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ON THE MAXIMIZATION OF CONTROL POWER IN LOW-SPEED FLIGHT FOR A BWB CONFIGURATION

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Abstract

The maximization of control power is considered for an aircraft with multiple control surfaces, which may be decoupled or coupled; the force and moment coefficients are specified by polynomials of the control surface deflections, of any degree not less than two. The optimal deflections, which maximize and minimize any force or moment coefficient are determined subject to constraints on the range of deflection of each control surface. The results are applied to a flying wing configuration to determine: (i/ii) the pitch trim, at the lowest drag for the fastest climb, and at the highest drag for the steepest descent; (iii) the maximum and minimum pitching moment and the associated range of trimmed c.g. positions for several engine configurations; (iv) the maximum and minimum yaw control power and the fraction needed to compensate an outboard engine failure for several propulsion configurations; (v) the maximum and minimum rolling moment and the strongest wake vortex which can be compensated. The optimal case of all control surfaces has significant advantages over using just one, e.g., the range of drag modulation with pitch trim is much wider and the maximum and minimum available control moments larger.

Keywords: yaw control power, multiple control surfaces, flying wing configuration

1. Introduction

There is a substantial literature on the optimization of the deflection of several control surfaces [1-5], for example to minimize trim drag in cruise; this is particularly relevant to the flying wing configuration or blended wing body [6-9], that has the whole span available at the trailing-edge for control and high-lift surfaces, and a large fraction of the leading-edge. For example, the trailing-edge surfaces may consist of a body flap inner and outer flaps and ailerons, that may be used for lift, pitch or roll control. The present work extends existing knowledge on the subject in two directions: (Part I) on theoretical side [10-13] by allowing for the interactions among different control surfaces, in the sense that the deflection of one control surface may affect the forces or moments produced by neighboring ones; (Part II) on the application side [14-17] by considering not only minimum drag but also maximum drag (e.g. for fast descent) and maximum control moments for emergencies, such an engine-out condition or a wake vortex encounter. These applications are made to a flying wing, extending the scope of the literature [18-22] that concentrates mostly on minimum drag for pitch trim in cruise. Thus, the present paper is also a contribution to the expanding literature on various aspects of the Blended Wing Body (BWB) aircraft [23-26].

The theory (Part I) concerns the maximization of control power in low-speed flight for a flying wing aircraft configuration and is considered as concerns several components of control forces and moments. First a method of finding the minimum and maximum forces and/or moments is presented (§1); it finds the extrema (§3) of forces and moments (§2), considering the range of possible control surface deflections (§4). The method is presented first (§2-4) assuming (i) decoupled controls specified by (ii) polynomials of second degree in the deflections. These two restrictions are removed in turn, viz. (i) in §5 and (ii) in §6, and both in §7. Thus, the optimization theory applies to coupled controls specified by polynomials of arbitrary degree (§7); the number of input data points increases with the degree of the control polynomials and the extent of cross-coupling (§8).

The theory (Part I) is applied to a flying wing aircraft configuration (Part II) with five control surfaces: a body flap, wing inner, middle and outer flaps, and rudders. A baseline low speed straight and level steady flight condition (§9) is considered in several situations: (i) the minimum drag decrease (10%) to achieve pitch trim with highest climb rate after take-off (§10); (ii) maximum drag (§11) increase (135%) with constant lift and pitch trim to achieve the steepest descent to land; (ii) the maximum and minimum pitching moment (§12); (iv) this has implications on the c.g. range for several aircraft configurations with engines overwing or underwing relative to the aircraft datum as concerns pitch-down or pitch-up (§13); (v) the maximum and minimum yawing moment (§14); (vi) the implications for the worst case of outboard engine failure (§15); (vii) the maximum and minimum rolling moment (§16); (viii) the implications as concerns the maximum difference in vertical velocity at the wing tips which can be compensated in a wake vortex encounter (§17). The conclusion (§18) summarizes the low-speed control capabilities which result from the optimization method.

PART I – OPTIMIZATION OF HIGH-ORDER INTERDEPENDENT CONTROLS

Considering an aircraft with multiple control surfaces, the choice of deflections which maximize and minimize any force or moment is considered (§2) first under two assumptions: (a) the controls are independent or decoupled; (b) the control forces and moments are specified by quadratic functions of the deflections. Using these assumptions (a, b) leads to the simplest application of the method to find the extrema (maxima and minima) of the control forces and moments (§3), whether they lie inside the interval of possible deflections or at the limit deflections (§4). The method is then extended to the case of coupled or interdependent controls (§5) and to the case of control forces and moments specified by a polynomial or arbitrary degree of the deflections (§6), thus removing one at a time of the two restrictions (a) and (b) above; both restrictions can be removed in the general case of coupled controls specified by high-order polynomials (§7), which requires more aerodynamic data (§8).

2. Forces and moments due to multiple control surfaces

The forces and moments along the three axes are denoted (1b) by C_α with the index α running (1a) from one to six:

$$\alpha = 1, 2, ..., 6: \qquad C_\alpha = \{X, Y, Z, L, M, N\}; \tag{1a,b}$$

the index *i* numbers the *N* control surfaces.

$$i = 1, ..., N = 5: \quad i = \{\text{body flap,inner flap,middle flap,outer flap,rudder}\}; \tag{2a,b}$$

The forces and moments due to the deflection δ_j of a surface are assumed to: (a) depend only on the deflection of that surface; (b) to be specified by a polynomial of the second degree:

$$C_\alpha(\delta_i) = C_{\alpha 0} + a_{\alpha i}\delta_i + b_{\alpha i}(\delta_i)^2, \tag{3}$$

$$C_{\alpha 0} = C_\alpha(\delta_i = 0), \qquad a_{\alpha i} = \frac{dC_{\alpha i}}{d\delta_i}, \qquad 2b_{\alpha i} = \frac{d^2 C_{\alpha i}}{d\delta_i^2}, \tag{4a-c}$$

which involves: (i) the static coefficient (4a); (ii) the control slope (4b); (iii) the control curvature (4c). The assumptions (a) and (b) will be removed in the sequel (§4-6), first one at a time, and then the two together (§7). The presentation of the method of maximization of control power is made first under the assumptions (a) and (b). The control effect for the surface j is the deviation from the static value.

$$\Delta C_{\alpha i}(\delta_i) = C_{\alpha i}(\delta_i) - C_{\alpha 0} = a_{\alpha i}\delta_i + b_{\alpha i}(\delta_i)^2. \tag{5}$$

The sum over all control surfaces of (3)/(5) specifies:

$$C_\alpha = \sum_{i=1}^{N} C_{\alpha i} = C_{\alpha 0} + \sum_{i=1}^{N}\left[a_{\alpha i}\delta_i + b_{\alpha i}(\delta_i)^2\right], \tag{6}$$

$$\Delta C_\alpha = \sum_{i=1}^{N} \Delta C_{\alpha i} = \sum_{i=1}^{N}\left[a_{\alpha i}\delta_i + b_{\alpha i}(\delta_i)^2\right], \tag{7}$$

the total force/moment (6) and the total control effect (7).

3. Extrema (maximum and minimum) of forces and moments

The extremum of a force or moment is obtained for the deflection which leads to a zero derivative (8a):

$$0 = \frac{dC_{\alpha i}}{d\delta_i} = a_{\alpha i} + 2b_{\alpha i}\,\bar{\delta}_i, \quad \bar{\delta}_i = -\frac{a_{\alpha i}}{2b_{\alpha i}}. \tag{8a,b}$$

In the case of quadratic control polynomial (3) there is only one extremum (8b). If the second-order derivative is positive/negative the extremum is a minimum/maximum:

$$\frac{d^2C_{\alpha i}}{d\delta_i^2} = 2b_{\alpha i} = \begin{cases} >0 & \text{implies } \bar{\delta}_i \text{ is minimum of } C_{\alpha i}, \\ <0 & \text{implies } \bar{\delta}_i \text{ is maximum of } C_{\alpha i}. \end{cases} \tag{9a,b}$$

The value of the extremum is:

$$\bar{C}_{\alpha i} = C_{\alpha i}\left(\bar{\delta}_i\right) = C_{\alpha 0} + a_{\alpha i}\bar{\delta}_i + b_{\alpha i}\left(\bar{\delta}_i\right)^2 = C_{\alpha 0} + a_{\alpha i}\left(-\frac{a_{\alpha i}}{2b_{\alpha i}}\right) + b_{\alpha i}\left(-\frac{a_{\alpha i}}{2b_{\alpha i}}\right)^2, \tag{10}$$

which simplifies to (11b):

$$\delta_{i_{\min}} \le \bar{\delta}_i \le \delta_{i_{\max}}: \qquad \bar{C}_{\alpha i} \equiv C_{\alpha 0} - \frac{\left(a_{\alpha i}\right)^2}{2b_{\alpha i}}. \tag{11a,b}$$

This extremum (11b) is achievable only if the value (8b) lies within the limits of deflection of the control surface (11a). If not, then the extremum lies at one of the deflection limits, as detailed next.

4. Extrema in the deflection range or at the limits

Consider first that a maximum of $C_{\alpha i}$ is sought. It will occur in the range of deflection if (9b,11a) are met:

$$\delta_{i_{\min}} \le \bar{\delta}_i = -\frac{a_{\alpha i}}{2b_{\alpha i}} \le \delta_{i_{\max}}; \quad b_{\alpha i} < 0: \qquad C_{\alpha i_{\max}} = C_{\alpha i}\left(\bar{\delta}_i\right) = C_{\alpha 0} - \frac{\left(a_{\alpha i}\right)^2}{2b_{\alpha i}}. \tag{12a-c}$$

If (12a) is not met the maximum (12c) is not achievable, because it lies outside the deflection range; if (12b) is not met there is no maximum, and there is a minimum instead. In both cases the maximum will be at one of the extreme deflections. If the extreme defections are symmetric (13a):

$$\delta_{i_{\min}} = -\delta_{i_{\max}}: \qquad C_{\alpha i_{\max}} = \begin{cases} C_{\alpha i}\left(\delta_{i_{\max}}\right) & \text{if} \quad a_{\alpha i} > 0, \\ C_{\alpha i}\left(\delta_{i_{\min}}\right) & \text{if} \quad a_{\alpha i} < 0, \end{cases} \tag{13a,b}$$

the maximum will be at: (i) the largest deflection (13a) for positive slope; (ii) the lowest deflection (13b) for negative slope. The reason is that symmetric deflection (14a) implies (14b).

$$\delta_{i_{\min}} = -\delta_{i_{\max}}: \qquad b_{\alpha i}\left(\delta_{i_{\min}}\right)^2 = b_{\alpha i}\left(\delta_{i_{\max}}\right)^2, \tag{15a,b}$$

and thus:

$$C_{\alpha i}\left(\delta_{i_{\max}}\right) - C_{\alpha i}\left(\delta_{i_{\min}}\right) = a_{\alpha i}\left(\delta_{i_{\min}} - \delta_{i_{\max}}\right). \tag{15c}$$

Thus, the maximum is at $\delta_{i_{\max}}$ if $a_{\alpha i} > 0$ and at $\delta_{i_{mim}}$ if $a_{\alpha i} < 0$.

