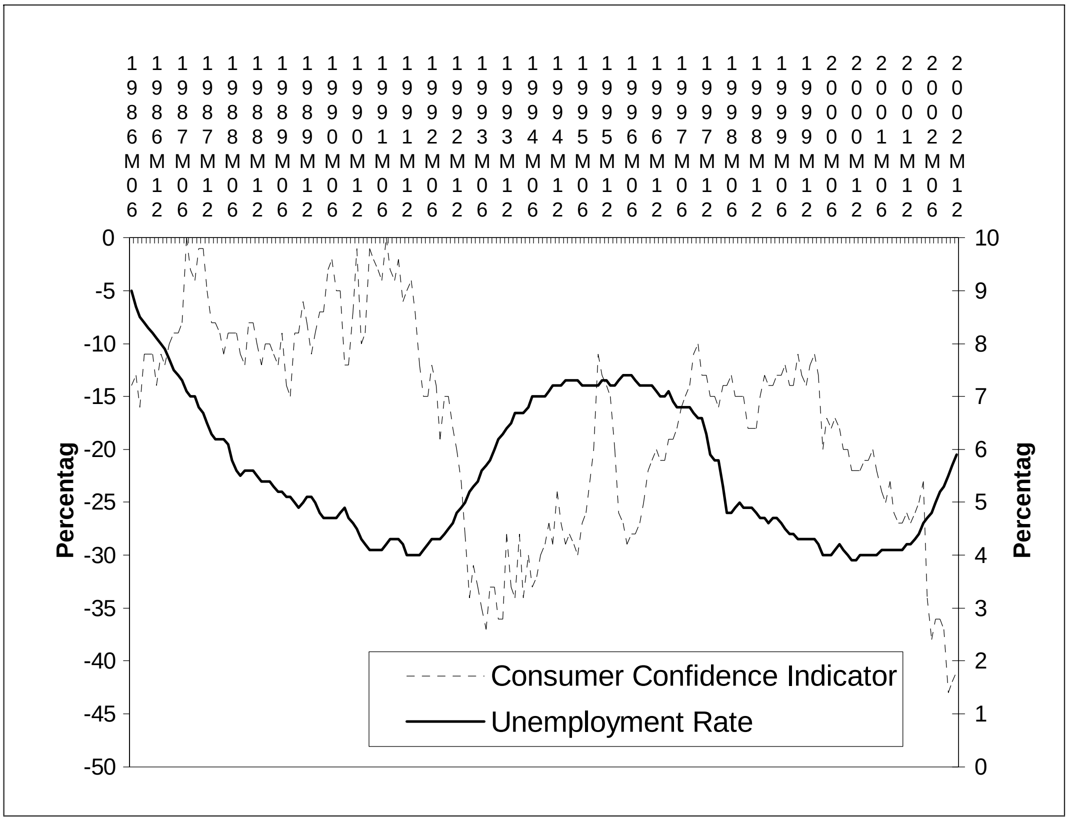

Figure 1: The objective *versus* the subjective measures

As the previous figure clearly shows there is no apparent connection between the values assumed by the consumer confidence indicator and those assumed by the unemployment rate. Concerning the unemployment rate, it follows a clear cycle whereas consumer confidence presents what one may consider an idiosyncratic trajectory. In fact, after being roughly constant at the beginning of the period, it stepped down during around two years and finally it followed a hump shaped curve. The correlation coefficient for the two series is as low as -0.17. Hence, at first sight it may be

[2] In fact, as acknowledged in European Commission (2003), "the consumer confidence indicator is the arithmetic average of the balances (in percentage points) of the answers to the questions on the financial situation of households, the general economic situation, unemployment expectations (with inverted signs) and savings, all over the next 12 months."

[3] The source of the data, which is montly and covers the period 1986M06 to 2002M12, is the OECD.

considered challenging to obtain values based on the unemployment rate that approximate the ones registered by the consumer confidence indicator, *without using some optimization technique*, as it is the case with fuzzy logic.

