Stochastic differential equation models for population and individual growth and for harvesting in randomly varying environments

Carlos A. Braumann
Department of Mathematics and CIMA (Centro de Investigação em Matemática e Aplicações), Universidade de Évora, Portugal

Abstract

The growth of a population living in a randomly varying environment (that affects birth and death rates) can be modelled by a stochastic differential equation (SDE) describing the dynamics of population size (number of individuals, biomass of a fishery,…). Stochastic differential equations can also be used to model the growth in size (weight, volume, length,…) from birth to maturity of individual animals or plants living in a randomly varying environment. Many SDE models have been proposed in the literature, some of them for both phenomena.

It is worth saying that the traditional regression models are appropriate to model observational errors but totally inadequate to model these phenomena. In fact, they do not keep memory of past sizes and a population (or individual) with a size substantially below model average has an equal probability of having a size above or below model average in the immediate future. This is clearly a non realistic property. SDE models, on the contrary, always consider the present situation and project it into the future using the dynamics of the growth process (and also how it is affected by the random environmental fluctuations).

Contrary to what is customary in the literature, instead of considered specific SDE models, we have obtained results on extinction and existence of a stationary density valid for a general class of SDE models (with mild assumptions mostly dictated by biological considerations). Such results are therefore model robust. Models for populations subjected to harvesting were considered as well. We have also studied the time to population extinction (for the population growth models) and the time to reach a maturity size (for the individual growth models). Here, we review those results.

Examples of application to real data (for specific models) will be shown, including the issues of parameter estimation and prediction.