In the case a minimum is sought the same reasonings apply. The minimum lies within the range of deflection if (9a)=(16a) and (11a)=(16b) are met, and then takes the value (11b)=(16c):

$$\delta_{i_{\min}} \le \bar{\delta}_i = -a_{\alpha i}/2b_{\alpha i} < \delta_{i_{\max}}, \quad b_{\alpha i} > 0: \qquad C_{\alpha i_{\min}} = C_{\alpha i}\left(\bar{\delta}_i\right) = C_{\alpha 0} - \left(a_{\alpha i}\right)^2/2b_{\alpha i}. \tag{16a-c}$$

If one of the conditions (16a,b) is not met the minimum is at an extreme of the range of deflection. In the case of symmetric maximum deflections, the minimum is: (i) at the lowest deflection (17a) for positive slope; (ii) at the largest deflection (17b) for negative slope:

$$\delta_{i_{\min}} = -\delta_{i_{\max}}: \qquad C_{\alpha i_{\min}} = \begin{cases} C_{\alpha i}\left(\delta_{i_{\min}}\right) & \text{if} \quad a_{\alpha i} > 0, \\ C_{\alpha i}\left(\delta_{i_{\max}}\right) & \text{if} \quad a_{\alpha i} < 0; \end{cases} \tag{17a,b}$$

these conclusions follow from (15a-c) mutatis mutandis.

5. Effect of mutual interaction between controls

In the preceding analysis it was assumed that the deflection of one control surface does not create forces or moments upon the others; the assumption of independent or decoupled controls implies that the global extremum (maximum or minimum) is the sum of the extrema (maximum or minimum) for each control surface:

$$C_\alpha = \sum_{i=1}^{N} C_{\alpha i} = C_{\alpha 0} + \sum_{i=1}^{N}\left[a_{\alpha i}\bar{\delta}_i + b_{\alpha i}\left(\bar{\delta}_i\right)^2 \right], \tag{18}$$

where the optimal deflections for each control surface are determined separately for each of them. If all deflections were maxima (19a) within the range of deflection (19b) the maximum force/moment would be (19c):

$$b_{\alpha i} < 0, \quad \delta_{i_{\min}} \le \bar{\delta}_i = a_{\alpha i}/2b_{\alpha i} \le \delta_{i_{\max}} : \qquad C_{\alpha \max} = C_{\alpha 0} - \sum_{i=1}^{N}\left(a_{\alpha i}\right)^2 / 2b_{\alpha i}\ . \tag{19a-c}$$

If some $\bar{\delta}_i$ are not maxima or lie outside the range of possible deflections, then the corresponding terms in (19c) must be substituted as in (13a,b). Likewise for a minimum.

In the case of interdependent or coupled controls, the force and moment on one surface are affected at most by the deflections of all others:

$$C_{\alpha i} = C_{\alpha i 0} + \sum_{i=1}^{N} a_{\alpha ij}\delta_j + \sum_{j,k=1}^{N} b_{\alpha ijk}\delta_j\delta_k\ , \tag{20}$$

assuming still a polynomial of second degree: (i) the static values are unchanged (4a); (ii) the slopes are specified by a matrix (21a) indicating the effect of the deflection of control surface δ_j on the forces and moments on control surface $C_{\alpha i}$:

$$a_{\alpha ij} = \frac{\partial C_{\alpha i}}{\partial \delta_j}; \qquad 2b_{\alpha ijk} = \frac{\partial^2 C_{\alpha i}}{\partial \delta_j \partial \delta_k}; \tag{21a,b}$$

(iii) the curvatures (21b) involve a multiplicity with three indices.

The extremum cannot be found separately for each surface, but only for all together:

$$0 = dC_\alpha = \sum_{i=1}^{N} dC_{\alpha i} = \sum_{i,j=1}^{N} d\delta_j \left(a_{\alpha ij} + 2\sum_{k=1}^{N} b_{\alpha ijk}\,\bar{\delta}_k \right), \tag{22}$$

where the multiplicity of slopes is symmetric in the last two indices (23a)

$$b_{\alpha ijk} = b_{\alpha ikj} : \qquad \frac{\partial^2 C_{\alpha i}}{\partial \delta_j \partial \delta_k} = \frac{\partial^2 C_{\alpha i}}{\partial \delta_k \partial \delta_j}\ , \tag{23a,b}$$

on account of (21b). The deflections for the extrema are given by (23)

$$\bar{\delta}_{i,\alpha} = -\frac{1}{2}\sum_{j,k=1}^{N} a_{\alpha ij}\left(b_{\alpha ijk}\right)^{-1}\ , \tag{23c}$$

where $\left(b_{\alpha ijk}\right)^{-1}$ is the inverse matrix of $b_{\alpha ijk}$ with regard to jk. The extremum will be a maximum/minimum if the two-dimensional differential form is positive/negative:

$$d^2 C_\alpha = \sum_{i=1}^{N} d^2 C_{\alpha i} = \sum_{i,j,k=1}^{N}\left(\frac{\partial^2 C_{\alpha i}}{\partial \delta_i \partial \delta_j}\right) d\delta_i d\delta_j = 2\sum_{i,j,k=1}^{N} b_{\alpha ijk}\, d\delta_j d\delta_k \begin{cases} > 0 & \text{minimum,} \\ < 0 & \text{maximum;} \end{cases} \tag{24a,b}$$

this is equivalent to the matrix $b_{\alpha ijk}$ being positive/negative definite in (j,k) for every i, i.e., having all eigenvalues real and positive (negative):

$$b_{\alpha ijk} = \begin{cases} \text{is positive-definite implies } \bar{\delta}_i \text{ is local minimum,} \\ \text{is negative-definite implies } \bar{\delta}_i \text{ is local maximum.} \end{cases} \tag{25a,b}$$

If these conditions [(24a) or (24b)] are not all met for some (i,α) or the values (23) lie outside the range of possible deflections, the maximum/minimum occur at the extreme deflections. The case of coupled controls (20) reduces to independent controls (3) when for each α the matrix $a_{\alpha ij}$ and multiplicity $b_{\alpha ijk}$ are replaced by two vectors $\left(a_{\alpha i}, b_{\alpha i}\right)$.

6. Controls specified by polynomials of high degree

Next the assumption of decoupled controls is retained, but the control forces and moments are not restricted to polynomials of degree two in (3); polynomials of any degree *M* are considered:

$$C_{\alpha i}=\sum_{m=1}^{M}A_{\alpha i}^{(m)}\left(\delta_i\right)^m , \tag{26}$$

where: (i) the static values (27a), slopes (27b) and curvatures (27c) are retained:

$$A_{\alpha i}^{(0)}\equiv C_{\alpha i}\left(0\right)\equiv C_{\alpha 0},\quad A_{\alpha i}^{(1)}=\frac{dC_{\alpha i}}{d\delta_i}\equiv a_{\alpha i},\quad 2\,A_{\alpha i}^{(2)}=\frac{d^2C_{\alpha i}}{d\delta_i^{\,2}}\equiv 2b_{\alpha i},\quad m!A_{\alpha i}^{(m)}\equiv\frac{d^mC_{\alpha i}}{d\delta_i^{\,m}}, \tag{27a-d}$$

and derivatives of order *m* up to *M* are added. The extremum can be obtained independently for each decoupled surface. The total extremum is the sum of partial extrema. Thus, only partial extrema need to be considered. The extremum corresponds to the vanishing of the derivative of a polynomial:

$$0=\frac{dC_{\alpha i}}{d\delta_i}=\sum_{m=1}^{M}A_{\alpha i}^{(m)}m\left(\bar{\delta}_i\right)^{m-1}. \tag{28}$$

In the case of second-degree (8a) there was only one root (8b), whereas in the case of degree *M* there are *M-1* roots. Of these roots, those with positive/negative second-order derivative can be local minima/maxima:

$$\frac{d^2C_{\alpha i}}{d\bar{\delta}_i^{\,2}}=\sum_{m=2}^{M}A_{\alpha i}^{(m)}m\left(m-1\right)\left(\bar{\delta}_i\right)^{m-2}\begin{cases}>0 & \text{implies } \bar{\delta}_i \text{ is local minimum,}\\ <0 & \text{implies } \bar{\delta}_i \text{ is local maximum;}\end{cases} \tag{29a,b}$$

if there are several local minima/maxima then the lowest/highest is chosen as the overall minimum/maximum.

It may happen that the second order derivative vanishes, and some higher-order derivatives vanish also. Not all derivatives up to order *M* can vanish otherwise the polynomial would reduce to a constant. Suppose that the first derivative which does not vanish at $\bar{\delta}_i$ is of order *s*; if s is odd the $\bar{\delta}_i$ is an inflexion point, not a maximum or a minimum (30a):

$$C_{\alpha i}'\left(\bar{\delta}_i\right)=\ldots=C_{\alpha i}^{(s-1)}\left(\bar{\delta}_i\right)=0\neq C_{\alpha i}^{(s)}\left(\bar{\delta}_i\right)\begin{cases}s\text{ is odd} & \text{implies } \bar{\delta}_i \text{ is an inflexion-point,}\\ s\text{ is even} & \text{implies } \bar{\delta}_i \text{ is an extremum;}\end{cases} \tag{30a-b}$$

if *s* is even then $\bar{\delta}_i$ is an extremum and the sign decides if it is a maximum or minimum.

$$C_{\alpha i}'\left(\bar{\delta}_i\right)=\ldots=C_{\alpha i}^{(2s-1)}\left(\bar{\delta}_i\right)=0\neq C_{\alpha i}^{(2s)}\left(\bar{\delta}_i\right)\begin{cases}>0 & \text{implies } \bar{\delta}_i \text{ is local minimum,}\\ <0 & \text{implies } \bar{\delta}_i \text{ is local maximum.}\end{cases} \tag{31a-b}$$

All local maxima/minima, arising at second-order derivative (29a,b) or at any derivative s of higher even order (31a,b) are considered for the overall maximum and minimum:

$$C_{\alpha i\max}=\sup\left\{C_{\alpha i}\left(\bar{\delta}_i\right)\right\}\geq\inf\left\{C_{\alpha i}\left(\bar{\delta}_i\right)\right\}=C_{\alpha i\min}. \tag{32}$$

If the deflection $\bar{\delta}_i$ lies outside the range of possible deflections, then the corresponding extremum is at one of the deflection limits. The extrema for each decoupled control specify by addition the overall extrema.

7. High order polynomials together with coupled controls

The optimization method applies most simply (§2-4) to decoupled controls specified by second-order polynomials. These two restrictions were relaxed to allow either coupled controls (§5) or high-order polynomials (§6). Next both constraints are removed, so that the controls can be coupled and specified by polynomials of any degree. If the highest degree is M, all polynomials can be written, possibly with some zero coefficients, in the form:

$$C_{\alpha i}=A_{\alpha i}+\sum_{j=1}^{N}A_{\alpha i}\delta_i+\sum_{j,k=1}^{N}A_{\alpha ik}\delta_i\delta_k+\ldots=\sum_{m=0}^{M}\sum_{j_1,\ldots,j_m=1}^{N}A_{\alpha ij_1\cdots j_m}\delta_{j_1}..\delta_{j_m}. \tag{33}$$

where: (i) the static values (4a) are unchanged; (ii) the slopes (34a) and curvatures (34b) are as before in the coupled case (21a,b).

$$a_{\alpha ij}=\frac{\partial C_{\alpha i}}{\partial\delta_j}=A_{\alpha ij},\qquad 2b_{\alpha ijk}=\frac{\partial^2C_{\alpha i}}{\partial\delta_j\partial\delta_k}=2A_{\alpha ijk},\qquad \frac{\partial^mC_{\alpha i}}{\partial\delta_{j_1}..\partial\delta_{j_m}}=m!A_{\alpha ij_1\cdots j_m}; \tag{34a-c}$$

(iii) the higher-order derivatives (34c) may all be designated by the same letter A and distinguished by the number of indices $\left(j_1,\ldots,j_m\right)$ for $0\leq m\leq M$.