A correct understanding of those values may be of crucial importance, for instance, to economic policies designed to increase perceived well-being or satisfaction by the inevitable use of objective aggregates. In particular, these issues seem to be fundamental for an incumbent which, due to re-election goals, wants the electorate to feel particular confident at the day of the elections. Therefore, as a subsidiary objective of this note, we analyse the link between the position in time for the last five elections that took place in Portugal and the trajectories registered by the unemployment rate and the consumer confidence indicator.

The rest of the paper has the following structure. Section 2 analyses the fuzzy logic process from which the consumer confidence indicator, as a *subjective measure*, can be related with the unemployment rate, as an *objective measure*. Section 3 offers then an informative picture of the Portuguese election results in terms of trajectories undertaken by those two measures. Section 4 concludes.

2 The Fuzzy Logic Process

We start with a brief presentation of the fuzzy logic approach. Consider $\mathcal{U}$ to be a universal set and another set $\mathcal{A} \subseteq \mathcal{U}$, *i.e.* $\mathcal{A}$ being a subset of $\mathcal{U}$ in the classical sense. $\mathcal{A}$'s characteristic function $\mu_{\mathcal{A}} : \mathcal{U} \to \{0, 1\}$ is defined by $\mu_{\mathcal{A}}(x) = 1$ for $x \in \mathcal{A}$, and $\mu_{\mathcal{A}}(x) = 0$ for $x \notin \mathcal{A}$. Following the logic of crisp sets, we then say that the *degree* to which an element of the universal set belongs to $\mathcal{A}$ is (in this case) either 0 or 1. The *characteristic function* thus discriminates between members and non-members of the crisp set. The generalisation to a *fuzzy set* is made by relaxing the strict separation between elements belonging or not to $\mathcal{A}$, allowing the degree of belonging/membership to take more than these two values, typically by allowing any value in the closed interval $[0, 1]$. See, *inter alia*, Zimmermann (1991).

The values then assigned by the *membership function* of a fuzzy set to the elements in the universal set indicate the membership grade of each element in the set.[4] Larger values indicate higher membership grades, degrees, or consistency between an element

[4] The most considered types of *membership functions* are:

1. Bell membership functions given by

$$f(x) = \frac{1}{1 + \left(\frac{x-c}{a}\right)^{2b}},$$

or some gaussian function, as will be the case analysed below.

of the set and the full characteristics that the set describes.[5] Hence, using fuzzy logic, one can deal with reasoning like: "the observed value for the unemployment rate, say 5%, can be considered high, normal or low with some *degrees of membership*".

In terms of fuzzy logic, '*high*', '*normal*' or '*low*' values (for the variable under question) are said to be *subjective categories*, as agents often evaluate those concepts differently. In what follows, it will be assumed that consumers use these kind of subjective categories to 'construct' an approximate indicator of their confidence. In terms of fuzzy logic, this is sometimes referred to as *approximate* or *qualitative reasoning*. In general, this corresponds to the assumption of an inference mechanism based on *if-then decision rules* described as follows:

$$\begin{array}{ll}
\mathcal{P}^1 : & \text{If } \mathcal{X} \text{ is } \mathcal{A}^1 \text{ then } \mathcal{Y} \text{ is } \mathcal{B}^1\text{, else} \\
\ldots & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\mathcal{P}^i : & \text{If } \mathcal{X} \text{ is } \mathcal{A}^i \text{ then } \mathcal{Y} \text{ is } \mathcal{B}^i\text{, else} \\
\ldots & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\mathcal{P}^n : & \text{If } \mathcal{X} \text{ is } \mathcal{A}^n \text{ then } \mathcal{Y} \text{ is } \mathcal{B}^n. \\
\mathcal{Q} : \mathcal{X} \text{ is } \mathcal{A}' & \mathcal{I} : \mathcal{Y} \text{ is } \mathcal{B}'.
\end{array}$$

The so-called *antecedent vector* $\mathcal{X}$ is constituted by linguistic variables, possibly connected together by an operator AND, in the universe of discourse $\mathcal{U}$ while the *consequent vector* $\mathcal{Y}$ is constituted by linguistic variables in the universe of discourse $\mathcal{V}$. From the set of rules $\mathcal{P}$ and the (fuzzy) observation $\mathcal{Q} : \mathcal{A}'$, it is then possible to make the *inference* "$\mathcal{Y}$ is $\mathcal{B}'$".

