Line transect sampling in mountainous terrains

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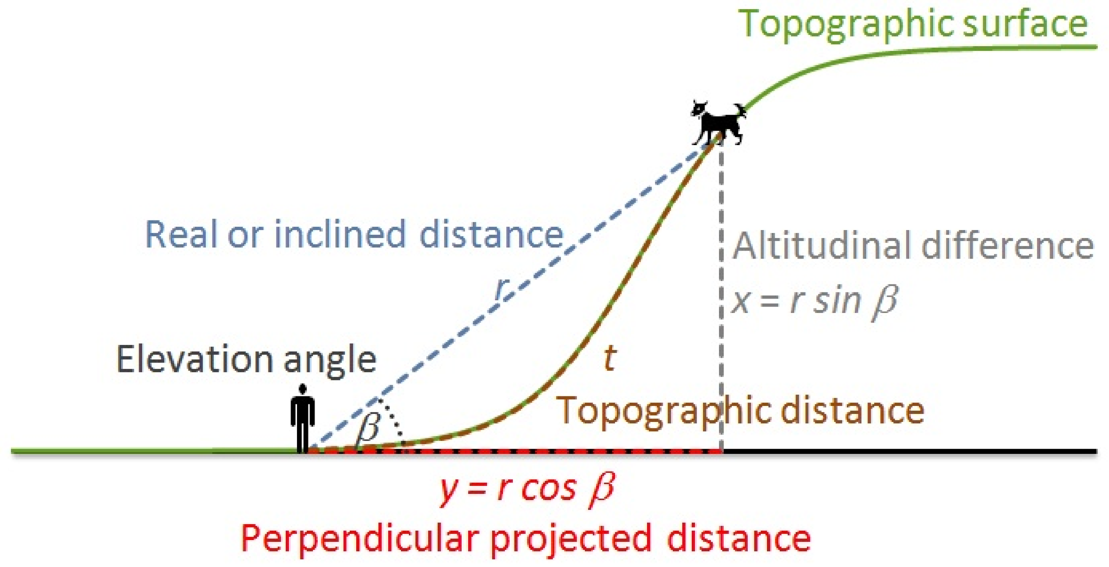
Abstract

Distance Sampling theory is based on measurements of perpendicular distances from a line (or radial distances from a point transect) to detected animals, in order to estimate their probability of detection. In plain terrains, these distances are often measured directly, with readily available instruments. In mountainous terrains, several types of distances could be measured. Buckland *et al.* [2] suggest horizontal projections of all measured distances (rather than the distances measured over the ground) for consistency and to avoid potential biases. However, simply projecting perpendicular distances on a horizontal plane can lead to biased estimates, even with the inclusion of altitude as a covariate to model detectability [1]. Here we show that this bias can be due to the violation of the uniformity of the projected perpendicular distances. We propose estimators of detection probability that could be useful when sampling in uneven terrains. Estimators' performance is compared with those used in conventional distance sampling.

Distances in mountainous terrains

Commonly used instruments are:

- GPS: to record the coordinates and the length of a given transect;
- rangefinder: to measure the distance between the observer and a detected animal;
- compass: to measure angles.



