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Solar Variability Induced in a Dynamo Code by Realistic Meridional Circulation Variations

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Abstract In this work we use an already-published method to infer a variation profile for the solar meridional circulation over the last 250 years. We feed this variation profile into a numerical dynamo code, and we reconstruct a sunspot time series that acts as a proxy for solar cycle activity. We perform three simulations with slightly different parameters, and the results are compared with the observational data. The medium and large correlation coefficients between reconstructed and observational time series seem to indicate that variations in meridional circulation play an important role in the modulation of solar activity.

Keywords Solar cycle: models · Sunspots: magnetic fields · Sunspots: statistics

1. Introduction

In an era when climate change is a very debated subject, telecommunication satellites are indispensable to civilization, and the advent of an active human solar system exploration takes shape, knowledge of the interplanetary space environment becomes an important asset. According to Mursula, Usoskin, and Maris (2007), since the concept of "space climate", was introduced more than ten years ago, we have learned a great deal about our local space environment. At the source of most of the observed phenomena within the vicinity of our planet we can find the Sun. Solar phenomena, mainly with origin in the solar cycle's variability, have a massive influence on the solar system's interplanetary space. Ongoing research on climate change shows evidence that the link between the Sun and the Earth is not as simple as was previously thought, leading to the belief that the Sun's variability has a non-negligible

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impact on the Earth's climate system (Haigh, 1996; Marsh and Svensmark, 2003). Therefore, in addition to their intrinsic interest, solar variability studies are important for several research fields.

Today the average behavior of the solar cycle can be fairly well explained by current dynamo theories, but, the variation in amplitude and period of the 11-year solar cycle is still hard to understand (Charbonneau, 2005).

The complex nature of the dynamo equations and the lack of knowledge about some of the physical parameters included in the models make it difficult to account for all possible effects (Parker, 1955; Dikpati and Charbonneau, 1999; Nandy and Choudhuri, 2002; Chatterjee, Nandy, and Choudhuri, 2004; Charbonneau, 2005; Dikpati, de Toma, and Gilman, 2006). One also has to account for the sensitivity of the intervening parameters since, according to Bushby and Tobias (2007), it has a large impact in the modeling. The authors also suggest that using phase-space attractor techniques to predict the behavior of some parameters presents better results than normal predictor techniques. Experimental, laboratory dynamo experiments aim to bring new light to some of the key processes active during dynamo action. Some of the low-order dynamics presented by dynamo action, such as magnetic-field reversal, have been recently observed in laboratory dynamo experiments (Berhanu *et al.*, 2007; Dubrulle *et al.*, 2007). These experiments suggest that some of main dynamics of the dynamo action can be explained by simplified models. This leads to an alternative approach to the study of solar variability where instead of solving the full set of dynamo equations, one tries to infer general properties of the system by using less complex models. These models, in turn, try to explain the variability of the solar cycle through stochastic fluctuations of some of the intervening physical processes (plasma flows, the α effect, *etc.*), or by assuming slow changes in these quantities (Choudhuri, 1992; Ossendrijver and Hoyng, 1996; Mininni, Gomez, and Mindlin, 2001; Pontieri *et al.*, 2003; Wilmot-Smith *et al.*, 2006; Passos and Lopes, 2008a). One aspect under study is the role of plasma flows in the behavior of the solar cycle. Although helioseismology revealed the flow profile due to differential rotation, it has not yet mapped all of the convective zone's plasma flows. Several authors agree that the meridional circulation, in particular, one of the less-known components of the plasma flow in dynamo modeling, has an important role in regulating the dynamo amplitude and period (*e.g.,* Nandy, 2004; Hathaway *et al.*, 2004; González Hernádez *et al.*, 2006; Rempel, 2006; Gizon and Rempel, 2008; Roth and Stix, 2008). Following these previous works, Passos and Lopes (2008b) used a method mixing dynamical system analysis with a simplified, low-order dynamo model and inferred discrete limit variations for the meridional circulation over the last 250 years. In this work we test the idea of Passos and Lopes (2008b), and we try to study to what extent these inferred changes in the meridional circulation, when applied to a computational solar dynamo model, can reproduce the variability observed in the Sun. In Section 2 we summarize the methodology used, and in Section 3 we explain how we implemented these results in the dynamo model. In Section 4 we present the results obtained and discuss some of their implications.

