On the minimum separation between any pair of flight trajectories

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The paper presents methods to determine the time, positions and distance of closest approach for two aircraft following arbitrary trajectories in two or three dimensions. The distance of closest approach of two aircraft following arbitrary curved trajectories is determined by two conditions: (i) the relative velocity must be orthogonal to the relative position in order for the distance to be a non-zero extremum; (ii) the radial acceleration including centripetal terms must have a direction that increases the separation for the extremum to be a minimum. This theorem on the distance of closest approach simplifies in the case of uniform motion along rectilinear trajectories. Three examples are given: (i) the two-dimensional motion of aircraft changing the velocity of one of them so as to enforce a given minimum separation distance; (ii) the three-dimensional motion of two aircraft, one flying horizontally and the other climbing, changing the vertical velocity of the latter to ensure a minimum separation distance set "a priori"; (iii) the case of an aircraft flying with constant velocity on a straight line so that its closest approach to another aircraft flying in a circular holding pattern in the same plane occurs at a given time chosen "a priori".

In the traffic of aircraft safety is identified with the absence of collisions or conflicts. A conflict occurs when the distance between the distance between the centroids of two aircraft is less than a safe separation distance (SSD) determined by their size. Thus (i) the absence of conflicts and (ii) the confirmation that a conflict has been resolved, depends on determining the distance of closest approach (DCA) that is not less than the SSD. The conflict resolution relies (iii) on trajectory modifications that change the DCA from smaller than the SSD to larger than (or equal to) the SSD. This paper presents methods to determine the time, positions and distance of closest approach for two aircraft following arbitrary trajectories in two or three dimensions. The two dimensional cases include aircraft ground movements at an airport. The three dimensional cases include all types of flying vehicles, like airplanes, helicopters, drones, rockets and satellites. The differences in conflict detection and resolution (CDR) between all these types of vehicles concern the speed, size, and distances that enter as parameters in the same methods of calculation of distance and time of closest approach.

The distance and time of closest approach are essential inputs for CDR methods. The collision risk applies to cars, ships and submarines and aircraft. Taking as example the case of Air Traffic Management (ATM) the problem may be divided into (i) prediction of flight paths, (ii) safety assessment and (iii) conflict resolution. Collision avoidance between aircraft starts with separation distances (e.g. longitudinal, lateral and altitude) leading to high Target Level of Safety (TLS: probability of collision less than 5E-9 per hour) for various aircraft encounter geometries, like level crossing or climb and descent. These safe separations ultimately determine airspace capacity. Many of these methods assume straight trajectories or approximate curved trajectories by straight segments. The purpose of the present paper is to determine the distance and time of closest approach for arbitrary curved trajectories without approximations of any kind. This may be used for CDR methods or to assess TLS.

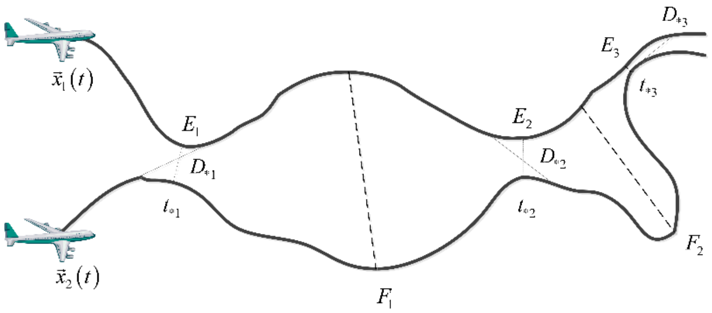

Fig. 1 For aircraft moving along curved trajectories there may exist several times of closest approach, corresponding to local minima of the relative distance that usually exceeds the minimum distances. The distance of closest approach is then the smallest of all local minima. There may also exist local maxima indicated by dotted lines.

The minimum is a local minimum for curved trajectories with non-uniform velocity, and an absolute minimum for straight trajectories with uniform velocity. In the latter case of straight trajectories with constant velocity the maximum distance is infinity after an infinite time, and there are no local maxima. In the case of non-uniform motion along curved paths there can exist local minima D_{*n} of the relative distance (*Figure 1*) as well as maxima F_*. The objective of maximizing the relative distance corresponds to: (i) relative position orthogonal to the relative velocity, as for any extremum; (ii) the local acceleration must overcome the centripetal acceleration to increase the distance.

There is a variety of CDR methods including multi-agent algorithms that apply to aircraft moving in two or three dimensions. All CDR methods: (i) start with the identification of a conflict; (ii) involve trajectory changes to resolve the conflict; (iii) end with the verification that the conflict has been resolved. The safety of traffic requires that a minimum SSD be held, for example ensuring that the "safety volumes" around two aircraft do not penetrate.

The conflict detection and resolution (CDR) requires that the trajectories of two aircraft lead to a relative distance not less than the safe separation distance (SSD) at all times. This can be ensured if the distance of closest approach (DCA) is not less than the SSD. The DCA has been calculated for two arbitrary trajectories, generally curved and with accelerated motion: in this case there can exist several local minima of the relative distance between two aircraft and the DCA is the smallest of them. In the case of two aircraft with constant velocity the rectilinear trajectories lead to one time of closest approach; the maximum distance is unbounded for large time. The preceding results apply to any type of vehicle (car, ship, aircraft) and to motion in two or three dimensions. They are illustrated by 2 examples with constant velocities: (i) conflict avoidance between two aircraft changing the modulus of the velocity of one of them; (ii) conflict avoidance between two aircraft changing the climb rate of one of them. A third example involves one straight and one curved trajectory and requires one velocity to be chosen to achieve a time of closest chosen "a priori".

Related Work:

Campos, L.M.B.C. and Marques, J.M.G. "On a dimensionless alternative to the ICAO target level of safety", *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, vol. 230, no. 9, pp. 1548 – 1557, 2016.

Campos, L.M.B.C. & Marques, J.M.G. "On the Probability of Collision for Crossing Aircraft", *Aircraft Engineering Aerospace Technology*, vol. 83, no. 5, pp. 306-314, 2011.

Campos, L.M.B.C. and Marques, J.M.G. "On the Probability of Collision Between Climbing and Descending Aircraft," Journal of Aircraft, vol. 44, no. 2, pp. 550-557, 2007.