Due to the coupling the extremum must be sought jointly for all controls together:

$$0 = dC_\alpha = \sum_{i=1}^{N} dC_{\alpha i} = \sum_{m=1}^{M} m \sum_{i=1}^{N} \sum_{j_1,\dots,j_m=1}^{N} A_{\alpha i j_1 \dots j_m} \bar{\delta}_{j_1} \dots \bar{\delta}_{j_m} d\delta_{j_1} \, ; \tag{35}$$

the roots of (35) specify the extrema. If the second variation is positive/negative the extremum is a minimum/maximum

$$d^2 C_\alpha = \sum_{i=1}^{N} d^2 C_{\alpha i} = \sum_{m=1}^{M} m(m-1) \sum_{i=1}^{N} \sum_{j_1,\dots,j_m=1}^{N} A_{\alpha i j_1 \dots j_m} \bar{\delta}_{j_1} \dots \bar{\delta}_{j_m} d\delta_{j_1} d\delta_{j_2}$$

$$\begin{cases} >0 & \text{implies } \bar{\delta}_i \text{ is local minimum,} \\ <0 & \text{implies } \bar{\delta}_i \text{ is local maximum;} \end{cases} \tag{36a-c}$$

this is equivalent to stating that the matrix (37a) is positive/negative definite, i.e., as all eigenvalues real and positive (negative)

$$X_{j_1 j_2} = \sum_{m=2}^{M} m(m-1) \sum_{i=1}^{N} \sum_{j_1,\dots,j_m=1}^{N} A_{\alpha i j_1 \dots j_m} \bar{\delta}_{j_3} \dots \bar{\delta}_{j_m} \begin{cases} \text{positive-definite implies local minimum,} \\ \text{negative-definite implies local maximum.} \end{cases} \tag{37a-c}$$

The matrix (37a) may be not definite, e.g. it may be indefinite, positive or negative semi-definite, i.e. have eigenvalues with opposite signs, or some zero eigenvalues, in this case $\bar{\delta}_i$ is not an extremum. If all eigenvalues of (37a) are zero, then it is necessary to proceed to higher order to know if $\bar{\delta}_i$ is an extremum.

If the differential form (38a) of lowest order which does not vanish is of odd order (38b) then $\bar{\delta}_i$ is an inflexion, not an extremum:

$$dC_{\alpha i}\left(\bar{\delta}_i\right) = \dots = d^{(s-1)} C_{\alpha i}\left(\bar{\delta}_i\right) = 0 \neq d^{(s)} C_{\alpha i}\left(\bar{\delta}_i\right) = X \begin{cases} s \text{ is odd} & \text{implies } \bar{\delta}_i \text{ is inflexion point,} \\ s \text{ is even} & \begin{cases} >0 & \text{implies } \bar{\delta}_i \text{ is local minimum,} \\ <0 & \text{implies } \bar{\delta}_i \text{ is local maximum;} \end{cases} \end{cases}$$

(38a-d)

if the first non-vanishing differential is of even order then $\bar{\delta}_i$ is an extremum (38c,d) viz. a local minimum/maximum if it is always positive/negative. If *X* does not have a fixed sign, then $\bar{\delta}_i$ is not an extremum. If the value $\bar{\delta}_i$ lies outside the range of possible deflections, then the extremum is at one of the extreme deflections. The set of values $\bar{\delta}_i$ is then used to calculate the overall maximum and minimum in (33).

8. Data points for coupled/uncoupled

The number of data points needed is lowest (6) for the three forces and moments (total 6), for *N* decoupled control surfaces ($6 \times N$), for second-degree polynomials with 3 coefficients each:

$$Q_0 = 6 \times N \times 3 = 18N \, . \tag{39}$$

If the controls remain decoupled but the polynomials are (26) of degree *M*, the 3 coefficients are replaced by *M*+1, and the number of data points need is:

$$Q_1 = 6 \times N \times (M+1) = 6N(M+1) \, ; \tag{40}$$

this reduces to (39) for M=2. In the case (20) of a polynomial of degree *M*=2 and *N* coupled controls, there are 6*N* static values, $6N^2$ first-order derivatives and $6N^3$ second-order derivatives; the latter are symmetric (23a), reducing their number to $6N^2(N+1)/2$ in:

$$M = 2: \qquad Q_3 = 6\left[N + N^2 + N^2(N+1)/2\right] = 3\left(2N + 3N^2 + N^3\right). \tag{41}$$

In the general case (33) of polynomials of degree *M* and *N* coupled controls for each force and moment there is one static value, N^2 slopes, $N^2(N+1)/2$ curvatures and $m = 1, 2, \dots$:

$$N \binom{N+m-1}{m} = (N+m-1)!\,N / \left[m!(N-1)!\right] = N^2(N+1)(N+2)\dots(N+m-1)/m! \, , \tag{42a}$$

coefficients of order *m* that are symmetric in all indices; the total number of data points is:

$$\mathrm{Q}_3 = 6N\left\{1+N+N\left(N+1\right)/2+...+N...\left(N+M-1\right)/M!\right\} = 6N\left\{1+\sum_{m=1}^{M}\binom{N+m-1}{m}\right\}. \qquad (42b)$$

For polynomials of second degree the number of data points (42b) agrees with (41), and it increases for polynomials of third degree to:

$$\text{M=3:} \qquad \mathrm{Q}_4 = 6N\left[1+N+N\left(N+1\right)/2+N\left(N+1\right)\left(N+2\right)/6\right] = 6N+11N^2+6N^3+N^4. \qquad (43)$$

As can be seen in the TABLE I the number of data points needed increases rapidly with the degree of the polynomial, more so in the coupled case.

The interaction between control surfaces farther apart would be weaker and could be neglected to reduce the number of data points needed. Suppose that there are *N* control surfaces each interacting only with its neighbors: (i) the first and last surfaces have each one neighbor, and thus one coefficient of degree zero, two of degree one $\left(\delta_1,\delta_2\right)$, three of degree two $\left[\delta_1^2,\delta_2^2,\delta_1\delta_2\right]$, four of degree three $\left[\left(\delta_1\right)^3,\left(\delta_1\right)^3\delta_2,\delta_1\delta_2^2,\left(\delta_2\right)^3\right]$, and $\left(m+1\right)$ of degree m; (ii) the $\left(N-2\right)$ control surfaces not at either end have two neighbors, and hence one coefficient of zero degree, three of first degree $\left(\delta_1,\delta_2,\delta_3\right)$, six of second degree $\left[\delta_1^2,\delta_2^2,\delta_3^2,\delta_1\delta_2,\delta_1\delta_3,\delta_2\delta_3\right]$ and $\left(m+1\right)\left(m+2\right)/2$ coefficients of degree m; (iii) these coefficients add for all degrees from 0 to M, for each of three components of forces and moments:

$$N \geq 2: \qquad \mathrm{Q}_2 = 6\left\{2\sum_{m=0}^{M}\left(m+1\right)+\left(N-2\right)\sum_{m=0}^{M}\left(m+1\right)\left(m+2\right)/2\right\}$$

$$= 6\left\{\left(M+1\right)\left(M+2\right)+3\left(N-2\right)\sum_{m=0}^{M}\left(m+1\right)\left(m+2\right)/2\right\}. \qquad (44)$$

This is the number of data points in the case of *N* control surfaces coupled only to the nearest neighbor, and polynomial of degree *M*. For two control surfaces the number of parameters is: (i) the same for neighboring interactions (44) or all interactions (42b); (ii) larger than for decoupled controls (40):

$$N=2: \ \mathrm{Q}_2 = 6\left(M+1\right)\left(M+2\right) = 12\sum_{m=0}^{M}\left(m+1\right) = 12\sum_{m=0}^{M}\binom{m+1}{m} = \mathrm{Q}_3 > 12\left(M+1\right) = \mathrm{Q}_1. \qquad (45)$$

For more than two control surfaces, the number of parameters for neighboring interaction (44) is: (i) larger than (40) for decoupled controls; (ii) smaller than (42b) for all controls coupled:

$$\mathrm{Q}_2 > \left(M+1\right)\left[12+6\left(N-2\right)\right] = 6N\left(M+1\right) > \mathrm{Q}_1, \qquad (46a)$$

$$N \geq 3: \qquad \mathrm{Q}_2 < 6\left(N-2\right)\sum_{m=0}^{M}\left(m+1\right)\left(m+2\right) = 6\left(N-2\right)\sum_{m=0}^{M}\binom{m+2}{2}$$

$$< 6N\sum_{m=0}^{M}\binom{m+2}{m} \leq 6N\sum_{m=0}^{M}\binom{N+m-1}{m}; \qquad (46b)$$

in (46b) it was considered that the first term on the l.h.s. of (44) is smaller than the second, because the control surfaces at one end have less parameters than the others. In the case of polynomials of degree two (47a) and (47b) the number of parameters for neighboring interaction:

$$M=2: \qquad \mathrm{Q}_2 = 72+60\left(N-2\right) = 60N-48, \qquad (47a)$$

$$M=3: \qquad \mathrm{Q}_2 = 120+120\left(N-2\right) = 120\left(N-1\right), \qquad (47b)$$

is indicated in TABLE I in comparison with decoupled (40) and fully coupled (41,43) controls. The case of each control surface interacting only with its neighbor(s) significantly reduces the number of data points needed and should account for control coupling with good accuracy. Since the method applies both to coupled and uncoupled controls, and polynomials of any degree, it can be illustrated for the simplest case, for which data is more readily available.

PART II – APPLICATION TO MULTIPLE CONTROL SURFACES ON A FLYING WING.

The preceding method of optimization of control power is particularly relevant to low-speed flight for which low dynamic pressure requires larger control deflections. The method applies to decoupled and coupled controls, specified by polynomials of second or higher degree. The method is illustrated most clearly and simply for decoupled controls specified by polynomials of second-degree for a reference large passenger BWB configuration. This configuration is considered for straight and level steady flight at low speed as the baseline equilibrium condition (§9). Pitch trim is achieved choosing control surface deflections for minimum drag (§10) e.g., for the fastest climb; the control surface deflections are also obtained in the opposite case of maximum drag (§11) e.g., for the highest possible deceleration for the steepest descent on approach to land. The minimum and maximum pitching moment are considered to establish center-of-gravity limits (§12); for several aircraft configurations with engines above and below the wing (§13). The minimum and maximum yawing moment are considered (§14); the available yaw control power is compared with that needed to compensate the failure of the outboard engine in a two-, three- or four-engined aircraft (§15). The maximum and minimum rolling moment are determined (§16); these are compared with the rolling induced by a wake vortex encounter (§17), to specify the maximum vertical velocity component at the wing tip which can be compensated. The conclusion (§18) discusses the general and iterative optimization methods.

9. Reference low-speed flight condition

A straight and level steady flight at M = 0.2 is assumed, for a reference large passenger BWB whose aerodynamic and control data indicated in the Tables II to IV. The data in TABLE II leads to a lift coefficient that balances the weight:

$$C_{L0} = 2W/\rho S V^2 = W/qS = 0.14916\,. \tag{48a}$$

The static lift coefficient is given in TABLE III for zero sideslip:

$$\beta = 0: \qquad C_{L0} = -0.0117 + 3.3516\alpha - 0.16966\alpha^2\,. \tag{48b}$$

For the value (48a) of the lift coefficient (48b) the AoA is a root of (49a):

$$\alpha^2 - 19.7548\alpha + 0.94813 = 0; \qquad \alpha = 4.8112\times10^{-2}\,rad = 2.7566^{\circ}\,, \tag{49a,b}$$

the only root with a reasonable value is (49b). The value (49b) of the AoA leads to the drag (50a), side force (50b), pitching (50c), yaw (50d) and rolling (50e) moment coefficients:

$$C_{D0} = 7.1036\times10^{-3}\,, \qquad C_{Y0} = 0.0\,, \qquad C_{m0} = 2.5121\times10^{-2}\,,$$
$$C_{n0} = -3.8997\times10^{-6}\,, \qquad C_{l0} = 3.9361\times10^{-5}\,, \tag{50a-e}$$

that are calculated from the formula:

$$C_{\alpha 0} \equiv C_{\alpha}\left(\delta_i = 0\right) = a_{0\alpha} + a_{1\alpha}\,\alpha + a_{2\alpha}\,\alpha^2\,, \tag{51}$$

using the static coefficients in TABLE III.