2. Reconstructed Meridional Circulation Variations

In the work of Passos and Lopes (2008b), we presented a possible reconstruction of the variations of the meridional circulation for the last 250 years, based on the analysis of the phase space of a proxy for the solar toroidal magnetic field coupled to a low-order dynamo model. This proxy is constructed by assuming a direct relation with the sunspot number (SSN). Since in this work we use the same methodology, we will just outline the steps

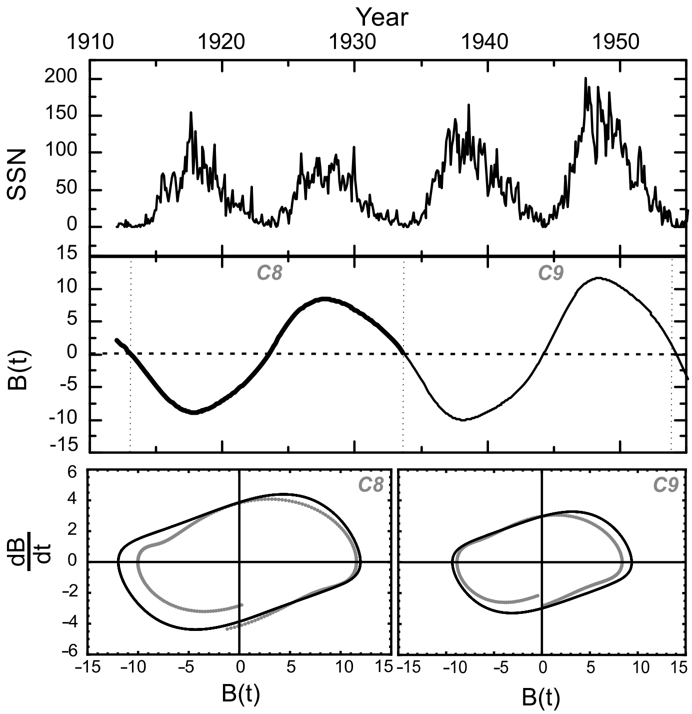

Figure 1 Application of the phase-space methodology exemplified for magnetic cycles eight and nine (years between 1912 and 1955). Top: Monthly sunspot number. Middle: $B(t)$, the magnetic field proxy. Bottom: Phase space of $B(t)$ in gray and the fitted cycle in black.

needed to construct the proxy and calculate the meridional-circulation variations but we will skip the details. For further details we suggest that the reader consult Passos and Lopes (2008b) and references therein.

We used the monthly-averaged international sunspot number (SSN) since 1750 to construct a proxy for the toroidal component of the magnetic field as $B(t) \propto \pm\sqrt{\mathrm{SSN}}$. The sunspot data was retrieved from NOAA database available at ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/. To account for the change of polarity of the field, we change the sign of $B(t)$ by hand every sunspot cycle. The proxy data is then smoothed by means of a FFT filter that smooths out variations shorter than 24 months. Afterwards, using this smoothed signal, we calculate the numerical derivative ($\mathrm{d}B/\mathrm{d}t$) using a time step of six months. The analyzed data encompasses 22 sunspot cycles, *i.e.*, 11 complete magnetic cycles. For each individual magnetic cycle, we plot the phase space $[B(t), \mathrm{d}B/\mathrm{d}t]$, and we fit it to an analytical function. In Figure 1, as an example, we show this procedure applied to magnetic cycles eight and nine.

The analytical function that we used to do the fit is a result from a low-order dynamo model presented in Passos and Lopes (2008b) and basically says that the toroidal magnetic-field component behavior over time can be approximated by a van der Pol – Duffing oscillator. We would like to point out that other functions could be used in this step, namely those presented by Mininni, Gomez, and Mindlin (2001) and Pontieri *et al.* (2003). The chosen function for $B(t)$ is defined as

$$\frac{d^2 B_\phi}{dt^2} + \omega^2 B_\phi + \mu\left(3\xi B_\phi^2 - 1\right)\frac{dB_\phi}{dt} - \lambda B_\phi^3 = 0, \tag{1}$$

where ω, μ, ξ, and λ correspond to spatially invariant coefficients and are defined as

$$\omega^2 = \left(\frac{\mu}{2}\right)^2 - \frac{\alpha R \Omega}{l_0^2}, \tag{2}$$

$$\mu = 2\left(\frac{v_p}{l_0} + \frac{\eta}{l_0^2} - \frac{\eta}{R_\odot^2}\right), \tag{3}$$

$$\xi = \frac{\gamma b_\phi^2}{4\pi\rho\mu}, \tag{4}$$

$$\lambda = \frac{\mu\gamma b_\phi^2}{16\pi\rho}, \tag{5}$$

where $R_\odot$ corresponds to the solar radius, Ω is the differential rotation of the Sun, α is a velocity associated with the regeneration of toroidal to poloidal field, the so-called α-effect, l_0 is a characteristic length of interaction for the magnetic field, v_p is the poloidal component of the plasma flow, *i.e.*, the meridional circulation, η is the solar convection zone (SCZ) average magnetic diffusivity, γ is a coefficient related to the magnetic buoyancy, b_ϕ is the spatial average of the toroidal magnetic field, and ρ is the average density of the SCZ.

