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A note on a class of problems for a higher-order fully nonlinear equation under one-sided Nagumo-type condition

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ABSTRACT

The purpose of this work is to establish existence and location results for the higher-order fully nonlinear differential equation

$$u^{(n)}(t) = f(t, u(t), u'(t), \dots, u^{(n-1)}(t)), \quad n \geq 2,$$

with the boundary conditions

$$\begin{aligned} &u^{(i)}(a) = A_i, \quad \text{for } i = 0, \dots, n-3, \\ &u^{(n-1)}(a) = B, \qquad u^{(n-1)}(b) = C \end{aligned}$$

or

$$\begin{aligned} &u^{(i)}(a) = A_i, \quad \text{for } i = 0, \dots, n-3, \\ &c_1 u^{(n-2)}(a) - c_2 u^{(n-1)}(a) = B, \qquad c_3 u^{(n-2)}(b) + c_4 u^{(n-1)}(b) = C, \end{aligned}$$

with $A_i, B, C \in \mathbb{R}$, for $i = 0, \dots, n-3$, and c_1, c_2, c_3, c_4 real positive constants.

It is assumed that $f : [a, b] \times \mathbb{R}^{n-1} \to \mathbb{R}$ is a continuous function satisfying one-sided Nagumo-type conditions which allows an asymmetric unbounded behaviour on the nonlinearity. The arguments are based on the Leray–Schauder topological degree and lower and upper solutions method.

1. Introduction

Let us consider the nth-order differential equation

$$u^{(n)}(t) = f(t, u(t), \dots, u^{(n-1)}(t)), \tag{1}$$

for $n \geq 2$, with the following boundary conditions

$$\begin{aligned} &u^{(i)}(a) = A_i, \\ &u^{(n-1)}(a) = B, \qquad u^{(n-1)}(b) = C, \end{aligned} \tag{2}$$

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