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MATHEMATICAL MODELS IN LIFE SCIENCES & ENGINEERING 2007

PLENARY SESSIONS

ABSTRACTS

Growth and Extinction of Populations in Randomly Varying Environments

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In a randomly varying environment, the *per capita growth rate* (abbreviately *growth rate*) of a population can been described by an "average" rate $g(N)$ (which usually depends on population size N) perturbed by a white noise (which is a reasonable approximation to a noise with low correlations). So, with $N=N(t)$ being the population size at time t, we consider the general model

$$\frac{1}{N}\frac{dN}{dt} = g(N) + \sigma\varepsilon(t),$$

where $\sigma>0$ is the noise intensity and $\varepsilon(t)$ is a standard white noise.

Denoting by $W(t)$ the standard Wiener process, we can write the model in the form of a *stochastic differential equation*

$$dN(t) = g(N(t))N(t)dt + \sigma N(t)dW(t).$$

These models have been studied in the literature for specific functional forms of the "average" growth rate" g (like, for example, the logistic model $g(N)=r(1-N/K)$). Since it is hard to determine the "true" functional form of g, one wonders whether the qualitative results (concerning population extinction or existence of a stationary density) are properties of the specific functional form or rather properties of real populations. We have managed to prove the usual qualitative results for a general function g satisfying some basic assumptions dictated by biological considerations and some reasonable technical assumptions.

Another relevant issue is that the two main stochastic calculus, Itô and Stratonovich, lead to apparently different qualitative results regarding important issues like population extinction and that led to a controversy in the literature on which calculus is more appropriate to model population growth. We have resolved the controversy by showing that g means different types of "average" growth rate according to the calculus used and the apparent difference was due to the wrong implicit assumption that g represented the same "average". Taking into account the different meaning of g, there is no difference (qualitative or quantitative) between the two calculi.

We have extended the above general results to harvesting models in which we subtract to the previous model a harvesting term (dependent on population size). We look for optimisation of harvesting (or profit) not in the usual accumulated terms but under stationary conditions, so that we get sustainable steady harvesting policies.