The fitting procedure uses the dynamical system's standard form (Mininni, Gomez, and Mindlin, 2001), and the free parameters to adjust are ω, μ, ξ, and λ. For each magnetic cycle, the fit returns a set of parameters that characterizes the equilibrium solution for the oscillator that best approaches the experimental data. The main assumption here is that each magnetic cycle can be approximated as a van der Pol oscillator near its equilibrium solution (Passos and Lopes, 2008a). This assumption can be applied most of the time but fails to capture the dynamics of rapid cycle variation. In some cases, where the variation within a magnetic cycle is very pronounced, this assumption does not work as well. In our case the fitting procedure became unstable in magnetic cycle three. This happens because the shape of the cycle in phase space departs from a van der Pol oscillator. We also found that the fitting procedure can sometimes capture solutions in which the oscillator has not yet reached the equilibrium solution defined by the attractor. This might happen because one of the underlying variability mechanisms has a relaxation time longer than a solar cycle.

One thing that we can do to improve the fits is to constrain the only parameter that can be calculated by hand, *i.e.*, the oscillation frequency $\omega = 2\pi/T_N$, where T_N corresponds to the period (in years) of the cycle N that is being fitted. Also as discussed in Mininni, Gomez, and Mindlin (2001) and Passos and Lopes (2008b), the λ parameter is in general very small and can, most of the time, be neglected. We decided to do the fits using all of these possibilities providing four sets of parameters: fit with three parameters (excluding λ) and ω fixed (constrained by the cycle period), fit with three parameters and ω free, fit with four parameters (including λ) and ω fixed, and fit with four parameters and ω free. The results are presented in Figure 2 and indicate that λ influences the oscillation frequency ω but not μ nor ξ. Also the ω values obtained using the set with four parameters and ω free depart from the real values (Figure 2 top panel). This indicates that the system with four parameters is somehow overparametrized, and that it can be correctly described by three parameters only. Furthermore, in the sets with three parameters we can see that the values of ω obtained are