Detection probability estimation

Conventional detection probability estimator

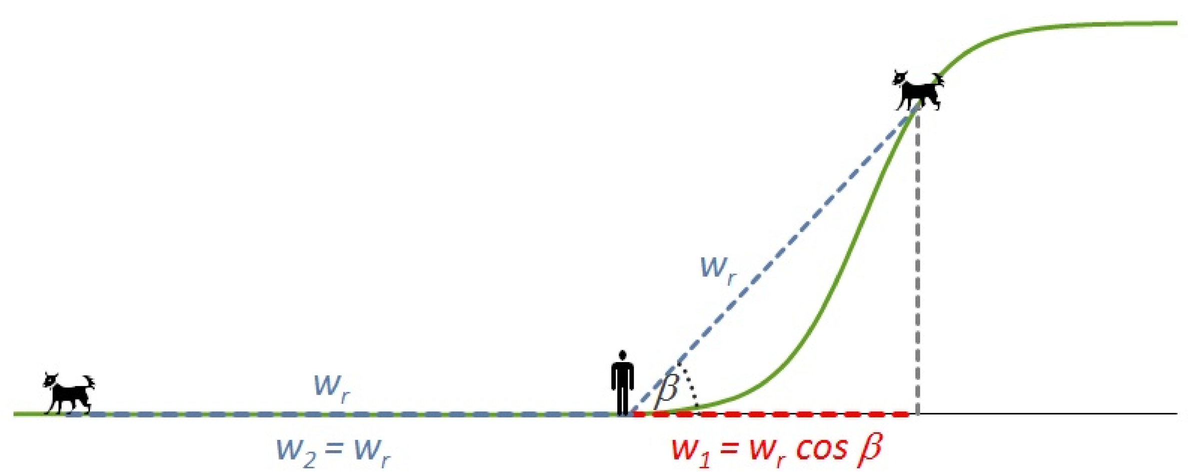
$$\hat{P}_a = \int_0^w \hat{g}(y) f(y) dy = \frac{1}{w}\int_0^w \hat{g}(y) dy = \frac{\hat{\mu}}{w}, \qquad (1)$$

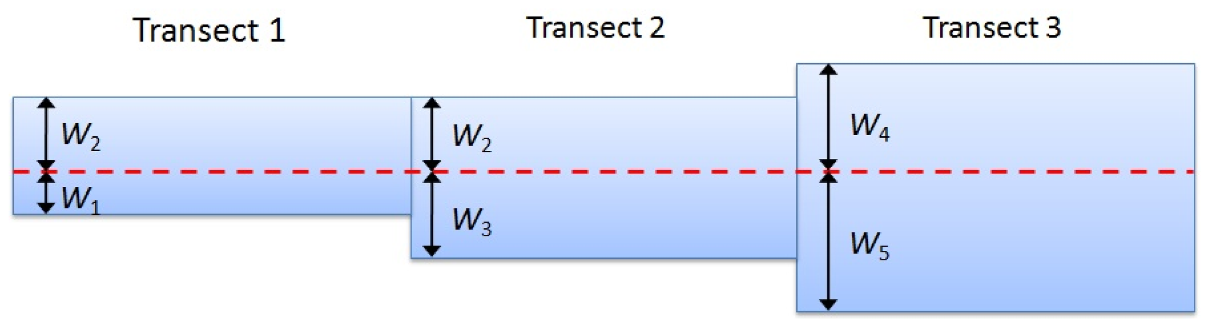
where

- w is the truncation distance;
- $g(y) = P(\text{detection}|\text{distance } y)$ is the detection function;
- $f(y) = \frac{1}{w}$ is the *pdf* of perpendicular distances y.

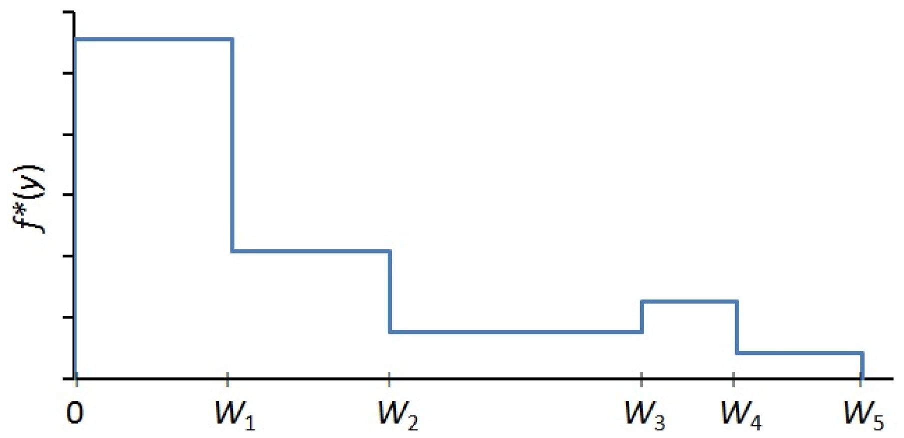
Common problems in mountainous terrains

- Projected transects with unequal width;
- Projected truncation distance is not necessarily the same for both sides of line transect.





