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OPTIMALITY TO FLOW AND DESIGN OF T SHAPE BRANCHING DUCTS

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1. INTRODUCTION

Tree-shaped flow networks have been the subject of numerous investigations due to their importance in understanding natural systems, and in the design of manmade systems (Bejan, 2000). Blood vessels supply cellular tissues with cells, nutrients and oxygen, and remove waste products of cellular activity, through branching vascular networks (Miguel, 2016). For fluid transport systems, the best flow configuration which connects a point-to-volume and vice-versa is a tree network with an arrangement of increasingly smaller descending vessels (Bejan, 2000). Assuming a minimum energy expenditure for blood flow and blood volume, Murray (1926) stated that the optimal branching is achieved when the cube of the diameter of a parent vessel equals the sum of the cubes of the diameters of the daughters (Hess-Murray law). This optimum way to connect large and small vessels together is only valid for steady, incompressible and fully developed laminar flow of Newtonian fluids through impermeable wall vessels (Miguel, 2016). This $2^{-1/3}$ rule is able to describe network of veins and arteries, the airways of conducting zones of the respiratory tract, etc. However, the smallest vessels and airways tend to deviate from this rule (Bejan et al.,2000). There is evidence that turbulent flows require an optimally $2^{-3/7}$ rule (Bejan et al.,2000; Uylings, 1977). However, fluid flow in living organisms is essentially laminar and evidences suggest that the exposure to turbulent flows might pose some health risk (Miguel, 2015). Blood includes erythrocytes (red blood cells), leukocytes (white blood cells) and thrombocytes (platelets) in an aqueous solution (plasma). Its rheology is largely influenced by the concentration of erythrocytes (Miguel, 2016a). Blood vessels exhibit diameters from 3 μm to 3 cm. In larger vessels, the flow is pulsatile due to heart pumping. Experimental studies suggest that if vessels experience high shear rates (higher than 100 s^{-1}), it is reasonable to consider blood flow as a Newtonian fluid (Miguel, 2016a). In small vessels, distant from the heart, the flow may be approached as steady. At shear rates lower than 100 s^{-1}, blood displays shear-thinning behavior, since its viscosity decreases with increasing shear rate. A power-law fluid model was employed by Revellin et al. (2009) and Miguel (2016) to derive expressions for these vessels.

Although first derived from the principle of minimum work, Hess-Murray law can be also obtained in the light of the Constructal law (Bejan, 2000). For minimum resistance under global size constraints of a Newtonian fluid under laminar flow, Bejan et al. (2000) showed that both diameter and length of the offspring vessels can be predicted conform a $2^{-1/3}$ rule. Other studies used the Constructal law to propose the rules of design for flows of non-Newtonian fluids through bifurcating vessels and for porous-wallvessels (Miguel, 2015; Pepe et al., 2017). These rules were reported to depend on fluid behaviour index and on wall permeability. It is important to note that the rules of design obtained based both on principle of minimum work and on Constructal law were based on one-dimensional (1D) and two-dimensional (2D) analytical approaches, and involve a series of assumptions and simplifications listed in Pepe et al. (2017). This study aims to obtain new insights into the dynamics of Newtonian and non-Newtonian flows in bifurcating vessels. A three-dimensional (3D) numerical analysis is performed to study fluid flow through T-shaped structures. The results are compared with analytical expressions presented by Murray (1926), Revellin et al. (2009), Bejan et al. (2000) and Miguel (2016). We point out similarities and distinctions, to provide a comprehensive view of the flow process and resulting designs.

2. CONSTRUCTAL LAW

The Constructal law is not a statement of optimization, maximization, minimization or any other form of conception of end or final destination. The Constructal law deals with the direction in which evolution takes place, to generate forms and structures that move their chains more easily. According to the Construtal law, any system where there is flow with finite dimensions will persist over time, evolving its geometry to facilitate the access of its internal currents [Bejan, 2000; Bejan and Lorente, 2006]. Thus, these systems evolve by developing their geometry in order to facilitate the access of their chains. Flow systems can be observed in nature at the most different scales, as shown in Figure 1 [Bejan and Lorente, 2006]. Trees, river basins, lava, roads and cities, respiratory system and lightning, are examples of flow systems that evolve in order to facilitate the access of their chains [Bejan, 2000; Bejan and Zane, 2012]. The Construtal Design method is based on the Construtal Theory, which states that the geometry of the flow systems follows a physical principle, which is the Construtal law [Bejan, 2008]. In this way, the Construtal law did not begin with experimental observations, but with the idea of representing the general tendency of things to flow more easily. The Construtal law, then, uses this simple universal principle to predict a great amount of phenomena [Bejan and Lorente, 2013]. Figure 2 represents the Construtal Design method applied to the design in the T-shaped structure presented in this work.