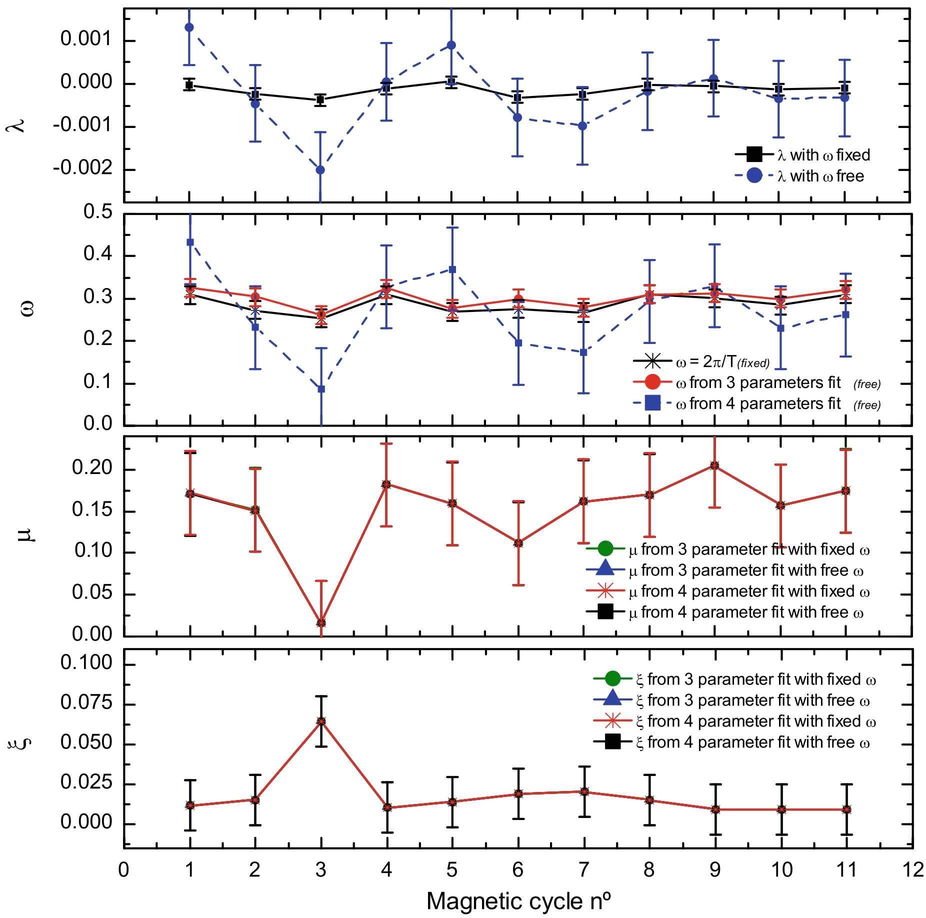

Figure 2 Evolution of the spatial parameters through the various magnetic cycles. Error bars represent 1 σ of the data.

just slightly overestimated when compared to the real values (curves in red and black from the ω panel in Figure 2).

The curves in Figure 2 show how the spatial parameters vary from cycle to cycle. As one can notice in Equations (2) to (5), all spatial parameters depend on μ and this coefficient, in turn, is defined just by physical quantities, namely the meridional circulation, the magnetic diffusivity, and some constants. This means that we can relate the variations of μ directly to changes in the physical parameters. Considering a dynamo scenario where the magnetic diffusivity is constant, we can interpret the variations in μ as variations in v_{p}. We can not, at this point, calculate absolute values since the proxy used involves an unknown proportionality, here assumed to be one for simplicity, and the obtained expressions are approximations. One can, however, calculate relative variations from cycle to cycle. To do so, we solve expression (3) in order to obtain the values for v_{p} of cycle 1, and calculate the variations of all the other cycles with respect to this one. For the calculations we used the values of μ, $l_0 = 0.1 R_{\odot}$ and $\eta = 10^6\ \mathrm{m}^2\,\mathrm{s}^{-1}$. Since we had stability problems in fitting cycle three and the μ value is very far from the mean, we decided to use half of the value in the calculation of v_{p} variation for this single case.

3. Dynamo Model Response to a Time Varying Meridional Circulation

In order to study if the inferred results for v_{p} variations could account for some of the variability observed in the solar cycle, we decided to test the results using a full dynamo model. Our intention here is to check if these results obtained through a simple low-order model can be applied to current dynamo models and retrieve plausible results. For the test we used the Surya dynamo code (Nandy and Choudhuri, 2002; Chatterjee, Nandy, and Choudhuri, 2004) which is publicly available upon request.

In Surya the meridional circulation is defined as a single cell flow per hemisphere, has v_0 amplitude at the surface at mid-latitudes, and acts as a clock controlling the output cycle period when the model is run in an advection-dominated regime (Choudhuri, Schussler, and Dikpati, 1995). While here we just tackle the problem of amplitude variations, there is also the possibility that the flow changes its geometrical distribution thus affecting the solar cycle. This problem is not taken into account here. Before we start our simulations, we first calibrated the model in order to reproduce the period of magnetic cycle one by choosing the appropriate v_0 and leaving all of the other model parameters with their default values. In our case we obtained the desired period of 20.8 years with a $v_0 = 36$ m s^{-1}. This value is the reference velocity to which we applied the previously calculated variations. By doing so we get the desired profile of varying velocity from one cycle to the next. The implemented values are shown in Figure 3 using the μ values from the three parameters with ω free (also fitted). We also decided to try another velocity-variation profile with a smaller initial value, smaller variations but the same behavior through time. The two simulated v_{p} variation profiles are called *full variation* and *smoothed variation* in Figure 3.

The simulation procedure adopted is the following: After evolving the dynamo to a stable solution corresponding to magnetic cycle 1, we stopped the run (corresponding to sunspot minimum), we changed v_0 by the same relative amount that our v_{p} changes from cycle one to two and we resumed calculations. This was successively done for all of the other cycles allowing us to see the impact of the *full-variation* and *smoothed-variation* profiles in the model. Using the *full-variation* profile we repeated the procedure but this time v_0 was changed at the maximum of activity of the cycle, prior to a sunspot minimum. The motivation behind this is that if magnetic-field feedback on the flow is the main cause of variation of the meridional circulation, then the maximum feedback level should occur when the Sun is at the maximum of its activity.