The *pdf* of the perpendicular distances, y, from the line transect is



Proposed detection probability estimator

$$\tilde{P}_a = \sum_{i=1}^{\kappa}\int_{w_{i-1}}^{w_i} \hat{g}(y) f^*(y) dy, \qquad (2)$$

where

- κ represents the number of distinct truncation distances w_i;
- $w_0 < w_1 < ... < w_\kappa$, $w_0 = 0$ and $w_\kappa = max(w_i) = w$;
- $f^*(y) = \frac{1}{(w_i - w_{i-1})}\frac{c_i}{\sum_{j=1}^{\kappa} c_j}$, $\quad w_{i-1} \le y < w_i, \quad i = 1, ..., \kappa$;
- c_i is the cumulative absolute distribution of step level

$$c_i = \begin{cases} 1, & \text{if } i = 1 \\ 1 - \frac{1}{C}\sum_{j=1}^{i-1}\frac{\alpha_j}{w_j}, & \text{if } i = 2, ..., \kappa, \end{cases}$$

- $0 < \alpha_i < 1$, is the contribution of strip with width w_i and $\sum_{i=1}^{\kappa}\alpha_i = 1$;
- $C = \sum_{j=1}^{\kappa}(\alpha_j / w_j)$.

We considered the following estimators for P_a:

- $\tilde{P}_{a_1}$: $\alpha_i = p_i$, $i = 1, ..., \kappa$ and p_i is the relative frequency of each w_i observed;
- $\tilde{P}_{a_2}$: $\kappa = 2$, $w_1 = \bar{w}$, $w_2 = \max(w_i)$ and $\alpha_i = 1/2$;
- $\tilde{P}_{a_3}$: $\kappa = 2$, $w_1 = (\max(w_i) + \min(w_i))/2$, $w_2 = \max(w_i)$ and $\alpha_i = 1/2$;
- $\tilde{P}_{a_4}$: $\kappa = 2$, $w_1 = \tilde{w}$, $w_2 = \max(w_i)$, where $\tilde{w}$ represents the median of the w and $\alpha_i = 1/2$.

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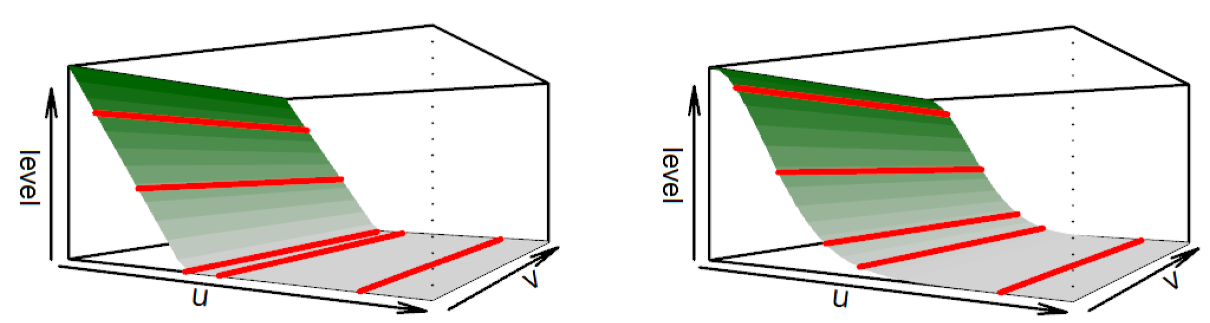
Simulation