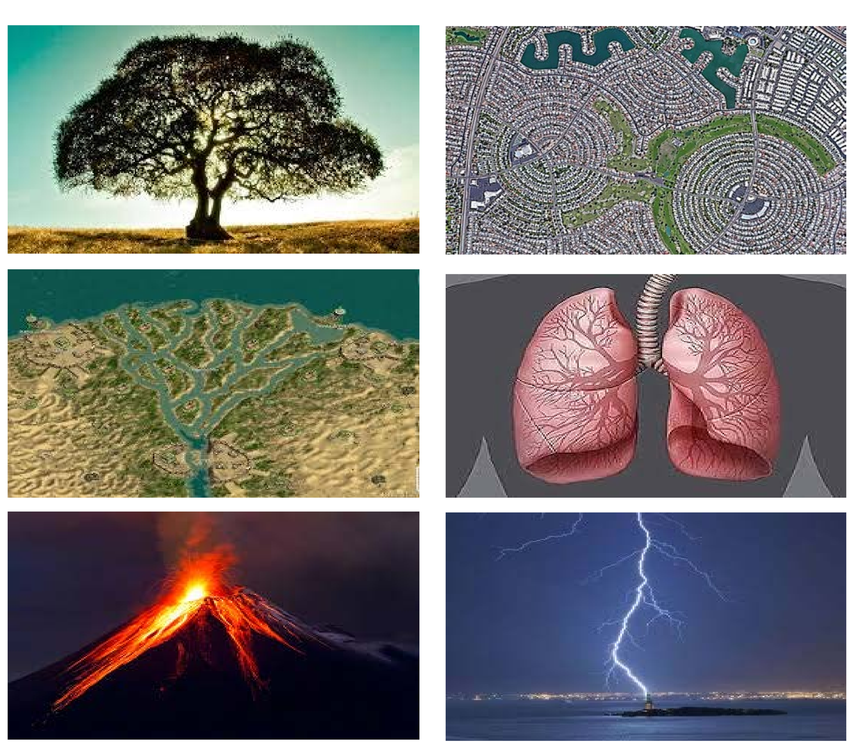


Figure 1. Flow systems involving chains moving from one point to an area or vice versa

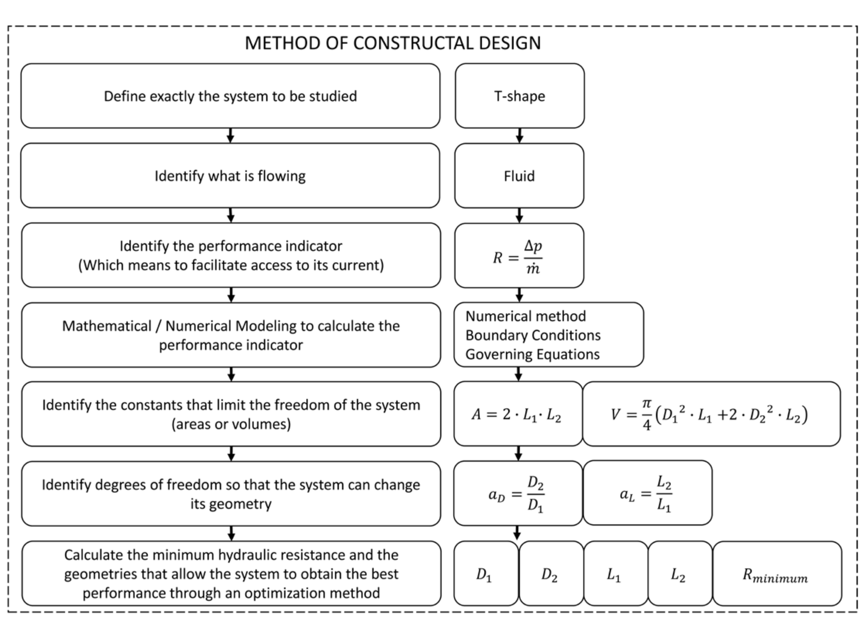


Figure 2. Scheme of the application of the Construtal Design Method

3. MATHEMATICAL FORMULATION

The emergence of configuration, defined by the Constructal law, requires that the entropy changes, rather than staying the same (Bejan, 2000; Miguel, 2016b). Consider that the fluid flow, Q, raised to the power of n is proportional to the pressure difference, ΔP. The rate of entropy generated, S_g, at absolute temperature, T, is given by