Afterwards, using the simulation's output poloidal-field component that erupts at the surface (B_ϕ) we were able to reconstruct a sunspot-like time series. Notice that this was the same hypothesis that we assume for building our field proxy $B(t)$, but used in the reverse way. To do this we monitored the average number of eruptions we got for an interval of one month and scaled these values to the same order of magnitude as sunspot number. In Figure 4 we present our three reconstructions of the sunspot record as an average over a year.

In order to study how these reconstructed time series compared to the observed values, we perform a quantitative analysis of the results. We calculated the linear correlation between observational and reconstructed time series for the maxima and minima amplitudes. Results are presented in Figure 5 and Table 1.

The simulated time series present several characteristics also found in the observed sunspot records. The low-activity periods between 1800 and 1840 and between 1870 and

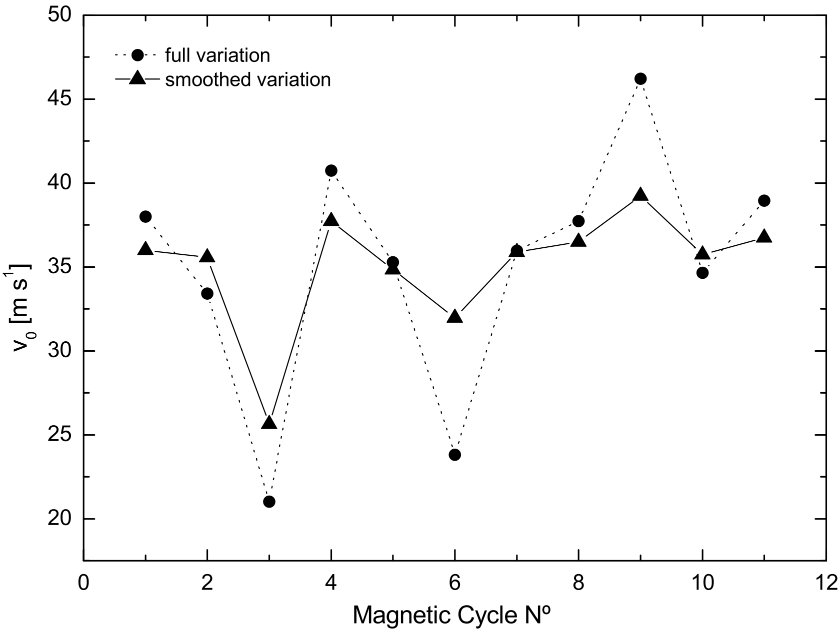

Figure 3 The two profiles used for v_{p}. Each point corresponds to a value of v_0 in the dynamo code.

1900 are recovered in the simulations. The higher correlations found occur for the *smooth variation* profile and for the *full-variation* profile changing v_0 on sunspot maximum.

For the *full-variation* profile we point out that the dynamo action does not stop, but the eruption from which we build our time series decreases markedly. This happens due to the fact that in Surya the eruptions occur through magnetic buoyancy. If the toroidal field, at the base of the convection zone (BCZ), increases above a certain threshold, then a certain part of it is made to erupt at the photosphere. In the cases where the meridional circulation decreases too much, the amount of advected poloidal field that is pushed down from the photosphere to the BCZ, is not enough to produce toroidal field strong enough to overcome this buoyancy threshold. This makes the amplitude of the next simulated cycle decrease and increases its period. On the other hand, this same buoyancy threshold coupled with the sampling interval for which eruptions occur, creates an upper limit for the maximum amplitude. During the simulation v_0 is kept constant through a complete magnetic cycle but the two corresponding sunspot peaks have different amplitudes due to the relaxation time of the dynamo.

Another thing that we were interested in was to check if the phase space of the magnetic field from the dynamo simulations has the same characteristics as the one built from observations. We applied the same methodology explained is Section 2 but this time, instead of using the observational SSN record, we used our reconstructed time series SSNrec[3]. Thus we were able to reconstruct the phase space of Surya's simulation. The phase space obtained with Surya's data shows the same characteristics as the phase space obtained with the experimental data (Figure 1 in Pontieri *et al.*, 2003 or Figure 1 in Passos and Lopes, 2008b).