To sum up, the structure of a fuzzy system, can be presented as follows:

1. *Fuzzification*: Transformation of real numbers into qualitative categories such as linguistic terms, *i.e.* the translation of crisp in fuzzified input(s);

2. Trapezoidal membership functions as follows

$$h(x) = \begin{cases} 0 & \text{if} \quad x < a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{if} \quad a_1 \leq x \leq a_2 \\ 1 & \text{if} \quad a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & \text{if} \quad a_3 \leq x \leq a_4 \\ 0 & \text{if} \quad x > a_4 \end{cases}.$$

In case of having only one singleton a_2, we then have triangular membership functions which are such that

$$g(x) = \begin{cases} 0 & \text{if} \quad x < a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{if} \quad a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{if} \quad a_2 \leq x \leq a_3 \\ 0 & \text{if} \quad x > a_3 \end{cases}.$$

[5] Although many authors defend that membership defines the degree of adherence rather than the probability of an event, Chang and Stekler (1977) consider that the membership function may be considered akin to a subjective probability distribution.

2. *Inference*: Construction of fuzzy rules from the membership functions of inputs-antecedents and outputs-consequences, followed by the determination of fuzzy output linguistic terms;

3. *Defuzzification*: Transformation of the previous qualitative/linguistic terms back into real numbers, *i.e.* the translation of fuzzy to crisp output(s).

2.1 Fuzzification

Let us assume that consumer confidence is related with unemployment and that the unemployment rate can be characterised by three qualitative categories (subjective terms) according to:

$$\text{'unemployment rate'} \in \{\text{low, normal, high}\}$$

The translation of the unemployment rates, measured on the real axis, into those categories is achieved through the use of membership functions associated with the fuzzy sets of 'low', 'normal' and 'high' values. In our case, these functions were derived from the empirical distributions of the Portuguese unemployment rates for the period 1986M06-2002M12. See figure 1.

Concerning the choice of the shape for the membership functions, we will use the *gaussian bell curve type* because, on the one hand, it does not exclude data from coming from all – '*low*', '*normal*', '*high*' – possible distributions, as might happen, in an 'ad-hoc' way, with trapezoidal/triangular membership functions and, on the other hand,, it is consistent with the data. This implies that, for each moment of time, we will have a situation which can be described graphically as follows:[6]

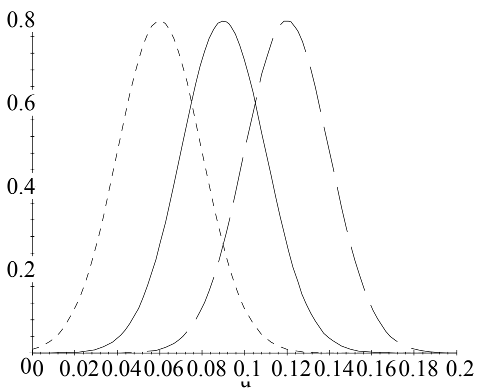

Figure 2: The gaussian type of membership functions (low-left; normal-middle; high-right)

As pointed out in Evans and Honkapohja (1999), following Sargent's (1993) suggestion, one possibility to substitute fully 'rational' agents with agents possessing

[6]The values in both axes, for the moment, have no particular significance.

bounded rationality is by considering that agents' memory is also bounded. In a sense, this suggestion was followed in the determination of the normalised membership values given that they were obtained using a rolling window with a fixed length. To put it more formal, we start by computing a moving average:

$$\overline{u}_t = \frac{\sum\limits_{i=t-(w+1)}^{t} u_i}{w+1}, \tag{1}$$

and a moving standard deviation

$$\sigma_t = \sqrt{\frac{\sum\limits_{i=t-(w+1)}^{t} (u_i - \overline{u}_t)^2}{w}}, \tag{2}$$

where $w+1$ is the length of the rolling window.[7]

As a starting conjecture we consider that consumers, when forming their confidence at a moment t, go through a fuzzification process applied to the unemployment rates observed during the last year, that is from $t-11$ until t, for t covering all the period under analysis. In other words, we start by considering a length of 12 months for the rolling window.