$$\frac{dS_g}{dt} = \frac{Q^n \Delta P}{T} \quad (1)$$

Here n is the power-law index (n < 1 fluid with shear-thinning properties, n > 1 fluid with shear-thickening properties, n = 1 Newtonian fluid). As $Q_n = R^{-1}\Delta P$, in terms of flow resistance R, Eq. (1) may be rewritten as:

$$R = \frac{\Delta P^2}{(dS_g/dt)} \quad \text{or} \quad R = \frac{T(dS_g/dt)}{Q^{2n}} \quad (2)$$

Minimum R for a specified potential (ΔP = constant) means maximizing of the entropy generation rate, but minimum R for a constant current (Q = constant), means minimizing the entropy generation rate. Consider a symmetric T-shaped flow system composed by cylindrical ducts designed according to

$$\frac{D_2}{D_1} = a_D \quad \text{and} \quad \frac{L_2}{L_1} = a_L \quad (3)$$

where D is the diameter, L is the length, the subscripts 1 and 2 mean parent and daughter ducts, and the scale factors a_D and a_L may vary between 0.1 to 1.0. The geometric constraints are:

$$V_{total} = \frac{\pi}{4}\left(D_1{}^2 L_1 + 2 D_2{}^2 L_2\right) \quad \text{and} \quad A_{planar} = 2 L_1 L_2 \quad (4)$$

which means the total volume occupied by the ducts and the total space occupied by the planar assembly of ducts are fixed. Consider a laminar, steady and incompressible flow. Continuity and the momentum equations are

$$\nabla \vec{v} = 0 \quad (5)$$

$$\rho \vec{v}(\nabla \vec{v}) = (\nabla P)\text{-}(\nabla \vec{\tau}) \quad (6)$$

Here v is the velocity, ρ is the density, τ is stress and

$$\tau_{ij} = \eta Z_{ij} \quad (7)$$

$$\eta = K\gamma^{n-1} \quad (8)$$

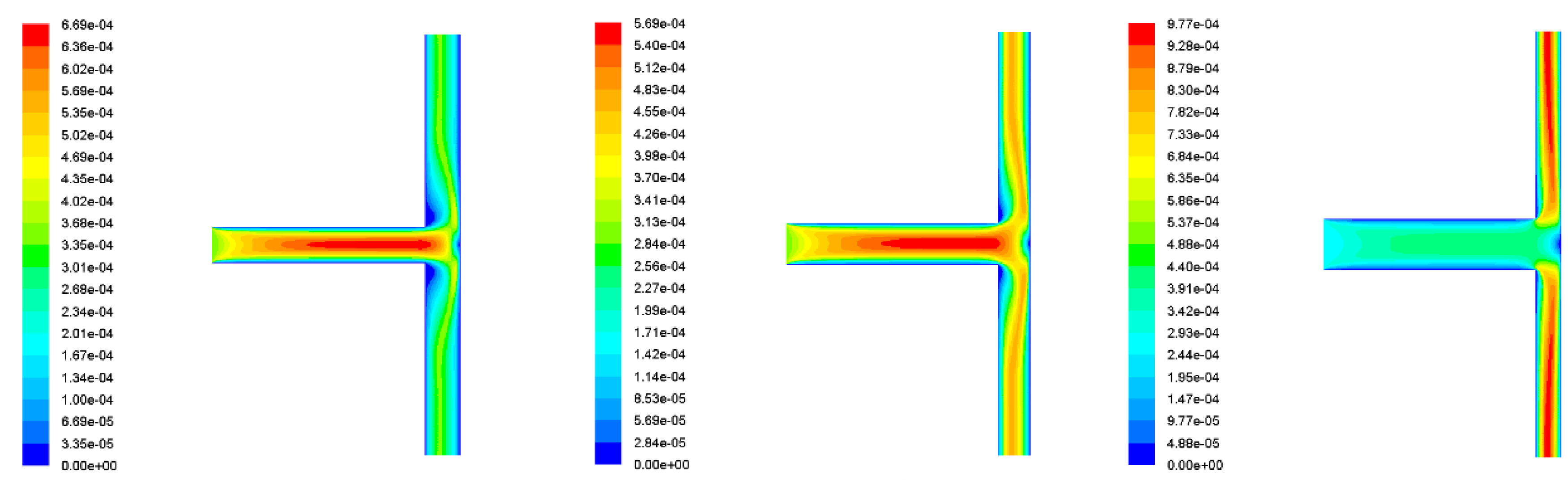
where τ is the strain rate tensor, η is the viscosity function as in Eq. (8), K is the consistency index, and n is the power-law index.
A general Reynolds number for flows of power-law fluids inside ducts is:

