

**Unified formulation, analysis, and improvement of all link-wise Dirichlet velocity boundary schemes**

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We propose two local corrections to a wide variety of link-wise boundary schemes: they can be generically built-in into any directional rule locally, for any collision operator. The first local adjustment provides parabolic exactness of the velocity to the linearly accurate schemes, like the FH, BFL, BFL-QI, YLI, LI, MGLI, and ELI (acronyms defined as in [Marson et al., Enhanced single-node lattice Boltzmann boundary condition for fluid flows, Phys. Rev. E (2021)]). As a result, the linear link-wise methods can now reproduce exactly the momentum field in an arbitrarily rotated Poiseuille Stokes flow. Concerning the link-wise approaches, this characteristic was previously a unique feature of the two and three-points multi-reflections (MR). The second alternative correction accounts for linear pressure and leading-order convective gradient. This local post-collision correction improves accuracy for finite Reynolds number Couette type flows, and it may reduce the pressure fluctuations around moving objects. Regarding common characteristics, both of them allow for steady-state viscosity independent momentum accuracy when applied to linear population interpolations. The MR combines all these characteristics but at the price of a larger stencil.

The corrections are part of a broader investigation that deals with other significant themes. We generalize and extend uniformly the well-established multi-reflections (linear LI and parabolic MR) and the recently proposed enhanced-local (ELI) boundary schemes. In particular, we include a weighted semi-implicit time treatment where the geometrical uniformity and locality of the ELI schemes can be made available to the LI approach leading to the same results. Besides, we raise the issue of the global error dependency upon the Reynolds number, and we relate it to the pressure, convective, and viscous terms in the boundary closure relation. Finally, we provide hints for the mass-balance properties of all examined methods.

All the theoretical investigations are validated in a set of test cases from creeping to laminar flows. These include regular arrays of impermeable cylinders to test the behavior for porous media applications and grid-aligned/inclined channel flows, like the Couette and Poiseuille, both with the Stokes and Navier-Stokes equilibrium. In each channel case, we identify the discrete solvability conditions of the exact profiles, built exact boundary schemes, and examine circular flows under similar regimes. Our results are helpful to identify the most suitable velocity schemes for static or moving boundary applications.