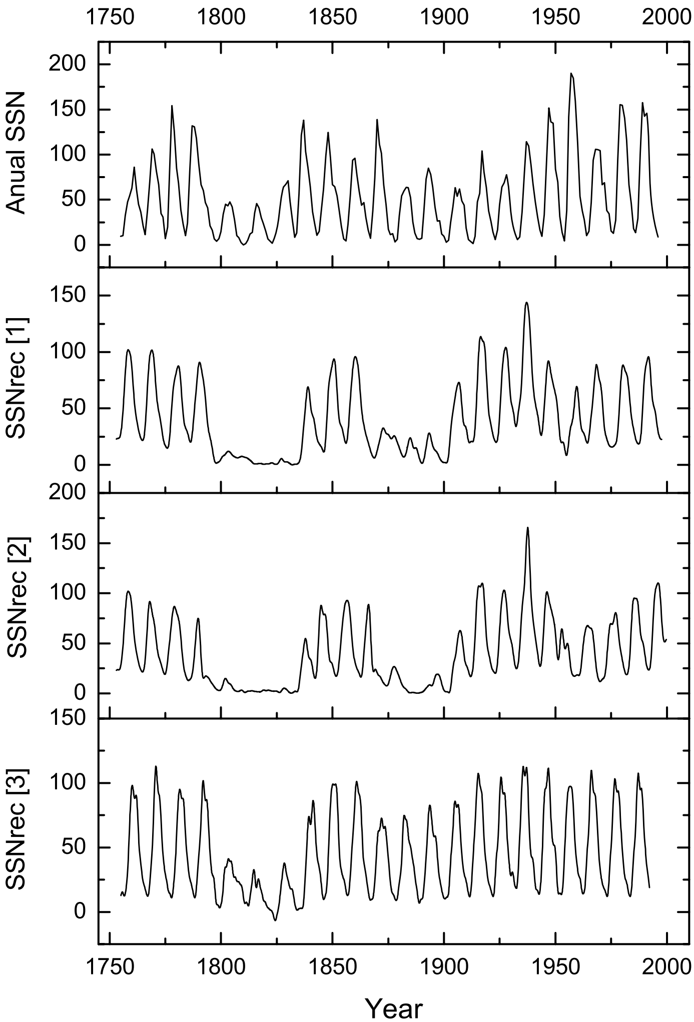

Figure 4 Top: the observational annual averaged SSN. Below the three reconstructed SSN time series. SSNrec[1] corresponds to the full-variation profile changing v_0 at sunspot minima, SSNrec[2] to full-variation profile changing v_0 at sunspot maxima, and SSNrec[3] to smoothed variation profile changing v_0 at sunspot maxima.

Table 1 Pearson's correlation coefficients between the three simulated data series and the observational sunspot records for the amplitude in solar maximum and the amplitude in solar minima.

Reconstructed SSN	Cor. Coef. for max. amplitude	Cor. Coef. for min. amplitude
SSNrec[1]	0.45	0.42
SSNrec[2]	0.51	0.51
SSNrec[3]	0.58	0.35