Given our choice for the type of membership function, i.e. a normal density function:

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \tag{3}$$

the non-normalised membership values for the unemployment rate associated with the categories '*low*', '*normal*' and '*high*' were determined as

$$f(u_t, \overline{u}_t - k\sigma_t, \sigma_t) \quad \rightarrow \text{low} \tag{4}$$

$$f(u_t, \overline{u}_t, \sigma_t) \quad \rightarrow \text{normal} \tag{5}$$

$$f(u_t, \overline{u}_t + k\sigma_t, \sigma_t) \quad \rightarrow \text{high} \tag{6}$$

where f is defined as in (3).

The normalised membership values for the low and high values of the unemployment rates were then obtained considering $k = 0.03$. See (4)-(6). The results can be visualised in the appendix on figure 6.

2.2 Inference

For making an inference, it will be assumed that the behaviour of consumers can be approximated by simple '*if-then*' decision rules as follows:

[7] This means that, for each new observation on the unemployment rate, consumers forget the 'oldest'. The length of the window thus can be associated with the number of observations that consumers keep in their memory. Consequently, throughout the period under analysis, the mean and standard deviation, in which are based the membershipness values, will vary.

$$\text{If } (u \in U) \text{ then } (z \in Z)$$

where U and Z are fuzzy sets and u and z are real numbers.

As we are using only one objective measure, this stage assumes a particularly simple form. Given that consumer confidence is plausibly inversely related with the unemployment rate, we simply consider that the only category to be considered is the 'low' one.[8] In other words, in this stage consumers transform the qualitative factors of unemployment rates into their confidence.

2.3 Defuzzification

The use of fuzzy rules during the inference stage produced fuzzy outputs, which have to be translated back into crisp outputs in order to be 'useful'. In our case, this means that inference output must be *defuzzified* into real numbers which will constitute the *approximate confidence index*. As the most simple case, we simply consider the values assumed by the degrees of membershipness of the 'low' category for unemployment. The following figure thus plots the original series of consumer confidence indicator and the normalised membership values associated with the 'low' category for unemployment.

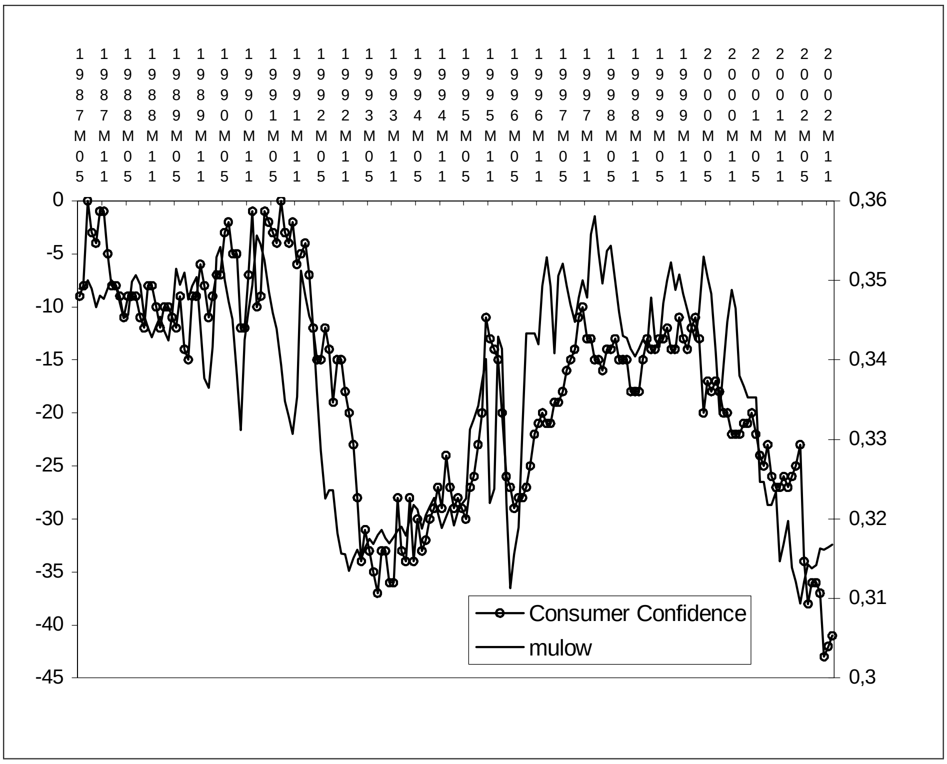

Figure 3: The subjective measure *versus* its approximation

[8] Given that symmetry was considered in determining the membership values, the 'high' category is just a mirror image of the 'low' one.