$$Re_{Dn} = \frac{4^{4-3n} \rho \dot{V}^{2-n}}{\pi^{2-n} K D_1^{4-3n} \left(\frac{3n+1}{4n}\right)^n} \quad (9)$$

where Re_{Dn} is the generalized Metzner–Reed Reynolds number, ρ is the density (kg/m³), V is the volumetric flow rate (m³/s), D_1 is the diameter parent duct (m), K is consistency index and n is power law index.
The governing Eqs. (5) – (7) were solved using a finite volume method and employing the segregated method with implicit formulation. A constant mass flow rate and an outflow boundary condition are used at the inlet and at the outlet, respectively. No-slip boundary conditions were applied at walls. Relaxation factors for momentum and pressure were set to 0.75. The residual values of the governing Eqs. (5) and (6) were all set to 10^{-6}. Details can be found in Pepe et al. (2017).

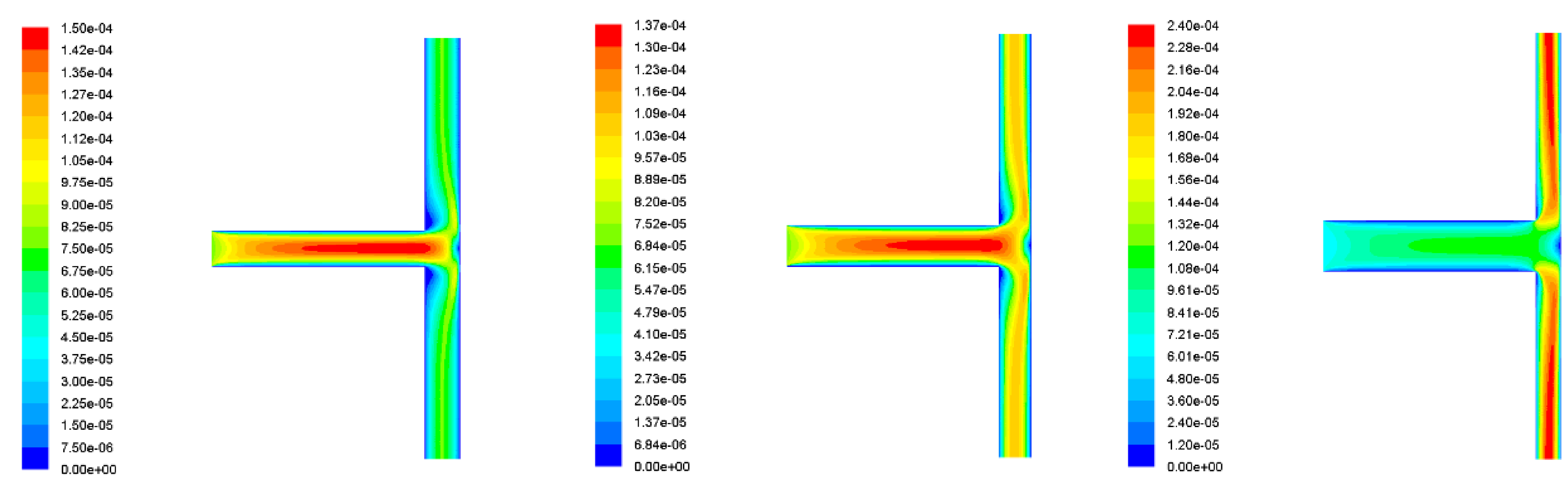
4. RESULTS AND CONCLUSIONS

Here we present a comprehensive set of results for laminar flow (Re_{Dn} = 100) and for power-law indices n < 1 (shear-thinning fluid) and n = 1 (Newtonian fluid). The numerical study was carried out using the following fluids with the following properties: n = 0.776: ρ = 1060 kg/m³ and μ = 1.47×10⁻⁴ Pa.sⁿ (blood) and n = 1: ρ = 1060 kg/m³ and μ = 0.00278 Pa.s (blood).