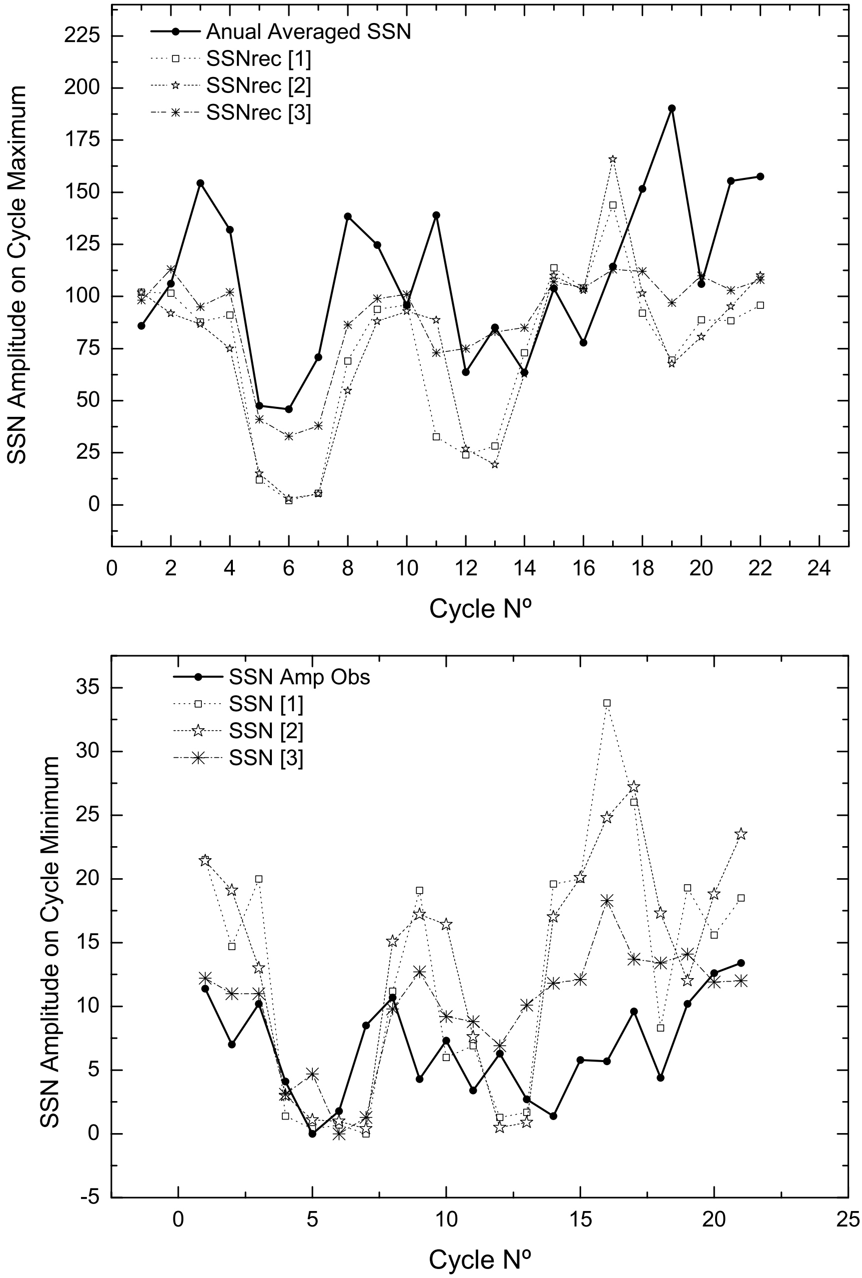

Figure 5 Maxima and minima behavior of the three reconstructed SSN time series with the mean annual observational values. Top: Amplitude of SSN on cycle maximum. Bottom: Amplitude of SSN on cycle minimum.