The horizontal and vertical force coefficients in body axis are specified by the lift and drag coefficients by:

$$\begin{bmatrix} C_{Z0} \\ C_{X0} \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} C_{L0} \\ C_{D0} \end{bmatrix} = \begin{bmatrix} 1.4933\times10^{-1} \\ -7.822\times10^{-5} \end{bmatrix}. \tag{52a,b}$$

The lift coefficient (48a) and the vertical force coefficient (52a) are close because the AoA is small; the horizontal force coefficient (52b) is much smaller than the drag coefficient (50c) and with opposite sign. The pitch trim is considered FIGURE 1 with: (i) the weight applied at the center of gravity at quarter chord (53a); (ii) the pitching moment, as all other aerodynamic forces at the reference point (53b); (iii) relative distance is (53c):

$$x_{cg} = 0.25c = 14.74m, \qquad x_{ref} = 0.565c = 33.31m,$$
$$\bar{x} = x_{ref} - x_{cg} = 0.315c = 18.57\,m. \tag{53a-c}$$

The balance of pitching moment for lift equal to weight (54a) is (54b):

$$L = W: \qquad M = M_0 - W\,\bar{x}\,, \qquad C_m = C_{m0} - 2W\bar{x}/\rho S U^2 \ell \equiv C_{m0} + \Delta C_m\,, \tag{54a-c}$$

corresponding to (54c) the reference length is (55a). It leads to the pitching moment coefficient (55b):

$$\ell = 36.416m\,; \qquad \Delta C_m = -2W\,\bar{x}/\rho S U^2 \ell = -7.6064\times10^{-2}\,, \tag{55a,b}$$

using also the values from TABLE II. From (50c) and (56b) it follows that the pitching moment coefficient to be trimmed is

$$C_m = C_{m0} + \Delta C_m = -5.0943\times10^{-2}\,, \tag{56}$$

that is a pitch down.

10. Pitch trim with minimum drag

The pitching moment coefficient (55b) must be balanced by deflection of the control surfaces:

$$-7.6064\times10^{-2}=\sum_{i=1}^{5}a_{Mi}\,\delta_i+b_{Mi}\,\delta_i^2\,, \tag{57}$$

where δ_i is the deflection of each flap, and all flap coefficients $a_{\alpha i}$ and $b_{\alpha i}$ are listed in TABLE IV; they were calculated from tables for zero angle-of-sideslip $\beta=0$ and angles-of-attack $\alpha=4^{\circ}$ and $\alpha=6^{\circ}$, interpolating linearly for the AoA (49b). These interpolated polynomial coefficients are then used to calculate forces and moments coefficients, e.g., the pitching moment (57) for the exact AoA (49b). The (i) linear interpolation of force and moment coefficients would have given an error $\mathrm{O}\left(\alpha^2\right)$ equivalent to neglecting the second-order term; the (ii) interpolation of the polynomial coefficients should give in the force and moment with an error $\mathrm{O}\left(\alpha^3\right)$, consistent with retaining the second-order term and omitting the third-order term. The former (i) would represent a degradation in accuracy whereas the latter (ii) does not, and hence is adopted in the present calculation. The trimming should not change the lift (58a)

$$0=\sum_{i=1}^{5}\left(a_{Li}\,\delta_i+b_{Li}\,\delta_i^2\right),\qquad \left(C_D\right)_{\min}=\sum_{i=1}^{5}\left(a_{Di}\,\delta_i+b_{Di}\,\delta_i^2\right), \tag{58a,b}$$

and should minimize the drag (58b). Using the values in TABLE IV, the optimization problem is: (i) to keep constant the lift (59) and pitching moment (60):

$$\begin{aligned}0=\Delta C_L&=0.23344\,\delta_1-0.00455\left(\delta_1\right)^2+0.19079\,\delta_2-0.02808\left(\delta_2\right)^2+0.12024\,\delta_3\\&-0.00674\left(\delta_3\right)^2+0.05995\,\delta_4-0.00478\left(\delta_4\right)^2-0.05101\,\delta_5-0.00246\left(\delta_5\right)^2;\end{aligned} \tag{59}$$

$$\begin{aligned}C_m&=2.5121\times10^{-2}-0.07267\,\delta_1+0.00037\left(\delta_1\right)^2-0.00340\,\delta_2+0.00590\left(\delta_2\right)^2-0.01656\,\delta_3\\&+0.00234\left(\delta_3\right)^2-0.01563\,\delta_4+0.00158\left(\delta_4\right)^2+0.01885\,\delta_5+0.00206\left(\delta_5\right)^2;\end{aligned} \tag{60}$$

$$\begin{aligned}\left(\Delta C_D\right)_{\min}&=0.00537\,\delta_1+0.01867\left(\delta_1\right)^2+0.003745\,\delta_2+0.01283\left(\delta_2\right)^2+0.00067\,\delta_3\\&+0.00930\left(\delta_3\right)^2-0.00005\,\delta_4+0.00596\left(\delta_4\right)^2-0.00093\,\delta_5+0.00824\left(\delta_5\right)^2.\end{aligned} \tag{61}$$

(ii) to minimize the drag (61). Eight optimization methods with constraints were presented in an earlier paper [6]; of this method VIII offered the best combination of accuracy, simplicity and clarity of interpretation. It is based on selecting the most effective control surfaces to generate each component of the aerodynamic force or moment of interest. This method is used in the sequel with suitable adaptations.

For a first iteration only terms linear in the deflections are considered. Concerning the drag it is reduced (Table IV) by: (i) negative deflection of the first three flaps, with the body flap being most effective; (ii) positive deflection or the outer flap and rudder, with the rudder being most effective. As concerns drag control the least effective surfaces are the middle and outer flaps; thus the optimization is performed most accurately if the deflections δ_4 and δ_3 are eliminated from the system of equations (59-61), as follows: (i) the linear form of (59) is used to express approximately the deflection of the outer wing flap in terms of the others:

$$\delta_4=-3.8939\,\delta_1-3.1825\;\delta_2-2.0057\,\delta_3+0.8509\,\delta_5; \tag{62}$$

(ii) this is substituted in the remaining constraint, i.e., constant pitching moment (60; 56), and the objective, i.e., minimum drag (61), which became independent of δ_4:

$$-7.6064\times10^{-2}=-0.01181\,\delta_1+0.04634\,\delta_2+0.01479\,\delta_3+0.005550\,\delta_5\,, \tag{63a}$$

$$\Delta C_D=0.005565\,\delta_1+0.003901\,\delta_2+0.0007703\,\delta_3-0.0009725\,\delta_5; \tag{63b}$$

(iii) the deflection δ_3 of the next least effective control surface is expressed in terms of the remaining more effective using the second constraint (63a) of constant pitching moment:

$$\delta_3=-5.1429+0.7985\,\delta_1-3.1332\,\delta_2-0.3753\,\delta_5; \tag{64}$$

(iv) substituting (64) into (63b) specifies the drag, satisfying both constraints on lift and pitching moment, in terms of the deflections of the three most effective control surfaces:

$$\Delta C_D=-0.003969+0.006173\,\delta_1+0.001488\,\delta_2-0.001262\,\delta_5; \tag{65}$$

(v) the quadratic terms are re-introduced in the drag in the same proportion to the linear terms as before in (61):

$$\begin{aligned}\Delta C_D = &-0.0003969 + 0.006173\,\delta_1\left(1+3.4767\,\delta_1\right)\\ &+0.001488\,\delta_2\left(1+3.4259\,\delta_2\right) - 0.001262\,\delta_5\left(1-8.8602\,\delta_5\right),\end{aligned} \tag{66}$$

leaving a formula for minimization without constraints, with three independent variables.

The next steps are: (vi) the deflections of the body δ_1 and inner δ_2 flaps must be negative to minimize the drag, whereas the deflection of the rudder must be positive to further reduce drag, as follows from (8b):

$$\bar{\delta}_1 = -1/\left(2\times 3.4767\right) = -0.14381\,rad = -8.2399^{\rm o}\,, \tag{67a}$$

$$\bar{\delta}_2 = -1/\left(2\times 3.4259\right) = -0.14595\,rad = -8.3621^{\rm o}\,, \tag{67b}$$

$$\bar{\delta}_5 = 1/\left(2\times 8.8602\right) = 0.056432\,rad = 3.2333^{\rm o}\,; \tag{67c}$$

(vii) the substitution of (67a-c) in (64) and (62) would lead to the deflection of the two least effective surfaces,

$$\bar{\delta}_3 = -4.8218\,rad = -276.27^{\rm o}\,, \quad \bar{\delta}_4 = 9.532\,rad = 546.16^{\rm o}\,; \tag{67d,e}$$

(viii) the values (67d,e) exceeds the limit $\delta_0 = 25^{\rm o}$; (ix) since these are the least effective control surfaces, the values (67d,e) are discarded, viz. these surface are not used $\delta_3 = 0 = \delta_4$, so the deflections are:

$$\bar{\delta}_i = \left(-0.1438, -0.1459, 0, 0, +0.05643\right) rad = \left(-8.239, -8.362, 0, 0, +3.233\right)^{\rm o}\,; \tag{68}$$

(viii) the corresponding drag (61) is (69a):

$$\left(\Delta C_D\right)_{\min} \equiv \Delta\bar{C}_D = \Delta C_D\left(\bar{\delta}_i\right) = -6.8567\times 10^{-4}, \qquad \left(\Delta C_D\right)_{\min} / C_{D0} = -0.096524\,, \tag{69a,b}$$

and thus the pitch trim decreases drag (50a) by 9.65% in (69b) to the value:

$$C_{D\min} = C_{D0} + \left(\Delta C_D\right)_{\min} = 7.1036\times 10^{-3} - 6.8567\times 10^{-4} = 7.035\times 10^{-3}\,, \tag{69c}$$

(x) the comparison with pitch trim by the body flap alone (70a) i.e., the first term of (60) leads (70b) to a large deflection (70c), close to the limit $\pm 25^{\rm o}$:

$$\bar{\bar{\delta}}_2 = \bar{\bar{\delta}}_2 = \bar{\bar{\delta}}_2 = \bar{\bar{\delta}}_2 = 0: \qquad \left(\bar{\bar{\delta}}_1\right)^2 - 196.41\bar{\bar{\delta}}_1 + 67.895 = 0, \qquad \bar{\bar{\delta}}_1 = -0.34633\,rad = -19.84^{\rm o}; \tag{70a-d}$$

(xi) the associated drag would be a penalty (70e) instead of a reduction for the optimal solution (69b), that is as increase (70f) of 57.69% instead of a reduction (69b) of 9.65%:

$$\Delta\bar{\bar{C}}_D = \Delta C_D\left(\bar{\bar{\delta}}_i\right) = 0.00537\,\bar{\bar{\delta}}_1 + 0.01867\left(\bar{\bar{\delta}}_1\right)^2 = 4.098\times 10^{-3}, \qquad \Delta\bar{\bar{C}}_D / C_{D0} = 0.57689. \tag{70e,f}$$

In conclusion the optimal pitch trim (68) compared with deflection of body flap alone (70a,c) gives: (a) smaller deflections of control surfaces (68) vs. (70d); (b) lower trim drag (69a) vs. (70e) by a difference (71a) that corresponds (71b) to 67.34% of the drag;

$$\Delta\bar{C}_D - \Delta\bar{\bar{C}}_D = -4.7837\times 10^{-3}, \qquad \left(\Delta\bar{C}_D - \Delta\bar{\bar{C}}_D\right) / \Delta C_{D0} = 0.67342\,; \tag{71a,b}$$

(c) a trim drag reduction of 9.65% in (69b) the optimal case vs. a drag penalty of 57.69% in (70f) using the body flap alone.

