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On the computation of option prices and Greeks under the CEV model

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Pricing options and evaluating Greeks under the constant elasticity of variance (CEV) model requires the computation of the non-central chi-square distribution function. In this article, we compare the performance, in terms of accuracy and computational time, of alternative methods for computing such probability distributions against an externally tested benchmark. In addition, we present closed-form solutions for computing Greek measures under the unrestricted CEV option pricing model, thus being able to accommodate direct leverage effects as well as inverse leverage effects that are frequently observed in options markets.

Keywords: Option pricing; Derivatives hedging; Computational finance; Statistical methods

JEL Classification: G1, G13

1. Introduction

Every option pricing model has to make a key assumption regarding the 'right' distribution to be used when discounting the option's expected payoff. This expectation is typically computed by integrating the payoff function over a risk-neutral density function. Under the log-normal models of Black and Scholes (1973) and Merton (1973) (the BSM model) it is assumed that the underlying asset price follows a geometric Brownian motion. Yet, this prediction has been convincingly rejected in the finance literature. For instance, it is well documented—see, for example, Jackwerth and Rubinstein (1996)—that the log-normal assumption is unable to accommodate the negative skewness and the high kurtosis that are usually implicit in empirical asset return distributions.

The constant elasticity of variance (CEV) model of Cox (1975) is consistent with two well-known facts that have found empirical support in the literature: the existence of a negative correlation between stock returns and realized volatility (leverage effect), as observed, for instance, by Bekaert and Wu (2000), and the inverse relation between the implied volatility and the strike price of an option contract (implied volatility skew)—see, for example, Dennis and Mayhew (2002). More importantly, being a 'local volatility' model, the CEV diffusion is consistent with a 'complete market' setup and, therefore, allows the hedging of short option positions only through the underlying asset.

Computing option prices under the CEV model typically involves the use of the so-called complementary non-central chi-square distribution function. There exists an extensive literature devoted to the efficient computation of this distribution function, with several alternative representations available (see, for instance, Farebrother (1987), Posten (1989), Schroder (1989), Ding (1992), Knüsel and Bablok (1996), Benton and Krishnamoorthy (2003), and Dyrting (2004)). The complementary non-central chi-square distribution function can also be computed using a method based on series of incomplete gamma functions. For certain ranges of parameter values, some of the alternative representations available are more computationally efficient than the series of incomplete gamma functions. Moreover, for some parameter configurations the use of analytic approximations (e.g. Sankaran (1963), Fraser *et al.* (1998), and Penev and Raykov (2000)) may be preferable.

The main purpose of this article is to provide comparative results in terms of accuracy and computation time for existing alternative algorithms for computing the non-central chi-square distribution function to be used for option pricing and hedging under the CEV model for a large set of parameter values. A similar study has been conducted by Ağca and Chance

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