Given that no attempt to choose the length of the rolling window, $w+1$, see (1), and the value for the parameter k, see (4)-(6), in order to obtain the best and not even a good fit, the two series follow trajectories remarkably close. In fact, the correlation coefficient is now as high as 0.75. This being said, a natural question that may arise is: what if other lengths for the rolling window were considered as well as other values for the parameter k. We then performed a simple exercise of this kind which is condensed on the following figure.

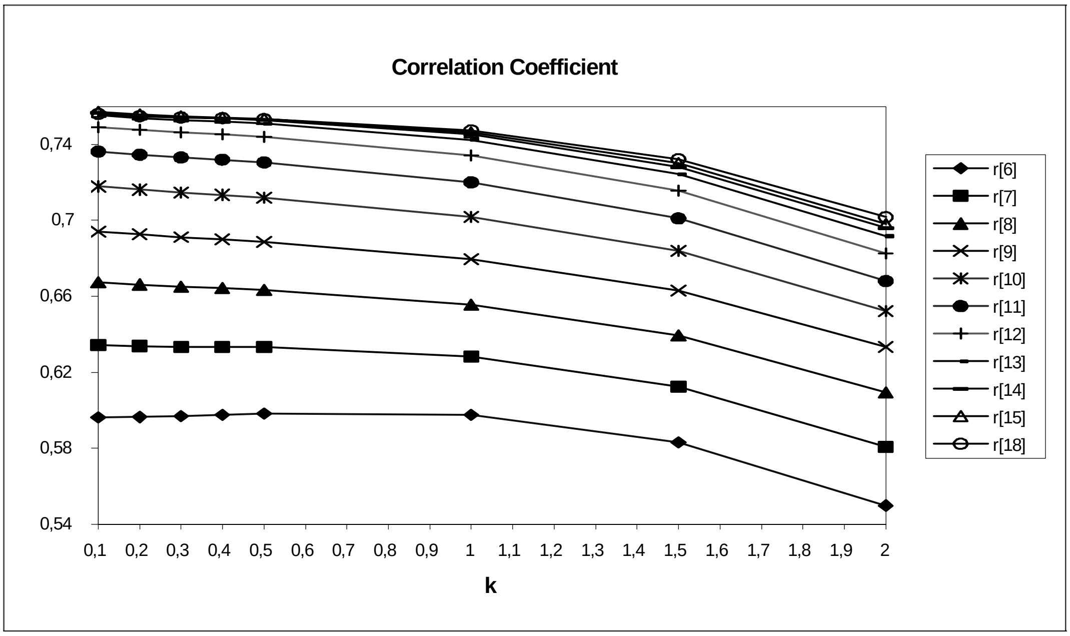

Figure 4: The influence of the length of the window and of the parameter k

As the figure clearly shows, an increase in k, which means higher separation between the average position for the three categories of unemployment, almost always leads to a decrease on the correlation coefficient. Moreover, an increase on the number of observations of the unemployment rate leads to an increase on the correlation coefficient but, in fact, inverts this tendency after the length of 15 months. In fact, for the considered grid, the maximum correlation was obtained for a length of 15 months and $k = 0.01$. The following figure shows the results associated with this parameterisation, together with the trends for both variables.