11. Maximum drag for greatest retardation

The minimum drag for pitch trim is desirable for the fastest climb after take-off; the reverse, the maximum drag with pitch trim, may be desirable for the steepest descent to land. The preceding analysis applies because in both cases lift balance and pitch trim must be maintained, and the most effective control surfaces should be used, leading to the total drag (66) without constraint. The only difference is that (66) should be maximized rather than minimized: (i) since $\left(\delta_1, \delta_2\right)$ lead to a minimum (67a,b), the maximum is at maximum positive deflection, taken to be 25º in (72a,b); (ii) conversely the maximum drag for δ_5 is at the smallest deflection in (72c):

$$\left\{\tilde{\delta}_1, \tilde{\delta}_2, \tilde{\delta}_3, \tilde{\delta}_4, \tilde{\delta}_5\right\} = \left\{25^{\rm o}, 25^{\rm o}, 0, 0, -25^{\rm o}\right\} = 0.43633\,rad\,\{1, 1, 0, 0, -1\} \equiv \left\{\delta_0, \delta_0, 0, 0, -\delta_0\right\}; \tag{72a-c}$$

(iii) the two least effective control surfaces are not used in the drag difference (66) leading to (73):

$$\left(\Delta C_D\right)_{\max} = -0.0003969 + 0.006399\,\delta_0 + 0.03774\,\delta_0^{\,2} = 9.5804\times 10^{-3}; \tag{73}$$

(iv) the maximum extra drag (73) with a limit of 25º on deflection is (74a) more than the baseline drag

(50a):

$$\left(\Delta C_D\right)_{\max} / C_{D0} = 1.3487 , \qquad \left(\Delta C_D\right)_{\max} / \left(\Delta C_D\right)_{\min} = 13.972 , \tag{74a,b}$$

and is over 13 times (74b) the minimum drag (69a) for pitch trim; (v) thus the maximum trimmed drag is more than twice the untrimmed drag (75):

$$0.9035 \le C_{D\min} / C_{D0} \le C_D / C_{D0} \le C_{D\max} / C_{D0} \le 2.3487 \equiv k ; \tag{75}$$

(vi) this shows the range of possible trimmed drag coefficients:

$$7.035 \times 10^{-3} \le C_{D\min} \le C_D \le C_{D\max} \le 1.6684 \times 10^{-2} . \tag{76}$$

In conclusion, there is a wide range of trimmed drag variations obtained by optimizing control surface deflections, from a reduction of 9.65% in (69b) to an increase of 134.87% in (74a). If the body flap alone had been used at maximum deflection (77a) the drag increase (66) would be smaller (77b):

$$\tilde{\tilde{\delta}}_1 = 25^{\text{o}} ; \tilde{\tilde{\delta}}_5 = \tilde{\tilde{\delta}}_2 = 0 : \qquad \Delta\tilde{\tilde{C}}_D = \Delta C_D \left(\tilde{\tilde{\delta}}_i\right) = 6.3829 \times 10^{-3}, \qquad \Delta\tilde{\tilde{C}}_D / C_{D0} = 0.89854, \tag{77a-c}$$

viz. 89.85% in (77b) instead of 134.87% in (74a). Thus, maximum deflection of the body flap alone would have less than doubled the trim drag (77a), whereas optimal deflections more than doubles it (74a). The range of drag modulation using optimal controls is much larger than using body flap alone because: (i) the minimum trimmed drag is smaller (78a) and the maximum trimmed drag is larger (78b):

$$k_1 \equiv \Delta\bar{\bar{C}}_D / \left(\Delta C_D\right)_{\min} = 9.309 ; \qquad k_2 \equiv \left(\Delta C_D\right)_{\max} / \Delta\bar{\bar{C}}_D = 1.501 ; \qquad k_1 k_2 = 13.972 ; \tag{78a-c}$$

(ii) the product (78c) $\equiv$(74b) shows that the range of drag modulation with optimal controls is more than 13 times larger than with body flap alone. This demonstrates the contrast between: (a) choosing “à priori” single control surface, even the most effective (body flap), which requires a large deflection (70d) for pitch trim with minimum drag, and would became saturated for not much larger drag, giving a small range of drag modulation; (b) using optimal deflections of all control surfaces, which allows pitch trim with a drag reduction (69b), and can also lead to a large drag increase (74a), providing a wide range of drag modulation (76). A wide range of drag is useful for steep descent allowing a continuous adaptation of the trajectory. The results on the range of possible drag modulation with pitch trim are compared in the TABLE V using total drag or thrust instead of drag coefficient by multiplying by:

$$D / C_D = \frac{1}{2} \rho S U^2 = 6.1343 \times 10^6 \, N . \tag{79}$$

It demonstrates the superiority of pitch control by optimal deflection of all surfaces versus body flap alone.

12. Maximum and minimum achievable pitching moment

The maximum and minimum pitching moment (§12) determines the extremes of the c.g. range which can be trimmed for several possible engine positions in the vertical plane relative to the aircraft datum (§13). The aim next is to find the maximum and minimum of the pitching moment (80a) keeping constant the lift (48a) $\equiv$(80b) and drag (50a) $\equiv$(80c) coefficients (80b,c):

$$C_{m\,\max,\min} : \qquad C_L = C_{L0} = 0.14916, \qquad C_D = C_{D0} = 7.1036 \times 10^{-3} . \tag{80a-c}$$

The TABLE IV shows that the least effective control surfaces for the pitching moment (60) are the inner δ_2 and outer δ_4 wing flaps; thus, these are eliminated using the constraints. The sequence of steps is as follows: (i) the deflection δ_2 of the least effective control surface is expressed in terms of all the other deflections from the condition (80b) of constant lift in the linearized form of (59):

$$\delta_2 = -\,1.22350\,\delta_1 - 0.63022\,\delta_3 - 0.31422\,\delta_4 + 0.26736\,\delta_5 ; \tag{81}$$

(ii) substitution of the condition (81) of constant lift in the drag (61; 69a) and in the pitching moment (60), eliminates the deflections δ_2, and leaves the remaining deflections:

$$-6.8567 \times 10^{-4} = 0.00078799\,\delta_1 - 0.0016902\,\delta_3 - 0.0012268\,\delta_4 + 0.000071263\,\delta_5 , \tag{82a}$$

$$C_m = 0.025121 - 0.068510\,\delta_1 - 0.014417\,\delta_3 - 0.014562\,\delta_4 + 0.017941\,\delta_5 ; \tag{82b}$$

(iii) the condition of constant drag (82a) is used to express the deflection δ_4 of the second least effective pitch control surface in terms of the remaining deflections.

$$\delta_4 = 0.55891 + 0.64224\,\delta_1 - 1.3777\,\delta_3 + 0.0058037\,\delta_5 ; \tag{83}$$

(iv) substitution of (83) in (82b) eliminates the deflection of the two least effective control surfaces:

$$C_m = 0.016982 - 0.077862\,\delta_1 + 0.0056451\,\delta_3 + 0.017681\,\delta_5\,, \tag{84}$$

from the pitching moment (84), which is thus unconstrained.

Next (v) the non-linear terms in the pitching moment are restored in the same proportion as in (60), leading to:

$$\begin{aligned} C_m &= 0.016962 - 0.077862\,\delta_1\left(1 - 0.0050915\,\delta_1\right) \\ &\quad - 0.0056451\,\delta_4\left(1 - 0.10109\,\delta_4\right) + 0.017681\,\delta_5\left(1 + 0.10928\,\delta_5\right); \end{aligned} \tag{85}$$

(vi) the extrema of the pitching moment relative to the three most effective pitch control surfaces (with the other two implicitly included through the constraints) are minima at the deflections (8b), leading to the values:

$$\left\{\bar{\delta}_1, \bar{\delta}_3, \bar{\delta}_5\right\} = \left\{98.202, 4.946, -4.575\right\}; \tag{86}$$

(vii) these values (86) are far outside the range of possible deflections (87a):

$$\left|\delta_i\right| \le 25^{\mathrm{o}} = 0.43633\,rad = \delta_0\,; \quad \delta_0 = -\hat{\delta}_1 = -\hat{\delta}_4 = \hat{\delta}_5\,; \quad \delta_0 = \breve{\delta}_1 = \breve{\delta}_4 = -\breve{\delta}_5\,, \tag{87a-c}$$

thus the maximum (minimum) pitching moment occur respectively for the deflections (87b) and (87c); (viii) these correspond to:

$$\left(C_m\right)_{\max} = C_m\left(\hat{\delta}_i\right) = 6.1686\times10^{-2}\,, \qquad \left(C_m\right)_{\min} = C_m\left(\breve{\delta}_i\right) = -2.8192\times10^{-2}\,, \tag{88a,b}$$

the maximum (88a) and minimum (88b) pitching moment.

13. Centre of gravity range as a function of the engine configuration

For a flying wing aircraft: (i) overwing engines provide noise shielding and cause a pitchdown; (ii) underwing engines have better inlet flow conditions and cause a pitch-up. Thus, the limits of trimmable engine configurations will be found both for two and four overwing (O2/O4) and underwing (U2/U4) engines and a mixed combination (M) of the over/underwing engines. The static pitch balance is made first ignoring the engines, which is equivalent to assuming that they lie at the aircraft horizontal datum. The balance of static pitching moment (FIGURE 1) leads (53c,54c) to:

$$C_m = C_{m0} - \left(2W/\rho S U^2 \ell\right)\left(x_{ref} - x_{cg}\right), \tag{89}$$

that can be solved for the c.g. position

$$x_{cg} = x_{ref} + \left(\rho S U^2 \ell / 2W\right)\left(C_m - C_{m0}\right). \tag{90}$$

Substituting the values from TABLE II and (55a) leads to:

$$x_{cg} = 33.31 + 2.4414\times10^2\left(C_m - C_{m0}\right). \tag{91}$$

The range of pitching moments (88a,b) together with (50c) leads to (92):

$$-5.3313\times10^{-2} \le \left(C_m\right)_{\min} - C_{m0} \le C_m - C_{m0} \le \left(C_m\right)_{\max} - C_{m0} \le +3.6565\times10^{-2}\,; \tag{92}$$

this leads by (90) to a trimmable c.g. range (93a):

$$20.294\,m \le x_{cg} \le 42.237\,m, \quad 0.3442 \le x_{cg}/c \le 0.7164, \tag{93a,b}$$

corresponding to (93b) in terms of the mean aerodynamic chord (53a).