a_D =1 a_D = $2^{-1/3}$ a_D = 0.5

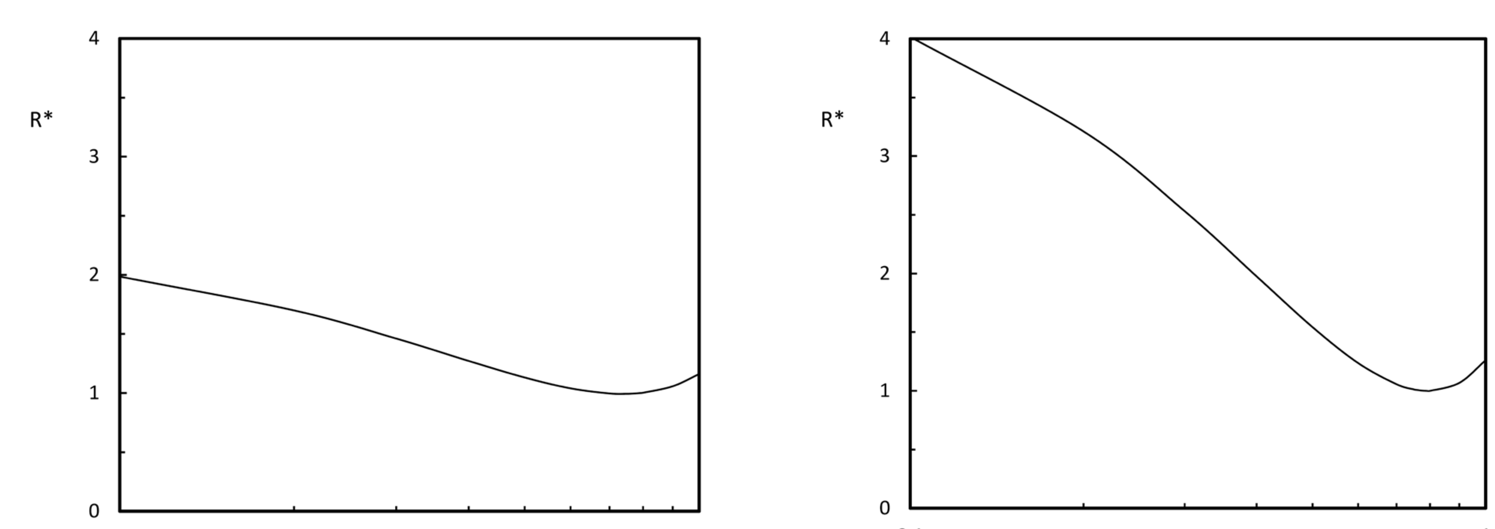
Figure 3. Velocity contours (middle plane) in a 3D T-structure (n = 1)



a_D =1 a_D = $2^{-1/3}$ a_D = 0.5

Figure 4. Velocity contours (middle plane) in a 3D T-structure (n = 0.776)

Figures 3 and 4 show the velocity contours for the Newtonian (n = 1) and non-Newtonian fluids (n = 0.776). Although fluids have different properties, velocity profiles are similar. It interesting to note in Figs. 1 and 2 the boundary layer detachment at the beginning of the daughter tubes originated by the transition of the flow direction at the junction for both fluids studied. This phenomenon is intense for the shear-thinning fluid (n = 0.776). It is possible to identify the existence of low shear separation zones, where the fluid has a lower velocity and high shear regions, and that the sizes of these zones are dependent on the power law index (n) and the diameters ratio (a_D).



n = 0.776 n = 1

Figure 5. Dimensionless total flow resistance, R*, of a T-flow structure.

According to Eq. (2), minimizing the entropy generation rate means minimizing the flow resistance under a constant fluid flow. Fig. 5 show the total dimensionless flow resistance, R*, for flows of Newtonian and shear-thinning fluids through T-shaped structures. The dimensionless resistance R* is defined by the ratio of total flow resistance in a T-shaped assembly of ducts designed according Hess-Murray law with a_D = $2^{-1/3}$. The scale factors a_D that allows a T-configuration with a minimum system-resistance are a_D = $2^{-1/3}$ for Newtonian fluids and a_D = 0.77 for the shear-thinning fluid (n = 0.776).
For Newtonian fluids, the optimal a_D is independent of fluid properties. For shear-thinning fluid, the optimal scale factors a_D depends on power-law index n. These numerical results agree very well with the prediction of analytical models presented by Murray (1926), Bejan et al. (2000), Revellin et al. (2009) and Miguel (2016).

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