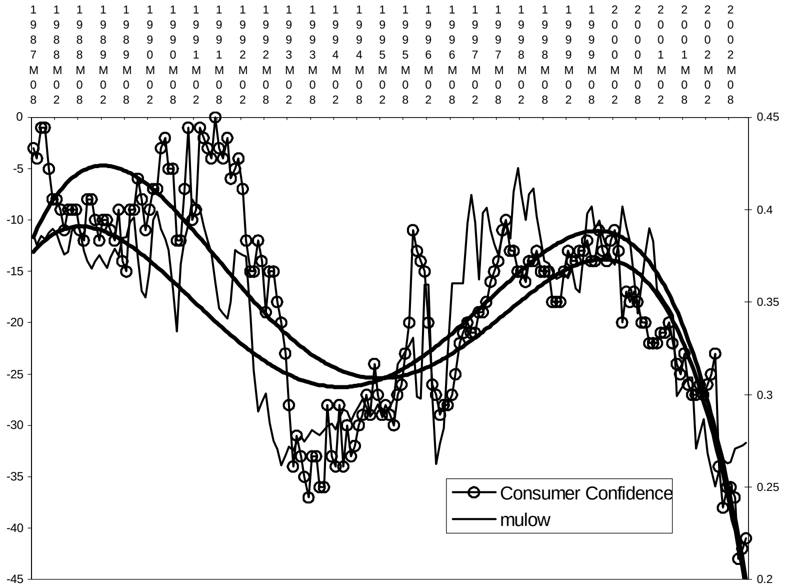

Figure 5: A better fit

3 Possible Consequences for the Electoral Cycle

Recalling the general recommendation of Nordhaus (1975), unemployment should decrease throughout all the mandate in order to maximise the level of popularity at the day of the elections. In other words, unemployment should be put at the highest level at the beginning of the mandate immediately after the elections. Moreover, still in accordance with Nordhaus (1975), the longer the mandate, the larger will be the cycle. From what our exercise on fuzzy logic has shown us, we should expect, at the beginning of the mandate, a readjustment of the level of confidence given a sudden increase on the average unemployment, which can indeed lead to an increase on confidence if the mandate is longer enough to allow a readjustment on the opposite direction.

In order to gain some understanding on the empirical side of those matters, let us consider figure 7, in the Appendix, which re-plots figure 1 considering also the position in time of the Portuguese elections. The figure shows that, in general, the moments of the elections – represented by the vertical lines – were indeed associated with peaks on consumer confidence, remarkably on the first two elections, which, as is well-known, represented absolute majorities for the incumbent. The second mandate on the figure, *i.e.* the one after the 1991 election, was, as the first one, characterised by a decrease followed by an increase on the consumer confidence but the disadvantageous trajectory of unemployment is clearly related to the electoral defeat of the incumbent. This fact indicates that, probably much more important than the *level* of the unemployment rate is the *direction* being followed. After the change in the government in 1995, another cycle on the confidence could be said to characterise the third mandate. As is well-known the 1999 election represented an electoral victory of the incumbent with almost the same percentage of votes as in the previous election, this being perfectly compatible with what happened to confidence. The forth mandate, *i.e.* the one after the 2002 election, is clearly one where the electoral defeat of the incumbent certainly reflected the loss in confidence.

4 Concluding Remarks

In the first place, we would like to highlight that, despite the use of only one objective measure, such as the unemployment rate, and absolutely no optimisation process behind, it was clearly possible to approximate the trajectory of a subjective measure, such as the consumer confidence, by an objective measure, such as the unemployment rate. In this sense, the paper reveals the apparent importance that unemployment should have on consumer confidence. Moreover, the paper illustrates that, in order to understand prospectiveness, one may use retrospectiveness.

In terms of the quality of the approximation, there were periods where the approximation was not so good but, more important than that, is the fact that there were no systematic 'errors'. A better approximation could easily be achieved by considering distinct consumer's memory and distinct separations between the membership functions throughout the period. For instance, a plausible improvement of the results seems possible to be achieved by considering that, when forming their confidence level, consumers possess decaying memory, in the sense of giving less importance to observations far away in time.