The pitch trim is considered next (FIGURE 2) for a total engine thrust T at a distance *z* from the horizontal datum, so that (54b) is replaced by (94b):

$$T = D: \qquad M = M_0 - W\,\bar{x} - T\,h = M_0 - W\left(x_{ref} - x_{cg}\right) - D\,z\,, \tag{94a,b}$$

where lift equals weight (54a), and thrust equals drag (94a) . The relation (94b) can be put in a dimensionless form like (54c):

$$C_m = C_{m0} + \left(2W/\rho S U^2 \ell\right)\left(x_{cg} - x_{ref}\right) - C_{D0}\,z/\ell\,. \tag{95}$$

This can be solved again for the c.g. position:

$$x_{cg} = x_{ref} + \left(\rho S U^2 \ell / 2W\right)\left(C_m - C_{m0} + C_{D0}\,z/\ell\right). \tag{96}$$

Using the same values as before in TABLE II plus (50a) yields:

$$x_{cg} = 33.31 + 2.4414\times10^2\left(C_m - 2.5121\times10^{-2} + 1.9507\times10^{-4}\,z\right). \tag{97}$$

The maximum (88a) and minimum (88b) pitching moment coefficients are used (FIGURE 3) for the configurations: (i/ii) four (O4) or two (O2) engines over the center-body at a height (98a) above the horizontal datum lead to the c.g. range (98b,c):

$$z_+ = 10\,m: \qquad 20.770\,m \le x_{cg} \le 42.713\,m, \qquad 0.3523 \le x_{cg}/c \le 0.7244\,; \tag{98a-c}$$

(iii/iv) four (U4) or two (U2) underwing engines at a height (99a) below the horizontal datum, lead to the c.g. range (99b,c):

$$z_{-} = -10m: \qquad 19.818m \le x_{cg} \le 41.761m, \qquad 0.3361 \le x_{cg}/c \le 0.7083; \tag{99a-c}$$

(v) the mixed configuration (M) with one engine over the center-body and two underwing at the same distance (98a)$\equiv$(99a) from the horizontal datum, corresponds to one-third distance (100a) below the horizontal datum

$$z_0 = -3.33m: \qquad 20.135m \le x_{cg} \le 42.078m, \qquad 0.3415 \le x_{cg}/c \le 0.7137; \tag{100a-c}$$

and the c.g. range (100b,c). These results are summarized in TABLE VI.

14. Maximum and minimum achievable yawing moment

The engine position relative to the horizontal datum and the maximum and minimum pitching moment (§12) determine the trimmable c.g. range (§13). The compensation of an outboard engine failure (§15) depends on the maximum and minimum yawing moment (§14), which are calculated next. The yawing moment coefficient is given by the data in TABLE IV:

$$C_n = -0.01396\,\delta_1 - 0.00537(\delta_1)^2 + 0.00464\,\delta_2 - 0.00821(\delta_2)^2 + 0.00136\delta_3 - 0.00966(\delta_3)^2$$
$$+0.00066\,\delta_4 - 0.00814(\delta_4)^2 + 0.03454\,\delta_5 - 0.00209(\delta_5)^2. \tag{101}$$

The maximum and minimum are sought at constant lift (48a)$\equiv$(102a), drag (50a)$\equiv$(102b) and pitching moment (56)$\equiv$(102c).

$$C_{L0} = 0.14916, \qquad C_{D0} = 0.0071036, \qquad C_{m0} = -0.050943. \tag{102a-c}$$

The three least effective yaw control surfaces are the outer δ_4, middle δ_3 and inner δ_2 flaps; the optimal deflections of the two most effective control surfaces (δ_1, δ_5) follow from (8b), and are given by (110a,b). Since they exceed the limit deflection (87a), the latter is used to calculate the maximum and minimum pitching moment (111-112a,b). As a preliminary illustration their deflections will be eliminated from the yawing moment (101) using the constraints (102a-c) of constant lift, drag and pitching moment. The sequence of steps is as follows: (i) the constraint of constant lift (102a) in linearized form (59) is used to specify the deflection of the least effective yaw control surface (the outer flap) in terms of the others:

$$\delta_4 = -3.8939\,\delta_1 - 3.1825\,\delta_2 - 2.0057\,\delta_3 + 0.85087\,\delta_5; \tag{103}$$

(ii) the deflection of the least effective yaw control surface is eliminated from the drag (61,69a)$\equiv$(104a), pitching (60,55b)$\equiv$(104b) and yawing (101)$\equiv$(104c) moment:

$$-6.8567\times10^{-4} = 0.0055647\,\delta_1 + 0.0039041\delta_2 + 0.0007703\delta_3 - 0.0009725\delta_5, \tag{104a}$$

$$-0.076064 = -0.011808\,\delta_1 + 0.046334\delta_2 + 0.014789\,\delta_3 + 0.0055509\,\delta_5, \tag{104b}$$

$$C_n = -0.01653\,\delta_1 + 0{,}0025396\,\delta_2 + 0.000362\,\delta_3 + 0.035102\,\delta_5; \tag{104c}$$

using (103).

The next step (iii) is to express the deflection δ_3 of the second least effective yaw control surface (middle flap) in terms of the remaining deflections, e.g., using (104a):

$$\delta_3 = -0.89013 - 7.2241\delta_1 - 5.0683\delta_2 + 1.2625\delta_5; \tag{105}$$

(iv) substitution on (105) eliminates the deflections of the two least effective yaw control surfaces from the condition of constant pitching moment (104b) and from the yawing moment (104c):

$$-0.0629 = -0.11865\,\delta_1 - 0.028621\delta_2 + 0.024222\,\delta_5; \tag{106a}$$

$$C_n = -0.0003222 - 0.019145\,\delta_1 + 0.0007049\,\delta_2 + 0.035559\,\delta_5; \tag{106b}$$

(v) the deflection of the third least effective yaw control surface (inner flap) is expressed in terms of the remaining two using (106a).

$$\delta_2 = 2.19769 - 4.14556\,\delta_1 + 0.8463\,\delta_5; \tag{107}$$

(vi) substitution of (107) in (106b) specifies the yawing moment in terms of the deflections of the two most effective yaw control surfaces:

$$C_n = 0.001227 - 0.022067\,\delta_1 + 0.036156\,\delta_5; \tag{108}$$

(vii) the non-linear terms in the yawing moment (101) are restored in the same proportion to the linear terms:

$$C_n = 0.001227 - 0.02067\,\delta_1(1 + 0.38467\,\delta_1) + 0.036156\,\delta_5(1 - 0.06051\delta_5); \tag{109}$$

(viii) the extrema are (8b) a minimum yawing moment for the deflection of the body flap (110a) and a maximum for the deflection of the rudder (110b):

$$\bar{\delta}_1 = -1.29981, \qquad \bar{\delta}_2 = 8.26316; \tag{110a,b}$$

(ix) the values (110a,b) are far outside the range of possible deflections, so the maximum and minimum yawing moments are obtained:

$$\delta_1 = -\delta_5 = 25^o = 0.43633\,rad: \qquad (C_n)_{\min} = -0.02621, \tag{112a,b}$$

$$-\delta_1 = \delta_5 = 25^o = 0.43633\,rad: \qquad (C_n)_{\max} = +0.024599, \tag{112c,d}$$

at the extreme opposite deflections of rudder and body flap.

15. Yaw control margin with outboard engine failure

The worst-case scenario for yaw control is an outboard engine failure. For an aircraft with *n* identical engines with thrust *T*, and the outer engine at a distance y from the aircraft axis, the engine-out yawing moment is (113b):

$$D = nT: \qquad N = yT = y\,D/n, \tag{113a,b}$$

where the total thrust (113a) is assumed to equal the reference drag (94a); this corresponds to the yawing moment coefficient.

$$C_n = 2N/\left(\rho U^2 S\,\ell\right) = \left(y/\ell n\right)\left(2D/\rho U^2 S\right) = C_D\, y/\ell n; \tag{114a}$$

using the values (50a), (55a) and (94a) leads to:

$$C_n = 2N/\left(\rho U^2 S\,\ell\right) = \left(y/\ell n\right)\left(2D/\rho U^2 S\right) = \left(C_{D0}/\ell\right) y/n = 1.9507\times10^{-4}\, y/n, \tag{114b}$$

showing that the yawing moment: (i) increases with the distance of the outboard engine from the centerline; (ii) decreases with the number of engines, since for the same total drag, each engine has less thrust. The same 5 engine configurations (FIGURE 3) are considered: (i) four engines (115a) above the center-body (O4) with failure of outboard engine at distance (115b) from centerline leads to a yawing coefficient (115c):

$$n=4,\ y_- = 8\,m: \quad C_n = 3.9014\times10^{-4}, \quad C_n/(C_n)_{\max} = 0.01586, \quad C_n/(C_n)_{\min} = -0.014885, \tag{115a-e}$$

which is less than 1.6% of the maximum available yaw control power; (ii) for two engines (116a) at the same outboard position over the center-body

$$n=2,\ y_- = 8\,m: \quad C_n = 7.8027\times10^{-4}, \quad C_n/(C_n)_{\max} = 0.03172, \quad C_n/(C_n)_{\min} = -0.02977, \tag{116a-e}$$

the yawing moment is doubled, because each engine has double thrust, and less than 3.2% of the maximum yaw control power is used; (iii) for four underwing engines (U4) and failure of the outboard engine at a distance (117b) from the axis:

$$n=4,\ y_+ = 35\,m: \quad C_n = 1.7068\times10^{-3}, \quad C_n/(C_n)_{\max} = 0.069387, \quad C_n/(C_n)_{\min} = -0.065122, \tag{117a-e}$$

the yawing moment is less than 7% of the maximum available; (iv) for two underwing engines (U2) at the same outboard position:

$$n=2, y_+ = 35m: \quad C_n = 3.4137\times10^{-3}, \quad C_n/(C_n)_{\max} = 0.13877, \quad C_n/(C_n)_{\min} = -0.13024, \tag{118a-e}$$

the yawing moment is still less than 14% of the maximum available, though this would be an extreme case of two very powerful engines far outboard; (v) for a mixed configuration (*M*) with one engine over the center-body and two under the outer wings at the same distance from the axis, the yawing moment would be intermediate between the two preceding cases:

$$n=3,\ y_+ = 35\,m: \quad C_n = 2.2758\times10^{-3}, \quad C_n/(C_n)_{\max} = 0.092516, \quad C_n/(C_n)_{\min} = -0.086829, \tag{119a-e}$$

i.e., use less than 9.3% of the available control power. The results for the five engine configurations are summarized in TABLE VI.

If rudder alone had been used, then the maximum and minimum yawing coefficient would have been (101) given by:

$$C_n^{\pm} = \pm 0.01396\,\delta_0 - 0.00537\,\delta_0^2 = \left(+5.0688, -7.1136\right)\times10^{-3}; \tag{120}$$

the fraction of available yaw control power needed to compensate an outboard engine failure in the five cases would be:

$$O4, O2, U4, M, U2: \quad C_n/C_n^+ = \left\{0.076969, 0.15394, 0.33673, 0.44898, 0.67347\right\}, \tag{121a}$$

$$C_n/C_n^- = -\left\{0.054841, 0.10968, 0.23993, 0.31992, 0.47988\right\}. \tag{121b}$$

In all cases the percentage of available yaw control power needed to compensate an outboard engine out condition is much smaller for optimal controls than for rudders alone, because the yaw control authority:

$$(C_n)_{\max}/C_n^+ = 4.853, \qquad (C_n)_{\max}/C_n^- = 3.6845, \tag{122a,b}$$

is increased by a factor of 3 to 5 using optimal controls, and thus there is a much larger safety margin to cope with other events.