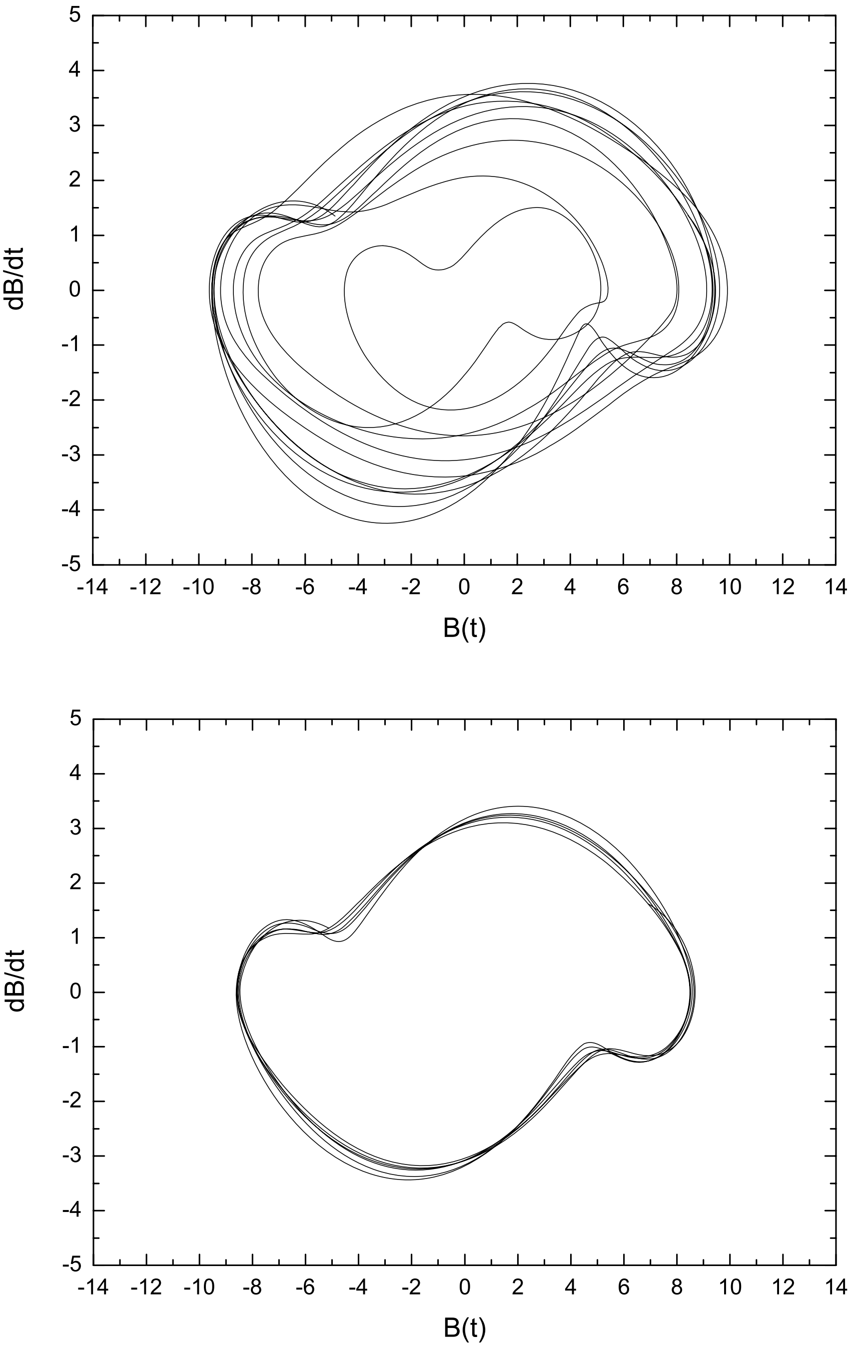

Figure 6 Top: The phase-space diagram for $B(t)$ based in Surya's SSN with changing v_{p}. Bottom: A 130-year simulation with constant v_{p} (right).

4. Discussion, Remarks and Conclusions

In this work we used a methodology recently proposed by Passos and Lopes (2008b) that can be used to extract additional information from the available 300 years of sunspot number observations. We used the sunspot number between 1750 and 2000 to construct the phase space of a quantity $[B(t)]$ that is representative of the toroidal component of the solar magnetic field. Then we used the phase space as a tool coupled to a low-order dynamo model to infer variations for the meridional circulation over 11 magnetic cycles. Here we intended to take this idea of a changing meridional circulation a step forward by using the derived results together with a computational dynamo model. The question we wanted to answer was if the inferred results applied to a dynamo model could reproduce, to some degree, the observed solar-cycle variability. To implement this idea, we used the publicly available Surya dynamo code. We calibrated the model in order to reproduce the period of magnetic cycle 1, and we retrieved the initial value for the amplitude of the meridional circulation (v_0). After simulating a full magnetic cycle, we paused the simulation, changed v_0 by the same amount that was calculated in the first part of the work and resumed the simulation for another complete cycle. Using the derived v_p-variation profile we performed two simulations: one changing the value of v_0 at sunspot minima and another changing v_0 at sunspot maxima (SSNrec[1] and SSNrec[2], respectively). A third simulation was done by using an "attenuated" v_p variation profile and changing v_0 at sunspot minima (SSNrec[3]). This was done for the 11 cycles. For each cycle simulation we looked at the number of eruptions that we got during a month and used that quantity to reconstruct a sunspot-like time series. Furthermore we performed a correlation analysis between the reconstructed and the observational time series in order to quantify the quality of the results.

We consider the medium and large correlations that we found for the maximum and minima amplitude values for the simulated time series and the observational one to be very encouraging. We are very aware of the differences that exist between the model implemented in the Surya code and the low-order dynamo model that was used to derive the results. While Surya solves the equations for a two-dimensional, kinematic dynamo in a spherical shell in the presence of a meridional circulation, using radial and azimuthal dependencies for several physical parameters, the low-order model deals with mean spatial quantities and averages. We also know that no computational code is able to reproduce accurately the observed solar variability. In this context, the fact that we found this level of correlation, leads us to believe that the main idea behind the work and the variation profile inferred for v_p is correct. Meridional circulation has a strong impact on the dynamo behavior and can explain part of the observed variability. It would be very interesting to see the implementation of this derived v_p profile by groups currently working in dynamo simulations. If we believe that these variations occurred then the implementation of other physical mechanisms in the models could present more realistic results.

The initial method used allowed us to sample the changes in v_p once every magnetic cycle, although in the real Sun we expect that these changes happen in a continuous way. We are currently improving the analysis method in order to retrieve values for v_p every sunspot cycle. If we succeed, this will allow us to refine the previous simulations and extend the study of variability to the cycle period in addition to the amplitude.

There is also room for improvement in terms of the model used for explaining the behavior of B_ϕ. The approximations used were perhaps too simplifying. The main goal would be to be able to use this phase-space methodology coupled to a more robust theoretical model to produce inversions and obtain more accurate information about the physical quantities relevant to dynamo mechanisms. The efforts by helioseismology to accurately map the meridional circulation profile below the photosphere should be continued and the inclusion of this experimental profile in current dynamo models should be pursued.

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