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5 Appendix

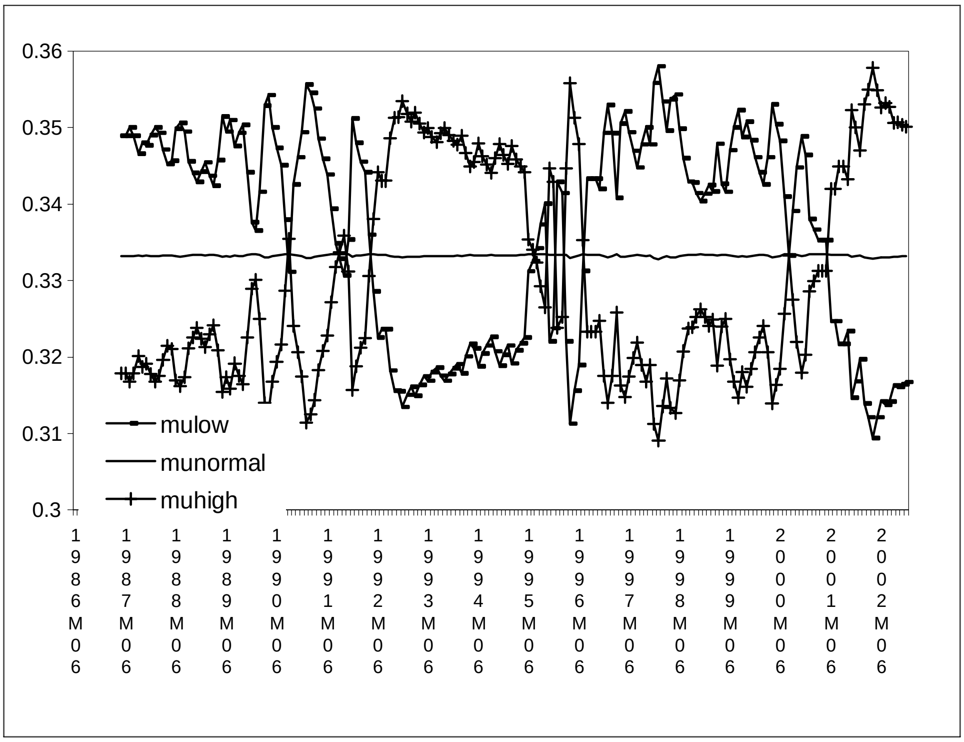

Figure 6: The normalised membership values for the unemployment rate

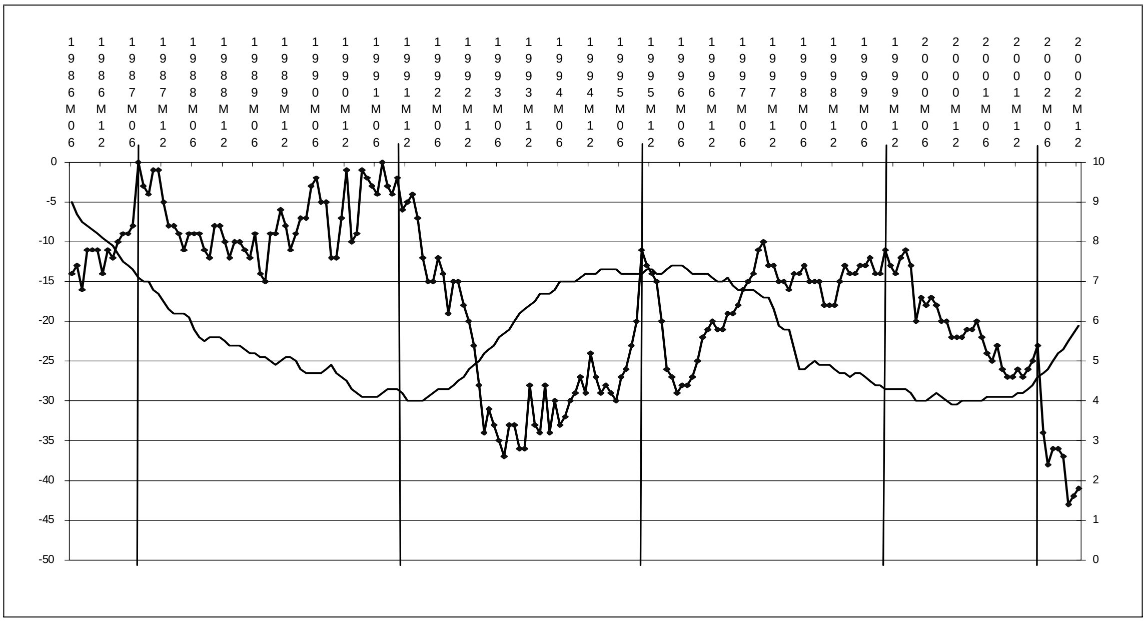

Figure 7: An informative picture about the Portuguese elections