16. Maximum and minimum rolling moment

The rolling moment is specified by the data in TABLE IV:

$$C_l = 0.01866\,\delta_1 + 0.00122\left(\delta_1\right)^2 + 0.09317\,\delta_2 - 0.00997\left(\delta_2\right)^2 + 0.09150\,\delta_3 - 0.00900\left(\delta_3\right)^2$$
$$+0.06039\,\delta_4 - 0.00597\left(\delta_4\right)^2 - 0.01552\,\delta_5 - 0.00093\left(\delta_5\right)^2. \tag{123}$$

The maximum and minimum rolling moment will be sought at constant lift, drag and pitching moment (101a-c) ≡ (124a-c) and no yawing moment (124d):

$$C_{L0} = 0.14916, \quad C_{D0} = 0.0071036, \quad C_{m0} = -0.050943, \quad C_{n0} = 0. \tag{124a-d}$$

The least effective roll control surfaces are the rudders $\left(\delta_5\right)$ and the body flap $\left(\delta_1\right)$; the outer flap $\left(\delta_4\right)$ is less effective than the middle flap $\left(\delta_3\right)$ and the inner flap $\left(\delta_2\right)$ marginally more effective. The most effective control surface δ_2 has (8b) optimal deflection (133a) which exceeds the limit (87a); the latter limit is thus taken to calculate the maximum and minimum rolling moment (133b,c). As a preliminary the deflections of the first four least effective control surfaces will be eliminated in this order using the constraints: (i) the rudder deflection δ_5 is expressed in terms of the remaining using the condition (124d) of zero yawing moment in linearized form (101), leading to (125):

$$\delta_5 = 0.40417\,\delta_1 - 0.13434\,\delta_2 - 0.039375\,\delta_3 - 0.019108\,\delta_4; \tag{125}$$

(ii) the deflection of the least effective roll control surface (rudder) is eliminated by replacing (125) in the conditions of constant lift (59) ≡ (126a), drag (61; 69a) ≡ (126b) and pitching moment (60, 55b) ≡ (126c):

$$0 = 0.21282\,\delta_1 + 0.19764\,\delta_2 + 0.12225\,\delta_3 + 0.060925\,\delta_4; \tag{126a}$$
$$-6.8567\times 10^{-4} = 0.0049941\,\delta_1 + 0.0038699\,\delta_2 + 0.0007066\,\delta_3 - 0.0000322\,\delta_4; \tag{126b}$$
$$-0.076064 = -0.065051\,\delta_1 - 0.0059323\,\delta_2 - 0.017302\,\delta_3 - 0.01599\,\delta_4; \tag{126c}$$
$$C_l = 0.012387\,\delta_1 + 0.095259\,\delta_2 + 0.092111\,\delta_3 + 0.060687\,\delta_4; \tag{126d}$$

and also in the rolling moment (123) ≡ (120d); (iii) the deflection δ_1 of the second least effective control surface (body flap) is expressed in terms of the remaining from the lift (126a) ≡ (127):

$$\delta_1 = -0.92867\,\delta_2 - 0.57443\,\delta_3 - 0.28627\,\delta_4; \tag{127}$$

(iv) substitution in the drag (126b), pitch (126c) and rolling moment (126d) coefficients eliminates the deflections of the two least effective roll control surfaces:

$$-6.8567\times 10^{-4} = -0.0007911\,\delta_2 - 0.0021622\,\delta_3 - 0.0014619\,\delta_4, \tag{128a}$$
$$-0.076064 = 0.054479\,\delta_2 + 0.020065\,\delta_3 + 0.0026321\,\delta_4, \tag{128b}$$
$$C_l = 0.083756\,\delta_2 + 0.085\,\delta_3 + 0.057141\,\delta_4; \tag{128c}$$

(v) the deflection δ_4 of the third least effective roll control surface (outer flap) is expressed in terms of the remaining from the condition (128a) of constant drag:

$$\delta_4 = 0.46903 - 0.54115\,\delta_2 - 1.479\,\delta_3; \tag{129}$$

(vi) substitution of (129) leaves the deflections of the two most effective roll control surfaces in the pitching (128b) ≡ (130a) and rolling moment (128c) ≡ (130b):

$$-0.077298 = 0.053055\,\delta_2 + 0.016172\,\delta_3, \quad C_l = 0.026801 + 0.052834\,\delta_2 + 0.00048846\,\delta_3; \tag{130a,b}$$

(vii) the deflection of the fourth least effective roll control surface (middle flap) is expressed from the pitching moment (130a) as a function of the deflection of the most effective:

$$\delta_3 = -4.7798 - 3.28067\,\delta_2; \qquad C_l = 0.024467 + 0.051232\,\delta_2; \tag{131a,b}$$

(viii) this specifies the rolling moment in terms of (131b) the most effective roll control surface (inner flap); (ix) the non-linear terms are restored to the rolling moment coefficient (131b) in the same proportion to the linear terms as in (123):

$$C_l = 0.24467 + 0.051232\,\delta_2\left(1 - 0.10701\,\delta_2\right); \tag{132}$$

(xi) the extremum of (132) corresponds (8b) to a deflection (133a) far outside the range of possible values:

$$\bar{\delta}_2 = 4.6725, \quad \bar{\delta}_0 = 60^\circ = 1.0472\,rad, \quad C_{l\max,l\min} = C_l\left(\pm\bar{\delta}_0\right) = \left(-0.035195,\ 0.072105\right); \tag{133a-d}$$

(xii) the maximum and minimum rolling moments (133b,c) thus occur at the extreme deflections, that are taken to be (133b).

17. Compensation of the wake vortex encounter

The wake vortex encounter is one of the events which places stronger demands on roll control power. The worst-case scenario is taken (FIGURE 4) of a vortex axis coincident with the aircraft axis and the whole wingspan within the vortex core. The question is the largest vertical component of the induced vortex velocity $\bar{u}$ at the wing tip which can be compensated using the maximum available roll control power. The velocity profile in the core of the vortex is assumed to be linear (134a):

$$u(x)=\bar{u}\,x/b, \qquad q(x)=(\rho/2)\bar{u}^2x^2/b^2\,, \tag{134a,b}$$

leading to a dynamic pressure (134b); the resulting rolling moment (135a):

$$l_0=2\int_0^b q(x)\,c(x)\,x\,dx\,, \qquad c(x)=2c(1-x/b)\,, \tag{135a,b}$$

involves the chord $c(x)$ at station x, specified by (135b) for a delta wing of mean chord c and root chord $2c$. The rolling moment is thus:

$$l_0=2(\rho/2)(\bar{u}/b)^2\,2c\int_0^b x^3(1-x/b)\,dx=2\rho\left(\bar{u}^2c/b^2\right)\left[x^4/4-x^5/5b\right]_o^b=\rho\bar{u}^2cb^2/10\,. \tag{136}$$

The corresponding rolling moment coefficient is:

$$C_{l_0}=l_0\Big/\left(\frac{1}{2}\rho U^2\ell S\right)=(\bar{u}/U)^2\left(b^2/S\right)(c/\ell)/5\,, \tag{137}$$

and involves the aspect ratio (138a) and ratio (138b) of mean chord (43a) to reference length (55a):

$$\lambda=4b^2/S=4.8344, \quad c/\ell=1.6191, \quad C_{l_0}=(\lambda/20)(c/\ell)(\bar{u}/U)^2. \tag{138a-c}$$

Thus, the vortex velocity at the wing tip:

$$\bar{u}=U\sqrt{20}\sqrt{C_{l_0}/\lambda}\sqrt{\ell/c}=1.5985U\sqrt{C_{l_0}}\,, \tag{139}$$

which can be compensated increases with: (i) the aircraft velocity and square root of the rolling moment coefficient, corresponding to a greater roll control power; (ii) the inverse square root of the aspect ratio showing that a wing with larger aspect ratio is more affected by a wake vortex encounter, since it increases the induced rolling moment. The substitution in (139) of the extreme values (133b,c) of the rolling moment coefficients specifies:

$$-0.29988\le\bar{u}/U\le 0.42924, \qquad -20.39\,m/s\le\bar{u}\le 29.19\,m/s\,, \tag{140a,b}$$

the maximum vortex velocities at the wing tip which can be countered in a wake vortex encounter. The results including the rolling moment:

$$l_0=2.2339\times10^8\,N.m, \qquad C_{l_0}=(-7.8621,16.107)\times10^6\,N.m, \tag{141}$$

are summarized in Table VIII.

18. Conclusion

The control limits of a flying wing configuration were explored in low-speed flight. The method used (Part I) can maximize or minimize (§3) any component of the aerodynamic forces or moments (§2) within the range of possible deflections of each control surface (§4); other components can be left free or constrained, e.g., by equilibrium conditions. The method applies to coupled or decoupled controls specified by polynomials of any degree (§5-7). The minimum set of data (§8) is second-degree polynomials for decoupled controls. Applying this method (Part II) to a BWB in (§9) a low-speed ($M=0.2$) configuration: (i) shows that pitch trim can be obtained with a minimum drag reduction of 10%, which has beneficial effect on climb performance after take-off (§10); (ii) conversely a maximum drag increase of 58% can be obtained with the same AoA and pitch trim for the steepest descent to land (§11); (iii) the maximum and minimum pitching moment (§12) demonstrate the center-of-gravity range for underwing or overbody engines or a combination of the two (§13); (iv) the maximum and minimum yawing moment (§14) specifies the yaw control authority available in the worst scenario of failure of an outboard engine (§15); (iv) the maximum and minimum rolling moment (§16) determine the vertical velocity at the wing tip of the strongest wake vortex that can be compensated (§17). The introduction (§1) outlines de problem, and the conclusion (§18) summarizes the results.

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TABLE I - *Number of parameters needed for control specification*

Number of control surfaces	N = 1	2	3	4	5	6	7
Polynomial of degree M=2 Q_1 - all surfaces decoupled: (40)	18	36	54	72	90	108	126
Q_2 - only neighboring surfaces coupled: (44)	18	72	132	192	252	312	372
Q_3 - all surfaces coupled: (41)	18	72	180	360	630	1008	1512
Polynomial of degree M=3 Q_1 - all surfaces decoupled: (40)	24	48	72	96	120	144	168
Q_2 - only neighboring surfaces coupled: (44)	24	120	240	360	480	600	720
Q_4 - all surfaces coupled: (43)	24	120	360	840	1680	3024	5040

TABLE II - *Reference flight condition*

Quantity	Symbol	Value	Unit
Mach number	M	0.2	-
Sound speed	s	340	m/s
Airspeed	$V = sM$	68	m/s
Air density	ρ	1.293	kg m^{-3}
Dynamic Pressure	$q = (\rho / 2) V^2$	2.989×10^3	N m^{-2}
Wing area	S	2052.0	m^2
Weight	W	9.15×10^5	N
Vertical force coefficient	C_{Z0}	0.14916	-
Angle-of-attack	α	2.7566	º
Angle-of-sideslip	β	0.0	º
Pitching moment coefficient	C_{M_0}	2.5555×10^{-2}	-
Yawing moment coefficient	C_{N_0}	-4.0213×10^{-6}	-
Rolling moment coefficient	C_{l_0}	4.0140×10^{-5}	-
Lateral force coefficient	C_{Y_0}	0.0	-
Horizontal force coefficient	C_{X_0}	6.5745×10^{-6}	-
Lift coefficient	C_{L_0}	0.14917	-
Drag coefficient	C_{D_0}	6.4451×10^{-3}	-
Half-span	b	49.8	m
Aspect ratio	λ	4.834	-
Reference position for aerodynamic forces and moments	x_{ref}	33.32	m

TABLE III - *Static force and moment coefficients:* $C_\alpha = a_{\alpha 0} + a_{\alpha 1}\,\alpha + a_{\alpha 2}\,\alpha^2$ at $\beta = 0$

Force/Moment	$c_{\alpha 0}$	$a_{\alpha 1}$	$b_{\alpha 2}$
C_L	-1.1700×10^{-2}	3.3516	-1.6966×10^{-1}
C_D	5.1880×10^{-3}	-6.8113×10^{-3}	6.9799×10^{-1}
C_Y	0.0	0.0	0.0
C_m	1.1630×10^{-2}	2.8915×10^{-1}	-1.8186×10^{-1}
C_n	4.4050×10^{-7}	-1.0404×10^{-4}	2.8744×10^{-4}
C_l	1.8360×10^{-5}	3.8850×10^{-4}	9.9777×10^{-4}