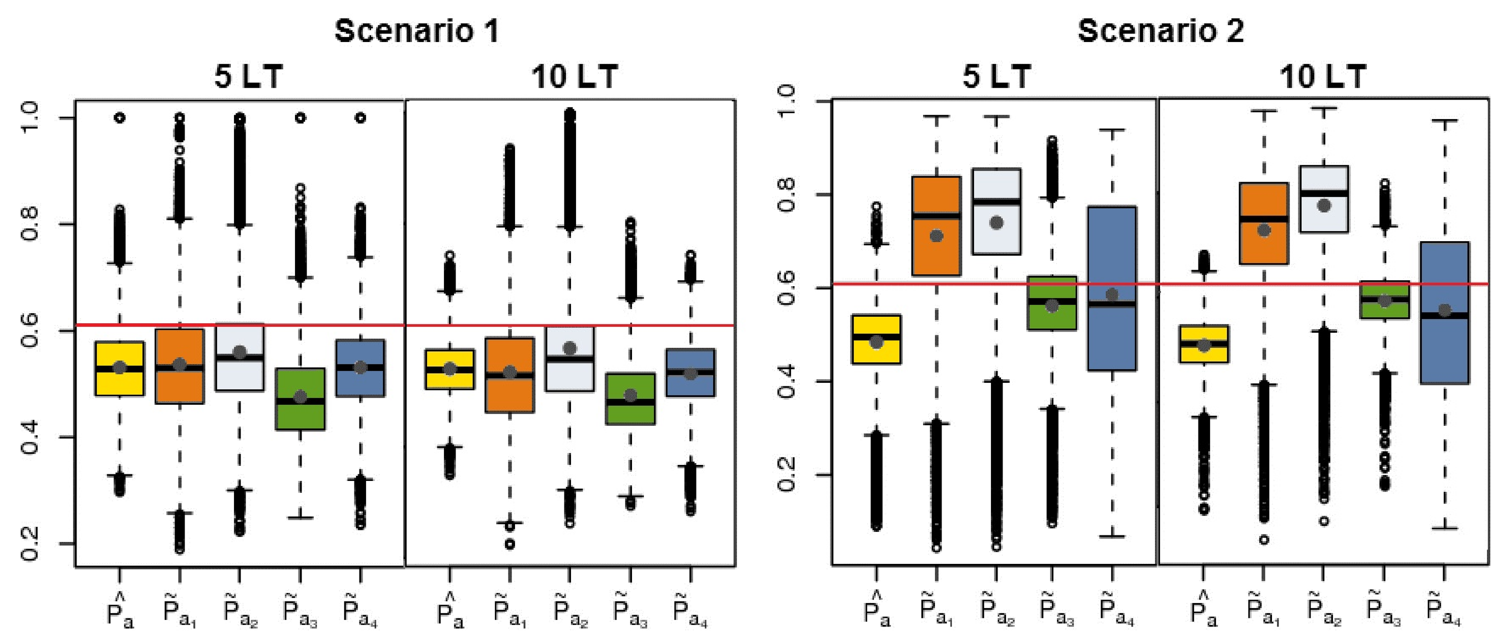
Surfaces generated with projected square area $A = [0;1] \times [0;1]$:



- Population with $N = 510$ animals randomly located;
- Random placement of $k = 5$ and $k = 10$ transect lines on u axis;
- Artificial sightings generated with $g(r) = exp\{-r^2/(2\theta^2)\}$, for $0 \le r \le w_r$, of the inclined distances;
- Probability of detection considered $P_{a_r} = 0.61$.

Scenario	k	w_r	θ	$\bar{a}$	$\bar{N}_c$	$\bar{n}$
1. Two slopes	5	0.0234	0.012	0.2013	102.4	62.7
	10			0.4020	204.9	125.3
2. Curve	5	0.0491	0.025	0.2972	156.0	95.5
	10			0.6075	312.3	191.4

$\bar{a}$: Mean covered area; $\bar{N}_c$: mean population size in the coverage area, $\bar{n}$: mean sample size.



Discussion

- In scenario 1 all estimators underestimate the true variance;
- High number of outliers, which contribute to an increase in the variance and also to the asymmetry of the distributions;
- For a highly varying slope surface (scenario 2):
  - Conventional estimator has a poor performance;
  - $\tilde{P}_{a_3}$, presents a much better performance (less biased and more precise) than the others.

References

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[2] Buckland, S.T., Anderson, D.R., Burnham, K. P., Laake, J.L., Borchers, D.L., Thomas, L.: Introduction to distance sampling. Oxford University Press, New York (2001).