TABLE IV - *Flap force and moment coefficients*

Flap number		1	2	3	4	5
Location		Body	Inner wing	Middle wing	Outer wing	Rudder
Pitching moment coefficients C_m	a_{mi}	–0.07267	–0.00340	–0.01656	–0.01563	0.01885
	b_{mi}	0.00037	0.00590	0.00234	0.00158	0.00206
Lift coefficients C_L	a_{Li}	0.23344	0.19079	0.12024	0.05995	–0.05101
	b_{Li}	–0.00455	–0.02808	–0.00674	–0.00478	–0.00246
Drag coefficients C_D	a_{Di}	0.00537	0.003745	0.00067	–0.00005	–0.00093
	b_{Di}	0.01867	0.01283	0.00930	0.00596	0.00824
Side force coefficients C_Y	a_{Yi}	–0.02489	0.00433	0.00338	0.00197	0.05493
	b_{Yi}	–0.00538	–0.00416	–0.00401	–0.00247	–0.00132
Yawing moment coefficients C_n	a_{Ni}	–0.01396	0.00464	0.00136	0.00066	0.03454
	b_{Ni}	–0.00537	–0.00821	–0.00966	–0.00814	–0.00209
Rolling moment coefficients C_ℓ	$a_{\ell i}$	0.01866	0.09317	0.0915	0.06039	–0.01552
	$b_{\ell i}$	0.00122	–0.00997	–0.00900	–0.00597	–0.00093

TABLE V - *Effect of pitch trim on drag*

Drag (kN)	Deflection of body flap alone	Optimal deflection of all surfaces
Untrimmed drag	43.575 kN	43.575 kN
Drag due to trim: - minimum - maximum	 + 25.138 kN + 39.155 kN	 −4.206 kN + 58.769 kN
Percentage of untrimmed drag - minimum - maximum -ratio	 + 57.69 % + 89.85 % + 1.56	 −9.65 % + 134.87 % −13.98
Total trimmed drag: - minimum - maximum -ratio	 68.713 kN 82.730 kN 1.204	 39.369 kN 102.340 kN 2.600
Deflections of control surfaces - minimum drag - maximum drag	 (–19.84, 0, 0, 0, 0) rad (+25, 0, 0, 0, 0)º	 (–8.24, +8.36, 0, 0, +3.13) rad (+25, +25, 0, 0, –25)º

TABLE VI - *c.g. range for five engine configurations*

Engine configuration	c.g. range	
	x_{cg} (m)	x_{cg}/c
along horizontal datum	20.294 – 42.237	34.42 – 71.64 %
$\left.\begin{matrix} O4-four \\ O2-two \end{matrix}\right\}$ above centerbody $z_+ = 10\,m$	20.77 – 42.713	35.23 – 72.44 %
M – one above center-body and two bellow wings: $z_\pm = 10\,m$	19.818 – 41.761	33.61 – 70.83 %
$\left.\begin{matrix} U4-four \\ U2-two \end{matrix}\right\}$ underwing $z_- = 10\,m$	20.135 – 42.078	34.15 – 71.37 %

TABLE VII - *Yaw trim compensate outboard engine failure*

Configuration with outboard engine out	Yawing moment	Percentage of maximum yaw control power need for compensation	
		with optimal controls	with rudders alone
O4- four above centerbody $n = 4$, $y_- = 8\,m$	3.9014×10^{-4}	-1.49 % , +1.59 %	-5.48 % , +7.70 %
O2- two above centerbody $n = 2$, $y_- = 15\,m$	7.8027×10^{-4}	-2.97 % , +3.17 %	-10.97 % , +15.39 %
U4- four underwing $n = 4$, $y_+ = 35\,m$	1.7068×10^{-3}	-6.51 % , +6.94 %	-23.99 % , +33.67 %
C – one over centerbody and two underwing $y_+ = 35\,m$	2.2758×10^{-3}	-8.69 % , +9.25 %	-31.99 % , +44.90 %
U2- two underwing $n = 2$, $y_+ = 35\,m$	3.4137×10^{-3}	-13.0 % , +13.9 %	-47.99 % , +67.35 %

Table VIII - *Compensation of a wave vortex encounter*

quantity	symbol	value		unit
		maximum	minimum	
Pitching moment	ℓ	-7.8621×10^{6}	$+16.107\times10^{6}$	N.m
coefficient	C_ℓ	– 0.035198	+ 0.072105	–
Vertical velocity at wing tip	u	– 20.39	+ 29.19	m/s

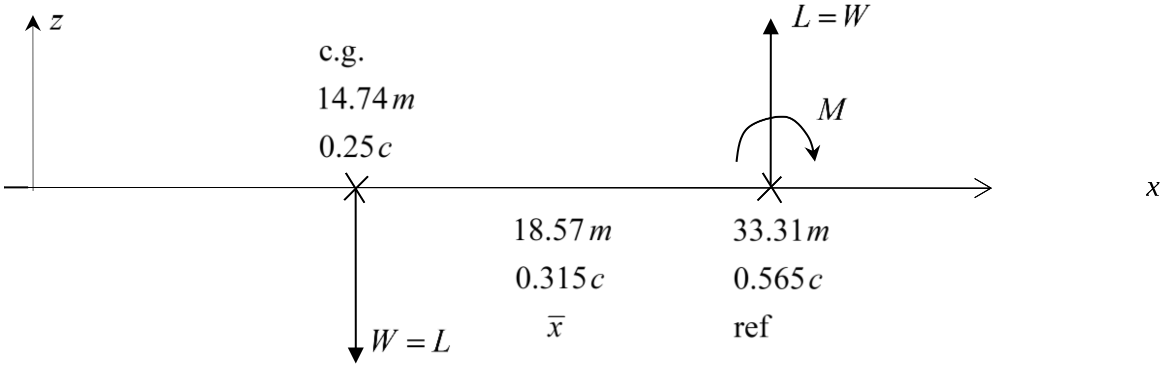

FIGURE 1

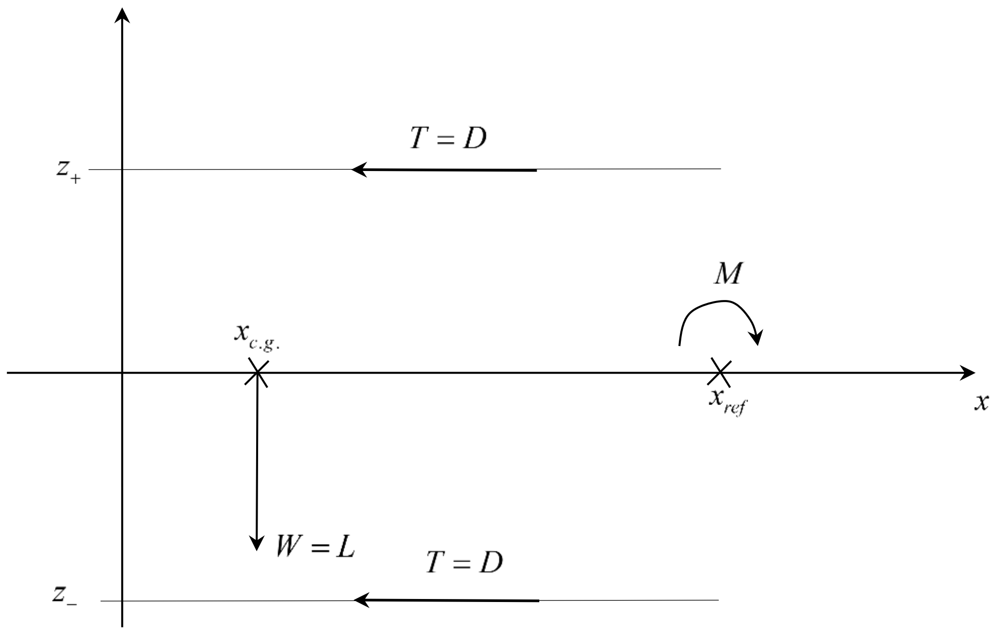

FIGURE 2

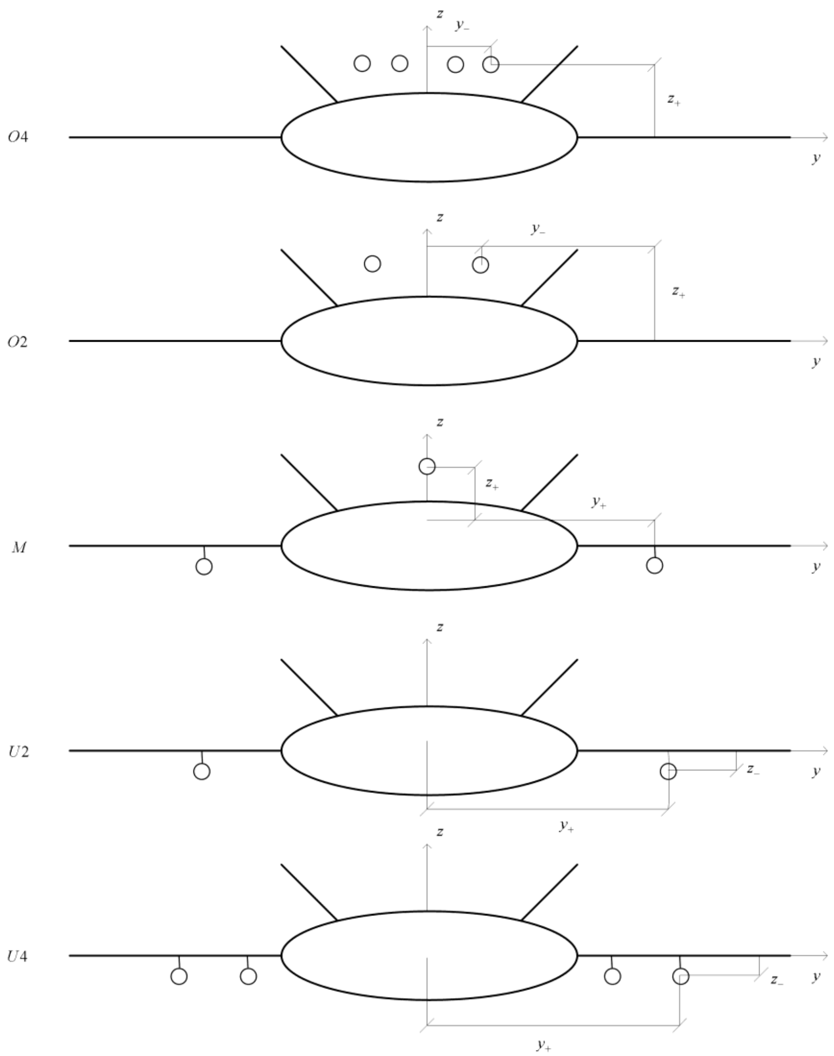

FIGURE 3

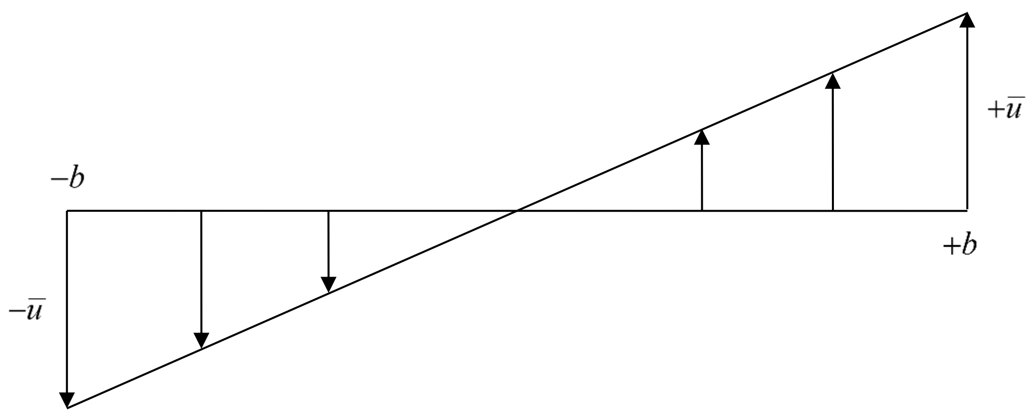

FIGURE 4